Intersection Line of Conical Surfaces and Its Application in the Blending of Tubes with Non-coplanar Axes

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Abstract: In this paper we give a further investigation of the problem about blending different radiuses circular tubes whose axes are in non-coplaner. On the premise of smooth blending axes, we construct an intersection line of conical surfaces, the line smoothly blends generatrix of two tubes. Furthermore, obtain that the conical intersection line rotates the axis to form a tube, which smothly blends two circular tubes whose axes are in non-coplaner. This is of theoretical significance and application value for the study on smooth blending different radiuses tubes whose axes are in non-coplaner.

Key words: Generatrix, non-coplaner tubes, rational bezier curve, smooth blending.

1. Introduction

Surface blending, having many classic methods, is a basic problem in computer geometry design and geometry modeling [1]-[5]. The smooth blending of tubes with non-coplanar axes is a basic yet difficult problem in the field of computer geometry. Lei na [6], [7] prove that two tubes whose axes are in non-coplaner can not be smoothly blended by cubic algebraic surface along any planar sections, and also construct auxiliary cylinders whose axes are coplanar with the axes of two given circular tubes. Moreover, by using Wu Wenjun's formula of two-axis coplanarity, cubic smooth blending between given circular pipes is realized in two segments. Gen-zhu Bai [7], [8] gave the necessary and sufficient conditions for the existence of a cubic blending surface between two tubes whose axes are in non-coplaner, and the relationship between the coefficients of the two tubes. However, no blending examples are given. Gen-zhu Bai [9]-[13] et al. used the method of smooth blending of axes to blend the non-coplaner tubes, and obtained some results. In [12], two special conical surfaces are constructed, whose intersection line is just tangent to the extension line of two non-coplanar edges of the regular hexahedron, and then a smooth blended tube is constructed based on the smooth blending of two axes with the same radius and 90 degrees of the different angle of the axes. In [13], the general case is discussed, that is, the smooth blended tubes between two non-coplanar tubes with same radius is constructed, when the smooth blending tubes of two axes with different plane angles not equal to 90 degrees.

This paper further discusses the smooth blending of two axes with different radius. Let CYL_1 and CYL_2 be the parameter representation of the two non-coplanar tubes, the surfaces determined by them are represented as $S(CYL_i)(i=1,2)$, respectively. L_1 and L_2 are parametric representations of their axes. We asume that the shortest distance from L_1 to L_2 is a. And choose X-axis to make the shortest distance line segement parallel. The Y-axis is parallel to L_1 and intersects with L_2. The right-hand rectangular coordinate system is established, so that the intersection coordinates of L_1 and X-axis is $V_0(a,0,0)$, and the intersection coordinates of Y-axis and L_2 is $V_2(0,b,0)$. Then, the point $V_0(a,0,0)$, points $V_1(a,b,0)$ on the L_1, the point $V_2(0,b,0)$, point $V_2(0,c,d)$ on the L_2 consist four points which are not coplanar. Then the parameter expressions Φ_1, Φ_2 of the two non-coplanar tubes are as follows.

$$\Phi_1: \begin{cases} x = X_1 + r_1\cos\varphi + r_1\sin\varphi, \\ y = Y_1 + r_1\cos\varphi + r_1\sin\varphi, \ \varphi\in[0,2\pi] \\ z = Z_1 + r_1\cos\varphi + r_1\sin\varphi. \end{cases} \quad \Phi_2: \begin{cases} x = X_2 + r_2\cos\varphi + r_2\sin\varphi, \\ y = Y_2 + r_2\cos\varphi + r_2\sin\varphi, \ \varphi\in[0,2\pi] \\ z = Z_2 + r_2\cos\varphi + r_2\sin\varphi. \end{cases}$$

where parametric representations L_1 and L_2 are the axes of the two non-coplanar tubes.

$$L_1: \begin{cases} X_1 = a, \\ Y_1 = B_1 s, s\in[t_1,0] \\ Z_1 = 0. \end{cases} \quad L_2: \begin{cases} X_2 = 0, \\ Y_2 = c + B_2 s, \ s\in[0,t_2] \\ Z_2 = d + C_2 s. \end{cases}$$

