# Explicit Formula of ARL for $SMA(Q)_L$ with Exponential White Noise on EWMA Chart

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Manuscript submitted July 10, 2016; accepted August 31, 2016.
doi: 10.17706/ijapm.2016.6.4.218-225

**Abstract:** The paper propose the method for analyze the explicit formula of Average Run Length (ARL) of Exponentially Weighted Moving Average (EWMA) when the random observations are seasonal moving average order q; $SMA(Q)_L$ with exponential white noise. The numerical results from explicit formula and the Gauss-Legendre quadrature rule are presented. The results show that ARL from both methods are in good agreement and useful to detect change in process. In addition, this paper show comparison between explicit formulas of ARLs from EWMA and CUSUM charts to monitor the process.

**Key words:** Exponentially weighted moving average (EWMA), cumulative sum chart (CUSUM), seasonal moving average, fredholm integral.

## 1. Introduction

The Exponentially Weighted Moving Average chart (EWMA) was proposed by Roberts [1] to detect small change in process mean of statistical process control. Some reviews are given in this paper of Han *et al.* [2] used EWMA and CUSUM control charts in economics and financial to detect turning point of IBM's stock. Nong *et al.* [3] implemented EWMA chart for monitoring the events intensity for intrusion network systems. The common characteristic of control chart is Average Run Length (ARL) which is the expectation of alarm time taken to trigger a signal about a possible change in parameters distribution. Ideally, an acceptable ARL of an in-control process should be large enough to detect a small change in process. There are many methods for evaluating ARL for EWMA and CUSUM chart, ie., Monte Carlo simulations (MC), Numerical Integral Equation (NIE), see, e.g., Crowder [4], Srivastava and Wu [5], Areepong [6] Markov chain Approximation (MCA), see, e.g., Brook and Evans [7] and Lucus and Saccucci [8] martingle approached, see, e.g Sukparungsee and Navikov [9]. EWMA and CUSUM control charts have been designed for independent and identically distributed (i.i.d.) observations. However correlated observations may be effect the properties of charts see e.g., Johnson and Bagshaw [10], they were investigate the effects of correlation on the run length distribution when the control variable in first-order autoregressive, AR(1) and moving-average process, MA(1). Lu and Reynolds [11] use integral equation to compute ARL when the observations can be model AR(1) and ARMA(1,1) process plus random error. In addition, Petcharat et al. [12], [13] used the Fredholm integral to derived the explicit formula for ARL of CUSUM and EWMA chart when observations are moving average order q, MA(q) process with exponential white noise. A number of studies compared the performance between EWMA and CUSUM under various situations. VCC de Vargas et al [14] compare performance of CUSUM with EWMA control charts for production process. They found that EWMA control chart is more powerful than CUSUM chart. In addition, Busababodin [15] present the explicit expression of ARL of seasonal AR(P) and MA(1) model for CUSUM chart and it is very useful for detect change in process. Recently, Petcharat [16] derived the explicit formula for ARL of seasonal AR(p)$_L$ with exponential white noise for EWMA chart and compared to CUSUM chart the results show that EWMA are more sensitive than CUSUM. This is the motivation to derived the explicit formula for seasonal moving average process with exponential white noise for EWMA chart and comparing the performance of explicit formulas between EWMA and CUSUM charts when observations are seasonal autoregressive process with exponential white noise at difference magnitude of the mean shift and various level of moving average coefficients.

## 2. The Average Run Length (ARL) for EWMA Chart of Seasonal Moving Average Model: *SMA(Q)*$_L$ Processes with Exponential White Noise

EWMA chart is usually use in monitoring and detecting small change in mean or variance of the process, which is define in recursive equation by

$$X_t = (1-\lambda)X_{t-1} + \lambda Z_t \ , \quad t = 1,2,\ldots \tag{1}$$

where $Z_t$ is sequence of independent and identically distribution (i.i.d.) nonnegative random variables, $\lambda$ is a exponential smoothing parameter, $0 < \lambda < 1$. The corresponding stopping time for (1) define as

