Research on Function Addition Decomposition and Function Moments

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Manuscript submitted January 8, 2019; accepted May 7, 2019.
doi: 10.17706/ijapm.2019.9.4.158-166

Abstract: In order to study some statistical characteristics of matrix addition decomposition, the concept of function addition decomposition and its moments is defined. Theoretically, if the function can be decomposed into the addition of the function sequence, the sum of the moments of the function sequence can be found to be equal to the moments of the original function under certain conditions. Then the property of function decomposition is applied to matrix addition decomposition. If a matrix can be decomposed into the addition of matrix sequence, the sum of matrix sequence moments is equal to that of the original matrix. Finally, the normalized central moments and local normalized central moments of image are calculated. These normalized central moments have scaling and translation invariance, which shows that the function sequence moments can be used to extract local features of images.

Key words: Function addition decomposition, function sequence, function moments, matrix sequence, matrix addition decomposition.

1. Introduction

At present, the common methods of function addition decomposition can be roughly classified into three categories:

The first decomposition method is for periodic function. According to the definition of Fourier series, any periodic function can be decomposed into the superposition of sine or cosine functions of each harmonic. The advantage of this decomposition is that the local characteristics of each harmonic can be studied and utilized. In the communication system, the transmitter can transmit different signals through each harmonic multiplexing, and then pass through the receiver. The filter extracts the harmonic signals to achieve the purpose of efficient communication.

The second decomposition method is to decompose the power series of any real function. The most commonly used method is Taylor series. The first derivative in Taylor series geometrically represents the slope of a function at a certain point, and the second derivative represents the slope change trend of a function at a certain point. It can explain the concavity and convexity of a function. The next point can be predicted by the slope change trend of a function at a certain point. Derivatives after the third order have no practical significance and can only be understood as a way of expression.

The third is that rational functions can be decomposed into partial fractions and forms. These methods include real root substitution, complex root substitution, limit method and derivation method. For detail methods refer to [1].

The methods mentioned above are all decomposition methods for one-dimensional function cases, which do not extend to two-dimensional and multi-dimensional function cases. The first and second methods are mostly used in the field of time series. For two-dimensional and multi-dimensional function decomposition and statistical properties are not involved.

The decomposition scheme in this paper can be applied to any addition decomposition scheme. The purpose is to study the local characteristics of arbitrary dimension function sequence and refine the local statistical characteristics of pattern sequence after addition decomposition.

2. Function Addition Decomposition and Relevant Properties of Moments

2.1. Decomposition of Function Addition and Definition of Function Moments

Definition 1 : Let $f(x,y)$ be a function on $R^2 \to R$, if there is a sequence of functions $f_l(x,y)$ ($l=1,2,\ldots,k$): $R^2 \to R$, such that $f(x,y)=\sum_{l=1}^{k} f_l(x,y)$, then the function $f(x,y)$ is the sum of the sequence of functions $f_l(x,y)$ ($l=1,2,\ldots,k$). $f_l(x,y)$ ($l=1,2,\ldots,k$) is the addition decomposition sub-function of $f(x,y)$.

It can be generalized to n-dimensional Euclidean space. Let $f(x_1,x_2,\ldots,x_n)$ be a function on $R^n \to R$ of n variables , if there exists a function sequence $f_l(x_1,x_2,\ldots,x_n)$ ($l=1,2,\ldots,k$) : $R^n \to R$, such that $f(x_1,x_2,\ldots,x_n)=\sum_{l=1}^{k} f_l(x_1,x_2,\ldots,x_n)$, then the function $f(x_1,x_2,\ldots,x_n)$ is the sum of the functions sequence $f_l(x_1,x_2,\ldots,x_n)$ ($l=1,2,\ldots,k$). $f_l(x_1,x_2,\ldots,x_n)$ ($l=1,2,\ldots,k$) is the addition decomposition sub-function of $f(x_1,x_2,\ldots,x_n)$.

The moments concepts description refer to [2]. This paper emphasized on the relationship of moments between the original function and its decomposition sequence.

Definition 2: Let $f(x,y)$ be a function on $R^2 \to R$,$m_{pq}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q f(x,y)dxdy$ is defined as *p*+*q* order moment.