We construct two cones. Let $r_1(s,t) = V_0 + (r_1(s) - V_0)t$ be a cone with cone roof $V_0(a,0,0)$, line segments $|V_0V_1|, |V_0V_3|$ as two generatrices and quadratic Bézier curve of the control vertices

$$V_1(a,b,0), V_2(0,b,0), V_3(0,c,d)$$

as directrix $r_1(s) = \sum_{i=1}^{3} B_{3,i}(s) w_i V_i$. And the other one is $r_2(s,t) = V_3 + (r_2(s) - V_3)t$ with cone roof $V_3(0,c,d)$, line segments $|V_3V_0|, |V_3V_2|$ as two generatrices and quadratic Bézier curve of the control vertices $V_0(a,0,0), V_1(a,b,0), V_2(0,b,0)$ as directrix $r_2(s) = \sum_{i=0}^{2} B_{3,i}(s) w_i V_i$. The equation intersection line of conical surfaces is denoted as $r(s)$.

Lemma [13] Intersection line $r(s)$ of conical surfaces is tangent to the points $V_0(a,0,0)$ and $V_3(0,c,d)$ of line segments $|V_0V_1|, |V_0V_3|$, the $r(s)$ namely, has similar properties to rational Bézier curves.

Next, we construct the blending tube between two non-coplanar tubes Φ_1 and Φ_2.

2. G^0 - Blending of Non-coplanar Tubes with Different Radiuses

In the study of [8]-[13], we also discussed the blending of non-coplanar circular tubes with different radius. Suppose $r(s)$ is an intersection line of conical surfaces, which is smoothly blending axes of the two non-coplanar tubes. Its parameter expression is $r(s) = (x(s), y(s), z(s))$, then let Φ be the parameter representation of the smooth blending tubes between two non-coplanar tubes with different radius.

$$\Phi: \begin{cases} x = x(s) + \alpha(s)N_1(u)\cos\varphi + \alpha(s)B_1(u)\sin\varphi, \\ y = y(s) + \alpha(s)N_2(u)\cos\varphi + \alpha(s)B_2(u)\sin\varphi, \quad s\in[0,1], \quad \varphi\in(0,\pi). \\ z = z(s) + \alpha(s)N_3(u)\cos\varphi + \alpha(s)B_3(u)\sin\varphi. \end{cases}$$

where $\alpha(u)$ is the mapping of R_1 to R_2, and R_1, R_2 are the radii of two non-coplanar circular tubes with different radius.

Example 1. Let Φ_1, Φ_2 be the parameter representation of two circular tubes with non-coplanar axes, where $R_1 = 2, R_2 = 1$ are the radii of the circular tube,

$$N_i = (N_{i1}, N_{i2}, N_{i3}), B_i = (B_{i1}, B_{i2}, B_{i3}), i = 1, 2$$

are the normal and binormal when $s = 1$ and $s = 0$, respectively.

$$\Phi_1 : \begin{cases} x = 6 + R_1 N_{11} \cos\varphi + R_1 B_{11} \sin\varphi, \\ y = s + R_1 N_{12} \cos\varphi + R_1 B_{12} \sin\varphi, \ \ \varphi \in [0, 2\pi] \\ z = R_1 N_{13} \cos\varphi + R_1 B_{13} \sin\varphi. \end{cases}$$

$$\Phi_2 : \begin{cases} x = R_2 N_{21} \cos\varphi + R_2 B_{21} \sin\varphi, \\ y = 4 - 2s + R_2 N_{22} \cos\varphi + R_2 B_{22} \sin\varphi, \ \ \varphi \in [0, 2\pi] \\ z = 6 + 6s + R_2 N_{23} \cos\varphi + R_2 B_{23} \sin\varphi. \end{cases}$$

the parameter expression $r(s)$ of the intersection line of conical surfaces is as follows.