$$\tau = \inf\{t > 0; X_t > b\} \quad , X_0 = u, \quad b > x. \tag{2}$$

where $b$ denote control limit. Let $Z_t$ be the observations of seasonal autoregressive process with exponential white noise denoted by *SMA(Q)$_L$*, the process can be defined as $Z_t = \xi_t - \theta_1\xi_{t-L} - \theta_2\xi_{t-2L} - \ldots - \theta_Q\xi_{t-QL}$ where autoregressive coefficient $|\phi_i| < 1$, for $\forall i = 1,2,..,Q$, $L$ is a period of time, and $\xi_t \sim Exp(\alpha)$. Assume process are in-control at time $t$ where $0 \le X_t \le b$. Let $j(u)$ denote the average run length for EWMA chart. We assume that, the process initially in-control $X_0 = u$. The integral equation defines in $j(u)$ as follow;

$$j(u) = 1 + \frac{1}{\lambda}\int_0^b j(y)\, f\left(\frac{y-(1-\lambda)u}{\lambda} + \left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} - \ldots - \theta_Q\xi_{t-QL}\right)\right) d(y)$$

Therefore,

$$j(u) = 1 + \frac{1}{\lambda\alpha}\int_0^b j(y)\, e^{\frac{y}{\lambda\alpha}}\, e^{\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} - \ldots - \theta_Q\xi_{t-QL}\right)}{\alpha}}\, d(y) \tag{3}$$

Let $C(u) = \exp\left[\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta\xi_{t-2L} + \ldots + \theta_P\xi_{t-QL}\right)}{\alpha}\right]$ then the function $j(u)$ in (3) can written as

$$L(u) = 1 + \frac{C(u)}{\lambda\alpha}\int_0^b L(y)\, e^{\frac{1}{\lambda\alpha}}\, d(y), \quad 0 \le u \le b. \tag{4}$$

The right hand side of (3) is continuous such that the solution of the integral equations (3) is continuous function.

Considering the complete metric space $\left(C(I),\|\ \|_\infty\right)$ where $C(I)$ denote the space of all continuous function on $I$, where $I$ is a compact interval, with the norm $\|j\|_\infty = \sup_{u\in I}|j(u)|$. Then operator $T$ is named as a contraction, if there exist a real constant $0 \le q < 1$ such that $\|j(L_1) - j(L_2)\| \le q\|j_1 - j_2\|$ for $\forall j_1, j_2 \in C(I)$. In this case let $T$ be an operation in the class of all continuous function $C(I)$ where $I = [0, b]$ defined by

$$T(j(u)) = j(u) = 1 + \frac{1}{\lambda\alpha}\int_0^b j(y) e^{\frac{y}{\lambda\alpha}} e^{\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} - \ldots - \theta_Q\xi_{t-QL}\right)}{\alpha}} d(\mathrm{y}). \quad (5)$$

According to Banach's fixed point theorem, if an operator $T$ is a contraction, then the fixed point equation $T(j(u)) = j(u)$ has a unique solution. To prove the uniqueness of solution of (5), then prove Theorem 1 that $T$ is contraction.

**Theorem 1**. On the metric space $\left(C(I),\|\ \|_\infty\right)$ with the norm $\|j\|_\infty = \sup_{u\in I}|j(u)|$ the operator T is a contraction

Proof

First, showing $T$ is a contraction for any $u \in I$, and $j_1, j_2 \in C(I)$. The inequality $\|T(j_1) - T(j_2)\| \le q\|j_1 - j_2\|$ for $\forall j_1, j_2 \in C(I)$ with $0 \le q < 1$. According to (5), then $\|T(j_1) - T(j_2)\|_\infty$

$$\le \sup_{u\in[0,b)}\left| j_1(0) - j_2(0)\frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} + \ldots + \theta_Q\xi_{t-QL}\right)}{\alpha}} \int_0^b j(y) e^{-\frac{y}{\lambda\alpha}} d(y)\right|$$

$$\le \sup_{u\in[0,b)}\left| \|j_1 - j_2\| \frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} + \ldots + \theta_Q\xi_{t-QL}\right)}{\alpha}} (-\lambda\alpha)\right|$$

$$= \|j_1 - j_2\|_\infty \left|1 - e^{-\frac{b}{\lambda\alpha}}\right| \sup_{u\in[0,b)}\left|\frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} + \ldots + \theta_Q\xi_{t-QL}\right)}{\alpha}}\right|$$

$$\le q\|j_1 - j_2\|_\infty,$$

where $0 \le q = \left|1 - e^{-\frac{b}{\lambda\alpha}}\right| \sup_{u\in[0,b)}\left|\frac{1}{\lambda\alpha} e^{-\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\left(\xi_t + \theta_1\xi_{t-L} + \theta_2\xi_{t-2L} + \ldots + \theta_Q\xi_{t-QL}\right)}{\alpha}}\right| < 1,\ 0 \le \lambda < 1,\ \alpha > 0$ and $\xi_i = 1$.