It can be generalized to the case of n-variable function. Let $f(x_1,x_2,\ldots,x_n)$ be an n-variable function of $R^n \to R$, the $\sum_{i=1}^{n} p_i$ order moment of $f(x_1,x_2,\ldots,x_n)$ is defined as $m_{p_1p_2,\ldots,p_n}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}(\prod_{i=1}^{n} x_i^{p_i}) f(x_1,x_2,\ldots,x_n)dx_1dx_2\ldots dx_n$.

Definition 3: Let $f(x,y)$ be a function on $R^2 \to R$, the point $\mathbf{C}(\bar{x},\bar{y}) \in R^2$, the central moment of p +q order of the function at the point $\mathbf{C}(\bar{x},\bar{y})$ is defined as $\mu_{pq}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q f(x,y)dxdy$, where *p*, *q* = 0, 1, 2,....

It can be generalized to the case of *n*-variable function. Let $f(x_1,x_2,\ldots,x_n)$ be an n-variable function of $R^n \to R$, the point $\mathbf{C}(\overline{x_1},\overline{x_2},\ldots,\overline{x_n}) \in R^n$,

$\mu_{p_1 p_2,\dots,p_n} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}(\prod_{i=1}^{n}(x_i-\overline{x_i})^{p_i}) f(x_1,x_2,\dots,x_n)dx_1dx_2\dots dx_n$ is defined as the $\sum_{i=1}^{n} p_i$ order central moment of $f(x_1,x_2,\dots,x_n)$ for the point $\mathbf{C}(\overline{x_1},\overline{x_2},\dots,\overline{x_n}) \in R^n$, where $p_1, p_2,\dots,p_n = 0,1,2,\dots$

Definition 4: Let $f(x,y)$ be a function on $R^2 \to R$, the point $\mathbf{C}(\overline{x},\overline{y}) \in R^2$, the normalized *p+q* central moments of the functions on the point *C* are defined as $\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}{}^{\gamma}}$, $\mu_{00} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)dxdy$ is normalized factor, where $\gamma = \frac{p+q}{2}+1,\ p+q = 2,3,\dots$.

For the case of n-variable functions, let $f(x_1,x_2,\dots,x_n)$ be an *n*-variable function, the point $\mathbf{C}(\overline{x_1},\overline{x_2},\dots,\overline{x_n}) \in R^n$, the normalized $\sum_{i=1}^{n} p_i$ order central moments of the function on the point $\mathbf{C}(\overline{x_1},\overline{x_2},\dots,\overline{x_n})$ are defined as

$$\eta_{p_1p_2,\dots,p_n} = \frac{\mu_{p_1p_2,\dots,p_n}}{(\mu_{\underbrace{00,\dots,0}_{n}})^{\gamma}} = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}(\prod_{i=1}^{n}(x_i-\overline{x_i})^{p_i}) f(x_1,x_2,\dots,x_n)dx_1dx_2\dots dx_n}{(\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} f(x_1,x_2,\dots,x_n)dx_1dx_2\dots dx_n)^{\gamma}}$$

$\mu_{\underbrace{00,\dots,0}_{n}} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} f(x_1,x_2,\dots,x_n)dx_1dx_2\dots dx_n$ is normalized factor.

where $\gamma = \frac{\sum_{i=1}^{n} p_i}{n} + 1, \sum_{i=1}^{n} p_i = n, n+1,\dots$

2.2. Properties of Moments of Functions and Function Sequences

Property 1: If $f(x,y)$ is a function on $R^2 \to R$,The *p+q* order moments of the function $f(x,y)$ are $m_{pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q f(x,y)dxdy$, if there exists a sequence of functions $f_l(x,y): R^2 \to R$, such that $f(x,y) = \sum_{l=1}^{k} f_l(x,y)$, the *p+q* order moments of sub-functions $f_l(x,y)$ are $m_{l,pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q f_l(x,y)dxdy$, where $l = 1,2,\dots,k$; $p, q = 0, 1, 2,\dots$, then $m_{pq} = \sum_{l=1}^{k} m_{l,pq}$.

Proof: Let $z = f(x,y): R^2 \to R$, $z_l = f_l(x,y): R^2 \to R$, $l = 1,2,\dots,k$,

$$\because \quad f(x,y) = \sum_{l=1}^{k} f_l(x,y).$$

$$\therefore \quad z = f(x,y) = \sum_{l=1}^{k} f_l(x,y) = \sum_{l=1}^{k} z_l ,$$

and $\because$ $\quad m_{l,pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q f_l(x,y)dxdy$

$$\therefore \quad \sum_{l=1}^{k} m_{l,pq} = \sum_{l=1}^{k}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q f_l(x,y)dxdy$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q \sum_{l=1}^{k} f_l(x,y)dxdy = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y^q f(x,y)dxdy \quad (1)$$
$$= m_{pq}$$

Thus, the conclusion is valid.