$$r(s) = \begin{cases} x(s) = \dfrac{-((12\sqrt{2}-1)s^2 - (\sqrt{2}-2)s - 1)((2\sqrt{2}+6)s^3 - (4\sqrt{2}+12)s^2 + (7\sqrt{2}+14)s - (5\sqrt{2}+8))}{(-7s^3 + (8\sqrt{2}+24)s^2 - (14\sqrt{2}+28)s + 10\sqrt{2}+16)((\sqrt{2}-2)s^2 - (\sqrt{2}-2)s - 1)}, \\ y(s) = \dfrac{-4s(-4+15\sqrt{2})s^4 - (45\sqrt{2}+2)s^3 + (61\sqrt{2}+29)s^2 - (51\sqrt{2}+48)s + 48)}{(-7s^3 + (8\sqrt{2}+24)s^2 - (14\sqrt{2}+28)s + 10\sqrt{2}+16)((\sqrt{2}-2)s^2 - (\sqrt{2}+2)s - 1)}, \\ z(s) = \dfrac{6(5+4\sqrt{2})s^3}{(8\sqrt{2}+24)s^2 - (14\sqrt{2}+28)s + 10\sqrt{2}+16}, \end{cases}$$

then let $p(s, \varphi)$ be the parameter representation of the smooth blending tubes between two non-coplanar tubes with different radius.

$$p(s, \varphi) = \begin{cases} x(s) + \alpha(s) N_1(s) \cos\varphi + \alpha(s) B_1(s) \sin\varphi, \\ y(s) + \alpha(s) N_2(s) \cos\varphi + \alpha(s) B_2(s) \sin\varphi, s \in [0,1], \ \ \varphi \in (0, \pi) \\ z(s) + \alpha(s) N_3(s) \cos\varphi + \alpha(s) B_3(s) \sin\varphi. \end{cases}$$

where $\alpha(s)$ is the mapping of R_1 to R_2, and $R_1 = 2, R_2 = 1$ are the radii of two non-coplanar circular tubes with different radius.

$$N_i = (N_{i1}, N_{i2}, N_{i3}), B_i = (B_{i1}, B_{i2}, B_{i3}), i = 1, 2$$

are the normal and binormal when $s = 1$ and $s = 0$, respectively. The blending effect diagram is as follows.

It is easy to verify that the unit normal vectors of the normal plane at the blending points of Φ between Φ_1 and Φ_2 are different, that is, the blending tube Φ between two non-coplanar tubes Φ_1, Φ_2 with different radius is not smooth.

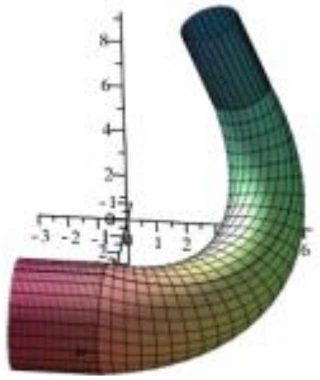

Fig. 1. G^0-blending of non-coplanar tubes with different radiuses.

In Example 1. Although we have obtained the G^0-blending of the non-planar tubes. However, smooth blending of tubes is required in application. Next, we discuss the construction of G^1-blending surfaces between two non-coplanar tubes Φ_1, Φ_2 with different radius.

3. G^1 - Blending of Non-coplanar Tubes with Different Radiuses

In order to realize the smooth blending tube between two non-coplanar tubes with different radius. We construct an intersection line of conical surfaces, the intersection line smoothly blends axes of two non-coplanar tubes. Secondly, construct an intersection lines of conical surfaces, the intersection lines smoothly blends generatrix of two non-coplanar tubes. Finally, we have constructed the blending tubes between two non-coplanar tubes with different radius. Parametric equations of smooth blending tube is as follows.

$$p(s,\varphi)=\begin{cases} x(s,\varphi)+d(s)N_1(s)\cos\varphi+d(s)B_1(s)\sin\varphi, \\ y(s,\varphi)+d(s)N_2(s)\cos\varphi+d(s)B_2(s)\sin\varphi, s\in[0,1], \quad \varphi\in(0,\pi). \\ z(s,\kappa)+d(s)N_3(s)\cos\varphi+d(s)B_3(s)\sin\varphi. \end{cases}$$

where $r(s)$ is an intersection line of conical surfaces, which is smoothly blending axes of two non-coplanar circular tubes with different radius. Its parameter expression is $r(s)=(x(s),y(s),z(s))$. Let $r^{'}(s)$ be conical intersection curve, the curve smoothly blends generatrix of two non-coplanar circular tubes with different radius. Its parameter expression is $r^{'}(s)=(x^{'}(s),y^{'}(s),z^{'}(s))$. Suppose $d(s)=\|r'(s)-r(s)\|$ is euclidean distance.