Triangular inequality has been used and the fact that is

$$|j_1(0) - j_2(0)| \le \sup_{u\in[0,b)}|j_1(u) - j_2(u)| = \|j_1 - j_2\|_\infty$$

Therefore, the uniqueness of solution is guaranteed via *Theorem 1* and the Banach Fixed Point Theorem. Then, using the Fredholm integral equation of second kind to derive the ARL for *SMA(Q)*$_L$ process.

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Therefore, the uniqueness of solution is guaranteed via *Theorem 1* and the Banach Fixed Point Theorem. Then, using the Fredholm integral equation of second kind to derive the ARL for *SMA(Q)*$_L$ process. The explicit formula of ARL of EWMA chart defined as follows:

$$ARL_0 = 1 - \frac{\lambda \exp\left(\frac{(1-\lambda)u}{\lambda\alpha_0}\right)\exp\left(-\frac{b}{\lambda\alpha_0}\right)-1}{\lambda \exp\left(\frac{\xi_t+\theta_1\xi_{t-L}+\theta_2\xi_{t-2L}+\ldots+\theta_Q\xi_{t-QL}}{\alpha_0}\right)+\exp\left(-\frac{b}{\alpha_0}\right)-1} \tag{6}$$

$$ARL_1 = 1 - \frac{\lambda \exp\left(\frac{(1-\lambda)u}{\lambda\alpha_1}\right)\exp\left(-\frac{b}{\lambda\alpha_1}\right)-1}{\lambda \exp\left(\frac{\xi_t+\theta_1\xi_{t-L}+\theta_2\xi_{t-2L}+\ldots+\theta_Q\xi_{t-QL}}{\alpha_1}\right)+\exp\left(-\frac{b}{\alpha_1}\right)-1}, \tag{7}$$

where process in-control parameter $\alpha=\alpha_0$ and process out-of-control parameter $\alpha=\alpha_1$, moving average coefficient $|\theta_i|\le 1$, for $\forall i=1,2,..,Q$, $\lambda$ is the smoothing parameter and $b$ is control limit.

## 3. Numerical Results

In this section, the approximation of ARL of *SMA(Q)*$_L$ processes for EWMA chart by using Gauss-Legendre quadrature rule to approximate numerical integral equation (NIE) (3) with 500 nodes. Then the numerical approximation with the explicit formulas and use the relative error to measure of accuracy of comparison defined as

$$\varepsilon_r = \frac{|Explicit - IE|}{Explicit}\times 100 \tag{8}$$

Evaluating ARL of *SMA(Q)*$_L$ of EWMA chart by used (6) and (7) are shown in Table 1 and Table 2 .

Table 1. Comparison of ARL$_0$ Values for *SMA(2)*$_4$ Process on EWMA Chart from Explicit Formula (Explicit) and Numerical Approximation (IE) for $\theta_1=0.1$ and $\theta_2=0.11, 0.15$ with $\lambda=0.02, 0.05, 0.2, 0.3$

| $\lambda$ | $\theta_1,\theta_2$ | $b$ | Explicit | NIE (CPU Time : second) | $\varepsilon_r$(%) |
|---|---|---|---|---|---|
| 0.02 | 0.1,0.15 | 0.0259756 | 370.367 | 370.367 (1.606) | $3.53\times10^{-5}$ |
| | 0.1, 0.11 | 0.0249435 | 370.46 | 370.367 (1.623) | $3.24\times10^{-5}$ |
| 0.05 | 0.1,0.15 | 0.0662486 | 370.155 | 370.155 (1.7) | $7.62\times10^{-5}$ |
| | 0.1, 0.11 | 0.0635645 | 370.018 | 370.017 (1.67) | $6.29\times10^{-5}$ |
| 0.2 | 0.1, 0.15 | 0.2962340 | 370.25 | 370.246 (1.638) | $9.30\times10^{-4}$ |
| | 0.1, 0.11 | 0.28279501 | 370.291 | 370.288 (1.684) | $8.32\times10^{-4}$ |
| 0.3 | 0.1, 0.15 | 0.4854113 | 370.091 | 370.083 (1.669) | $2.33\times10^{-3}$ |
| | 0.1, 0.11 | 0.4611871 | 370.075 | 370.068 (1.685) | $2.09\times10^{-3}$ |