Property 2: If $\mathbf{C}(\bar{x},\bar{y}) \in R^2$, $f(x,y)$ is a function on $R^2 \to R$, the p+q order centralized moments of $f(x,y)$ at the point $\mathbf{C}(\bar{x},\bar{y})$ are defined as μ_{pq}, ie. $\mu_{pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q f(x,y)dxdy$, if there exists a sequence of functions $f_l(x,y): R^2 \to R$, such that $f(x,y) = \sum_{l=1}^{k} f_l(x,y)$, the p+q order centralized moments of sub-functions $f_l(x,y)$ are $\mu_{l,pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q f_l(x,y)dxdy$, where $l = 1,2,\ldots,k$; p, q = 0, 1, 2,... , then $\mu_{pq} = \sum_{l=1}^{k}\mu_{l,pq}$.

Proof:

Let $z = f(x,y): R^2 \to R$, $z_l = f_l(x,y): R^2 \to R$, $l = 1,2,\ldots,k$,

$\because \quad f(x,y) = \sum_{l=1}^{k} f_l(x,y)$.

$\therefore \quad z = f(x,y) = \sum_{l=1}^{k} f_l(x,y) = \sum_{l=1}^{k} z_l$,

And $\because$ $\quad \mu_{l,pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q f_l(x,y)dxdy$

$$\therefore \quad \sum_{l=1}^{k}\mu_{l,pq} = \sum_{l=1}^{k}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q f_l(x,y)dxdy$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q \sum_{l=1}^{k} f_l(x,y)dxdy = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^p(y-\bar{y})^q f(x,y)dxdy \quad (2)$$
$$= \mu_{pq}$$

Thus, the conclusion is valid.

Property 3 If the point $\mathbf{C}(\bar{x},\bar{y}) \in R^2$, $f(x,y)$ is a function on $R^2 \to R$, the p+q order normalized central moment of $f(x,y)$ at the point $\mathbf{C}(\bar{x},\bar{y})$ are defined as η_{pq}, ie.

$$\eta_{pq}=\frac{\mu_{pq}}{\mu_{00}{}^{\gamma}}=\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^{p}(y-\bar{y})^{q}f(x,y)dxdy}{[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)dxdy]^{\gamma}}$$, if there exists a sequence of functions $f_l(x,y):R^2\to R$, such that $f(x,y)=\sum_{l=1}^{k}f_l(x,y)$, the p+q order normalized central moments of sub-functions $f_l(x,y)$ are $\eta_{l,pq}=\frac{\mu_{l,pq}}{\mu_{00}{}^{\gamma}}=\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^{p}(y-\bar{y})^{q}f_l(x,y)dxdy}{[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)dxdy]^{\gamma}}$, where $l=1,2,\ldots,k$; p, q = 0, 1, 2,... , then $\eta_{pq}=\sum_{l=1}^{k}\eta_{l,pq}$.

Proof:

Based on Property 1 and property 2,

$$,\because\quad \eta_{l,pq}=\frac{\mu_{l,pq}}{\mu_{00}{}^{\gamma}}=\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^{p}(y-\bar{y})^{q}f_l(x,y)dxdy}{[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)dxdy]^{\gamma}}$$

$$\begin{aligned}\therefore\quad \sum_{l=1}^{k}\eta_{l,pq}&=\sum_{l=1}^{k}\frac{\mu_{l,pq}}{\mu_{00}{}^{\gamma}}\\&=\sum_{l=1}^{k}\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^{p}(y-\bar{y})^{q}f_l(x,y)dxdy}{\mu_{00}{}^{\gamma}}\\&=\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^{p}(y-\bar{y})^{q}\sum_{l=1}^{k}f_l(x,y)dxdy}{\mu_{00}{}^{\gamma}}\\&=\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\bar{x})^{p}(y-\bar{y})^{q}f(x,y)dxdy}{\mu_{00}{}^{\gamma}}\\&=\eta_{pq}\end{aligned}\tag{3}$$

Thus, the conclusion is also valid.

The above properties can be extended to n-dimensional Euclidean space.