Example 2. The two non-coplanar circular tubes Φ_1 and Φ_2 with different radius, the intersection line of conical surfaces are mentioned in example 1 above, then we construct intersection line of conical surfaces, which smoothly blends generatrix $r'(s)$ of two non-coplanar circular tubes, parametric equations of the generatrix is as follows.

$$r'(s)=\begin{cases} x'(s)=\dfrac{-8((\sqrt{2}-2)s^2-(\sqrt{2}-2)s-1)((2\sqrt{2}+6)s^3-(4\sqrt{2}+12)s^2+(7\sqrt{2}+14)s-5\sqrt{2}-8)}{(-7s^3+(8\sqrt{2}+24)s^2-(14\sqrt{2}+28)s+10\sqrt{2}+16}, \\ y(s)=\dfrac{-s((-14+49\sqrt{2})s^4-(149\sqrt{2}+6)s^3+(202\sqrt{2}+95)s^2-(170\sqrt{2}+160)s+80\sqrt{2}+100)}{(-7s^3+(8\sqrt{2}+24)s^2-(14\sqrt{2}+28)s+10\sqrt{2}+16)((\sqrt{2}-2)s^2-(\sqrt{2}-2)s-1)}, \\ z(s)=\dfrac{6(5+4\sqrt{2})s^3}{-7s^3+(8\sqrt{2}+24)s^2-(14\sqrt{2}+28)s+10\sqrt{2}+16}. \end{cases}$$

Then, let Φ be the parameter representation of the smooth blending tubes between two non-coplanar tubes with different radius.

$$\Phi:\begin{cases} x = x(s)+\alpha(s)N_1(u)\cos\varphi+\alpha(s)B_1(u)\sin\varphi, \\ y = y(s)+\alpha(s)N_2(u)\cos\varphi+\alpha(s)B_2(u)\sin\varphi, \quad s\in[0,1], \quad \varphi\in(0,\pi). \\ z = z(s)+\alpha(s)N_3(u)\cos\varphi+\alpha(s)B_3(u)\sin\varphi. \end{cases}$$

$N_i=(N_{i1},N_{i2},N_{i3}), B_i=(B_{i1},B_{i2},B_{i3}), i=1,2$ are the normal and binormal at point $s\in[0,1]$ of blending tubes, $R_1=2, R_2=1$ are the radii of two non-coplanar circular tubes with different radius,

$$d(s)=\|r'(s)-r(s)\|$$

is euclidean distance. The blending effect diagram is as follows.

It is easy to verify that the unit normal vectors of the normal plane at the blending points of Φ between Φ_1 and Φ_2 are same, that is, the blending tube Φ between two non-coplanar tubes Φ_1,Φ_2 with different radius is smooth.

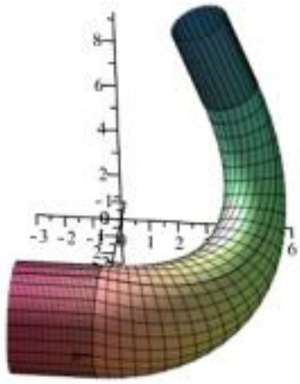

Fig. 2. G^1-blending of non-coplanar tubes with different radiuses.

4. Conclusion

In order to solve the problem of blending complex curves and surfaces in curve and surface modeling, conical intersection lines and blending surfaces are given. In this paper, two conical intersections are constructed. The corresponding conical intersection lines are used as the axes of circular tubes with different radius, finally, the G^0 – blending surface G^1 – blending surface are obtained. The construction example shows that the smooth blending surface has shape adjustability and can be used to generate many kinds of complex curves and surfaces, so it has a wide application prospect in engineering construction.

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