Table 2. Comparison of $ARL_0$ Values for *SMA(3)4* Process on EWMA Chart from Explicit Formula (Explicit) and Numerical Approximation (IE) For $\theta_1 = 0.1$ $\theta_2 = 0.11, 0.15$ and $\theta_3 = 0.25$ with $\lambda = 0.02, 0.05, 0.2, 0.3$

| $\lambda$ | $\theta_1, \theta_2, \theta_3$ | $b$ | Explicit | NIE (CPU Time :second) | $\varepsilon_r$ (%) |
|---|---|---|---|---|---|
| 0.02 | 0.1,0.15,0.25 | 0.0334848 | 370.039 | 370.039 (1.685) | $6.05\times10^{-5}$ |
| | 0.1, 0.11,0.25 | 0.0321495 | 370.497 | 370.367 (1.748) | $5.51\times10^{-5}$ |
| 0.05 | 0.1,0.15,0.25 | 0.085912 | 370.493 | 370.155 (1.763) | $1.40\times10^{-4}$ |
| | 0.1, 0.11,0.25 | 0.0823977 | 370.284 | 370.017 (1.778) | $1.27\times10^{-4}$ |
| 0.2 | 0.1, 0.15,0.25 | 0.399398 | 370.377 | 370.246 (1.701) | $1.93\times10^{-3}$ |
| | 0.1, 0.11,0.25 | 0.380315 | 370.194 | 370.288 (1.716) | $1.71\times10^{-3}$ |
| 0.3 | 0.1, 0.15,0.25 | 0.680997 | 370.246 | 370.083 (1.685) | $5.31\times10^{-3}$ |
| | 0.1, 0.11, 0.25 | 0.643415 | 370.42 | 370.068 (1.700) | $4.63\times10^{-3}$ |

From Table 1 and Table 2 show that *SMA(2)4* and *SMA(3)4* processes with $\alpha = 1$ and $\lambda = 0.02$, 0.05, 0.2, 0.3 there are good agreement between the values for $ARL_0$ computed from exact expression (Explicit) and from the numerical solution of the integral (NIE). The computational times for the explicit formula are less than 1 second while the numerical integral equation times are approximately 1.6 second.

## 4. A Comparison of Performance for EWMA and CUSUM Chart

In 2014, Busababodin [17] proposed the explicit formula of ARL of CUSUM chart for observations modeled as $SMA(1)_L$ with exponential white noise as follows;

$$ARL_0 = e^{\alpha_0 b}\left(1 + e^{\alpha_0\left(a+\theta_1 Z_{t-L}+\theta_1 Z_{t-2L}+...++\theta_1 Z_{t-QL}\right)} - \alpha_0 b\right) - e^{\alpha_0 x}, \tag{9}$$

$$ARL_1 = e^{\alpha_1 b}\left(1 + e^{\alpha_1\left(a+\theta_1 Z_{t-L}+\theta_1 Z_{t-2L}+...++\theta_1 Z_{t-QL}\right)} - \alpha_1 b\right) - e^{\alpha_1 x}, \tag{10}$$

where $|\theta_i| \le 1$, for $\forall i = 1,2,..,Q$, $a$ is reference value and $b$ is control limit. In this section, the numerical results of $ARL_0$ and $ARL_1$ for EWMA and CUSUM charts were calculated by (6) ,(7) and (9),(10), respectively. Table III and IV show a comparison of $ARL_0$ and $ARL_1$ between EWMA and CUSUM charts for *SMA(2)4* and *SMA(3)4* processes for given $ARL_0 = 370$. The parameter values for EWMA and CUSUM charts are $\theta_1 = 0.1$, $\theta_2 = 0.11, 0.15$ for *SMA(2)4* and $\theta_1 = 0.1$, $\theta_2 = 0.11, 0.15, \theta_3 = 0.25$ for *SMA(3)4*. When $\alpha_0 = 1.0$ indicates that the process is in control, but the process mean shift $\alpha = \alpha_1$, $\alpha_1 = \{1.1, 1.2, 1.3, 1.4, 1.5\}$ are out of control process.