3. Moments of Matrix

3.1. Decomposition of Matrix Addition and Definition of Matrix Moments

The detail description of moments is based on reference [2]. The method of addition decomposition of functions can be extended to matrix addition decomposition. Several decomposition schemes of matrices are studied in detail in references [3]-[5].

Definition 5: $A=(a_{ij})_{m\times n}$ represents $m\times n$ matrix, $m(A,p,q)=\sum_i\sum_j i^p j^q a_{ij}$ is defined as *p+q* order moments of *A*, where *p*, *q* = 0, 1, 2,...

Definition 6: If $a_{\bar{x}\bar{y}}$ represents an element of row $\bar{x}$ and column $\bar{y}$ inside the Matrix $A=(a_{ij})_{m\times n}$, The p+q order centralized moment of Matrix $A=(a_{ij})_{m\times n}$ centered on $a_{\bar{x}\bar{y}}$ is defined as $\mu(A,p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}(i-\bar{x})^p(j-\bar{y})^q a_{ij}$, where p, q = 0, 1, 2,...

Definition 7: Based on **definition 6**, the *p+q* order normalized central moments of Matrix $A=(a_{ij})_{m\times n}$ is defined as $\eta(A,p,q)=\dfrac{\mu(A,p,q)}{\mu(A,0,0)^{\gamma}}$, $\mu(A,0,0)=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}$ is normalized factor. where $\gamma=\dfrac{p+q}{2}+1,\ p+q=2,3,\ldots$.

3.2. Matrix Addition Decomposition and Properties of Moments

The properties of Moments of Functions and Function Sequences can be generalized to the matrices.

Property 4: If $A=(a_{ij})_{m\times n}$ is an $m\times n$ matrix, The *p+q* order moments of A are $m(A,p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}i^p j^q a_{ij}$, if matrix A can be denoted with addition $A=\sum_{l=1}^{k}A_l$, the p+q order moments of sub-matrix A_l are $m(A_l,p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}i^p j^q a_{l,ij}$, where $l=1,2,\ldots,k$; *p*, *q* = 0, 1, 2,... , then

$$m(A,p,q)=\sum_{l=1}^{k}m(A_l,p,q) \tag{4}$$

Property 5: If $a_{\bar{x}\bar{y}}$ represents an element of row $\bar{x}$ and column $\bar{y}$ inside the Matrix $A=(a_{ij})_{m\times n}$, The *p+q* order centralized moment of Matrix A centered on $a_{\bar{x}\bar{y}}$ is $\mu(A,p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}(i-\bar{x})^p(j-\bar{y})^q a_{ij}$, and $A=\sum_{l=1}^{k}A_l$, $A_l=(a_{l,ij})_{m\times n}$, the p+q order centralized moment of Matrix A_l centered on $a_{l,\bar{x}\bar{y}}$ is $\mu(A_l,p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}(i-\bar{x})^p(j-\bar{y})^q a_{l,ij}$, where $l=1,2,\ldots,k$; *p*, *q* = 0, 1, 2,... ; Then

$$\mu(A,p,q)=\sum_{l=1}^{k}\mu(A_l,p,q) \tag{5}$$

Property 6: If $a_{\bar{x}\bar{y}}$ represents an element of row $\bar{x}$ and column $\bar{y}$ inside the Matrix $A=(a_{ij})_{m\times n}$, The

$p+q$ order normalized central moment of Matrix A centered on $a_{\bar{x}\bar{y}}$ is

$\eta(A,p,q)=\frac{\mu(A,p,q)}{\mu(A,0,0)^{\gamma}}=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(i-\bar{x})^{p}(j-\bar{y})^{q}a_{ij}}{[\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}]^{\gamma}}$, and $A=\sum_{l=1}^{k}A_l$, $A_l=(a_{l,ij})_{m\times n}$, the $p+q$ order normalized centralized moment of Matrix A_l centered on $a_{l,\bar{x}\bar{y}}$ is

$\eta(A_l,p,q)=\frac{\mu(A_l,p,q)}{\mu(A,0,0)^{\gamma}}=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(i-\bar{x})^{p}(j-\bar{y})^{q}a_{l,ij}}{[\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}]^{\gamma}}$, $\mu(A,0,0)=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}$ is normalized factor.

where $l=1,2,\ldots,k$; $\gamma=\frac{p+q}{2}+1,\ p+q=2,3,\ldots$; Then

$$\eta(A,p,q)=\sum_{l=1}^{k}\eta(A_l,p,q) \tag{6}$$

4. Application of Matrix Addition Decomposition

Face recognition research has been popular in the past few years, and it is particularly important to extract the local features of human faces. Literature [6]-[8] give some current local feature extraction methods. These methods can represent the local features effectively but not provide the relationship between the local and the global features of image.

In this paper, the moment of decomposition sequence by function addition and the moment of block matrix sequence are applied to the local feature extraction of image, which provides a new solution for image local feature expression, and provide a relationship between local and global features. In order to recognize images more accurately, it is often necessary to describe the relationship between local features and global features.

A face image is used as the experimental object. First, the centroid of the image is calculated, then the image is divided into four parts along the centroid, and then the normalized moment of each part relative to the centroid is calculated to test the above conclusions. In addition, normalized central moments of localized sub-graphs can be used as local features of images to match local sub-graphs of images.

4.1. Image Segmentation

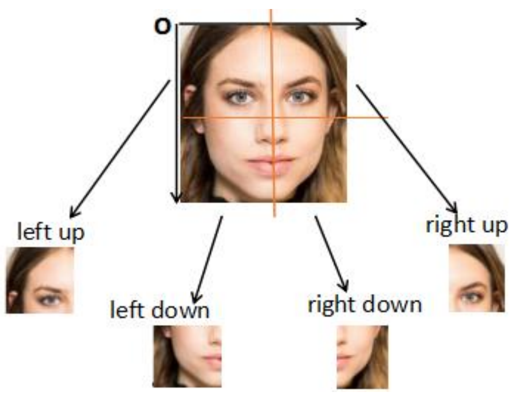

Fig. 1. The target image segmentation.

The centroid of the target image is calculated first, and then the image to be recognized is divided into four parts along the centroid, as shown in Fig. 1.

4.2. Calculation Results of Normalized Center Moments after Image Segmentation

The target image is divided into four parts along the center of mass: left-up, left-down, right-up and right-down. The normalized central moments of the image and the normalized central moments of the sub-images of each part are calculated respectively. Table 1 shows the result of p+q order normalized central moments of the above image and that of sub-images. This result shows the valid conclusion of

$$\eta(A, p, q) = \sum_{l=1}^{k} \eta(A_l, p, q) .$$

In this paper, the geometric segmentation and feature extraction of the image are simply carried out. Based on this algorithm, the image surface and depth can be segmented by any block according to different needs, and the local features of the image can be defined by the segmented sequence.

Table 1. The *p+q* Order Normalized Centralized Moments

	η_{pq}	$\eta_{1,pq}$	$\eta_{2,pq}$	$\eta_{3,pq}$	$\eta_{4,pq}$
	$\eta_{pq} = \sum_{l=1}^{4} \eta_{l,pq}$	left-up	left-down	right-up	right-down
η_{20}	5.62010736E-04	1.44647522E-04	1.38628058E-04	1.35431787E-04	1.43303369E-04
η_{02}	2.96992028E-04	8.16952823E-05	6.33160035E-05	7.22605655E-05	7.97201765E-05
η_{11}	1.00792062E-05	7.81111301E-05	-6.06365903E-05	-7.53949077E-05	6.79995741E-05
η_{30}	2.56362198E-07	-4.49770769E-06	4.39871265E-06	-4.15900486E-06	4.51436210E-06
η_{03}	2.70413611E-07	-2.37157369E-06	-1.31365627E-06	2.00516469E-06	1.95047888E-06
η_{12}	-1.44483521E-06	-1.92656032E-06	1.01404093E-06	-1.87560507E-06	1.34328924E-06
η_{21}	1.63730536E-07	-2.24311546E-06	-1.51367674E-06	2.25133828E-06	1.66918444E-06

5. Conclusion

We have researched some statistical characteristics of the moments of function and matrix addition decomposition, proved the relationship between function moments and function sequence moments, and extended it to n-function dimension space. Finally, the method of function addition decomposition and calculation of function sequence moments is applied to extract the local features of images. The central moments of functions and their sequences are invariant in scaling, rotation and translation. It shows that this method can be used to extract, calculate, and express the local features of patterns.

In a word, this paper only analyses the representation of global and local features from the angle of mathematical formula deduction. The performance of global and local features needs to be further studied in the future. This paper lists only one application of function sequence moments in image local feature representation. Our derivation here is not only for image patterns, but also for other patterns. All these need further study and discussion.

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