Table 3 to Table 4 show a comparison of $ARL$ between EWMA and CUSUM charts for *SMA(2)4*. and Table 5 to Table 6 show a comparison of $ARL$ for *SMA(3)4*. processes for given $ARL_0 = 370$. The parameter values for EWMA and CUSUM charts are $\phi_1 = 0.1, \phi_2 = 0.11, 0.15$ for *SMA(2)4*. and $\phi_1 = 0.1$, $\phi_2 = 0.11, 0.15$, $\phi_3 = 0.25$ for *SMA(3)4*, when $\alpha_0 = 1.0$ indicates that the process is in control, but the process mean shift $\alpha = \alpha_1$, $\alpha_1 = \{1.1, 1.2, 1.3, 1.4, 1.5\}$ are out of control process. Such that, EWMA chart is more sensitive than CUSUM chart for all magnitudes of shifts.

Table 3. Comparison of ARL Values for *SMA(2)₄* on EWMA and CUSUM Charts When Given $ARL_0 = 370$, $\theta_1 = 0.1, \theta_2 = 0.15$

| $\alpha$ | EWMA<br>$\lambda = 0.02, \quad b = 0.0259756$ | CUSUM<br>$a = 3, \quad b = 2.737$ |
|---|---|---|
| 1.0 | 370.367 | 370.398 |
| 1.1 | 8.84803 | 212.155 |
| 1.2 | 5.10838 | 133.281 |
| 1.3 | 3.83438 | 89.946 |
| 1.4 | 3.18749 | 64.237 |
| 1.5 | 2.79374 | 48.014 |

Table 4. Comparison of ARL Values for *SMA(2)₄* on EWMA and CUSUM Charts When Given $ARL_0 = 370$, $\theta_1 = 0.1$ and $\theta_2 = 0.11$

| $\alpha$ | EWMA<br>$\lambda = 0.05, \; b = 0.0635645$ | CUSUM<br>$a = 3, b = 2.781$ |
|---|---|---|
| 1.0 | 370.018 | 370.078 |
| 1.1 | 8.84042 | 211.764 |
| 1.2 | 5.09854 | 132.931 |
| 1.3 | 3.82406 | 89.6543 |
| 1.4 | 3.17712 | 63.9983 |
| 1.5 | 2.78349 | 47.8185 |

Table 5. Comparison of ARL Values for *SMA(3)₄* on EWMA and CUSUM Charts When Given $ARL_0 = 370$, $\theta_1 = 0.1, \theta_2 = 0.11, \theta_3 = 0.25$

| $\alpha$ | EWMA<br>$\lambda = 0.02, \quad b = 0.0321495$ | CUSUM<br>$a = 3, b = 2.505$ |
|---|---|---|
| 1.0 | 370.497 | 370.126 |
| 1.1 | 10.7219 | 213.043 |
| 1.2 | 4.49248 | 134.377 |
| 1.3 | 3.74043 | 90.9766 |
| 1.4 | 3.11446 | 65.1366 |
| 1.5 | 2.73345 | 48.7799 |

Table 6. Comparison of ARL Values for *SMA(3)₄* on EWMA and CUSUM Charts When Given $ARL_0 = 370$, $\theta_1 = 0.1, \theta_2 = 0.15, \theta_3 = 0.25$

| $\alpha$ | EWMA<br>$\lambda = 0.3, \quad b = 0.680997$ | CUSUM<br>$a = 3, \quad b = 2.737$ |
|---|---|---|
| 1.0 | 370.246 | 370.239 |
| 1.1 | 24.6538 | 213.270 |
| 1.2 | 12.5737 | 134.604 |
| 1.3 | 8.51542 | 91.177 |
| 1.4 | 6.52205 | 65.306 |
| 1.5 | 5.35107 | 48.922 |

## 5. Conclusion

In this paper derived the explicit expressions for ARL of EWMA charts for observations are seasonal moving average order q *(SMA(Q)$_L$)* with exponential white noise and we also used Gauss-Legendre quadrature solve the integral equations for ARL of EWMA chart for *SMA(Q)$_L$*. We have show the numerical results that explicit formula and the numerical approximation are excellent agreement. The computational times for the explicit formula are less than 1 second while the numerical integral equation times are approximately 1.6 second. A comparisons of performance of ARL between EWMA and CUSUM charts for observations modeled as the seasonal moving average order q *(SMA(Q)$_L$)*, we found that the performance of EWMA charts is better than CUSUM charts which all magnitudes of shifts.

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