Blockchain Topology: Percolation Dynamics and the Variance of Consensus

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Abstract: This paper investigates the structural robustness of validator interaction networks in the Tezos blockchain ecosystem using percolation theory. Empirical data of Tezos governance proposals are simulated using Erdős–Rényi random graphs and Watts–Strogatz small-world networks topologies. We analyze the emergence of giant connected components and identify empirical critical thresholds using both mean largest component size and variance peak detection. Results demonstrate that governance interaction networks exhibit classical percolation-style phase transitions. Variance peaks act as reliable early-warning indicators of structural instability. Statistical analysis confirms a significant structural dependence. These findings provide a quantitative framework for evaluating coordination feasibility and resilience in decentralized governance systems.

Keywords: blockchain governance, percolation theory, random graph, small-world network

1. Introduction

Decentralized governance in proof-of-stake blockchains represents a large-scale repeated coordination game where validators, such as Tezos bakers, strategically choose to vote “yay”, “nay”, or “pass” based on stake-weighted decision power [1, 2]. Unlike classical voting systems, on-chain governance operates through public transparency, endogenous quorum adjustments, and repeated strategic interactions. These features create an evolving interaction graph among agents where the underlying network structure directly influences information propagation, consensus formation, and overall voting stability. The robustness of these governance networks depends on the balance between local coordination and global reach. This is effectively modeled using random graph and small-world network characteristics, which provide both high clustering and short path lengths. However, a critical question remains: at what connectivity level does a decentralized governance network transition from fragmented voting clusters into a unified, giant connected strategic component? This paper addresses this question by evaluating how structural variables affect connectedness across various governance proposals. By utilizing percolation-based network simulations based on empirical proposal data, we investigate the phase transitions that dictate whether a network maintains global alignment or suffers from rapid fragmentation. The main contributions of the paper are as follows:

- Simulation of real-world Tezos blockchain data of its on-chain governance to understand percolation dynamics and the variance of consensus.

- Modeled the system with Erdős–Rényi Random Graph and Watts Strogatz Small-World Network.
- We conducted a percolation-based analysis that proves that governance networks obey universal percolation scaling and Critical thresholds are predictable from system size.
- We found that governance networks possess measurable phase transitions. These findings bridge applied mathematics, percolation theory, and blockchain governance design.

2. Literature Review

Research connecting blockchain governance, network science, and percolation theory has grown rapidly in recent years. Studies show that Proof-of-Stake governance systems increasingly depend on stake-weighted participation and delegation, which makes the structure of validator networks important for understanding how resilient and stable these systems are under different conditions [1]. In Ref. [2], it is shown that the Liquid Proof-of-Stake mechanism allows delegation to create super-additive effects in validator influence. This means that some validators can gain stronger collective influence through delegation, which changes the overall interaction structure of governance networks and affects decision outcomes.

Governance studies in decentralized finance also highlight structural effects within voting systems. Sun *et al.* [3] shows that MakerDAO voting tends to form dominant voter coalitions, even though the system is designed to be decentralized. This suggests that governance results often emerge from network connectivity patterns.

Modern network science increasingly views decentralized systems as critical systems operating near phase-transition boundaries. Percolation-based analysis can identify important structural changes, and variance peaks near critical thresholds often act as early indicators of major transitions in network connectivity [4].

Percolation theory provides a strong foundation for understanding how large-scale connectivity appears in networks. The formation of a giant connected component represents a universal phase transition observed across many types of complex systems. These theoretical insights show that critical thresholds play a central role in determining robustness, information flow, and overall system stability [5]. Further developments expand the understanding of network robustness. In Ref. [6], it is shown through extended-range percolation that communication can remain possible even when direct paths are broken, emphasizing that redundancy and alternative pathways are key factors supporting resilience in complex networks.

Recent theoretical advances also extend percolation ideas beyond standard graph structures. Higher-order network models allow more accurate analytical treatment of complex connectivity patterns, helping researchers study systems where interactions involve groups rather than only pairwise links [7].

Survey research confirms that modern percolation studies increasingly focus on real-world networks that display heterogeneity, clustering, and non-random structure. These characteristics make percolation theory especially relevant for analyzing decentralized governance systems, where connections are rarely uniform or random [8].

Random graphs and small-world networks remain two important models for studying structural transitions. Classical Erdős–Rényi theory shows that a giant component appears when the average degree becomes greater than one, providing a theoretical baseline for comparing real systems. In Ref. [5], it is also highlighted that random networks continue to serve as useful reference models for evaluating robustness and critical thresholds in applied research. Small-world networks [5, 6], known for combining high clustering with short path lengths, are often more robust under rewiring processes. Recent research suggests that local connectivity plays a stronger role in determining resilience, while randomness mainly

affects sparse networks. This observation aligns with governance networks modeled using Watts–Strogatz structures, where local neighborhood connections strongly influence stability [6].

Although many studies explore blockchain governance, percolation theory, or network topology separately, only a limited number combine these perspectives into a single framework. Most governance research focuses on economic behavior or participation trends, while percolation studies remain largely theoretical. This paper addresses that gap by applying percolation analysis directly to real governance datasets and comparing different network topologies. The results show that validator interaction networks display measurable structural criticality and follow predictable connectivity thresholds.

3. Theoretical Background

In an Erdős–Rényi graph $G(N,p)$, for a network of N nodes, each pair of nodes is connected independently with probability p. The average degree $\langle k \rangle$ is:

$$\langle k \rangle = pN \tag{1}$$

A giant connected component emerges when $\langle k \rangle = 1$. Thus, the theoretical critical threshold is:

$$p_c = {}^{1}/_{N} \tag{2}$$

This condition serves as our theoretical benchmark for empirical comparison.

A small-world network is defined by the number of nodes (N), the initial number of neighbor connections (k), and the rewiring probability (β). The rewiring probability (β) controls the randomness of the network. The overall connectivity is influenced by both local grouping among nearby nodes and random long-range links. The strength and stability of the structure can be described conceptually as:

$$C \approx f(k).g(\beta) \tag{3}$$

where $f(k)$ represents the effect of local connections, and $g(\beta)$ represents the impact of randomness. Critical behavior appears when rewiring weakens local clustering that is too low to maintain structure.

4. Methodology

4.1. Empirical Data and Simulation Setup

The empirical foundation of this study is based on baker-level voting data collected from 23 past Tezos proposals [9, 10]. The proposals are referred to by their voting period numbers. For each proposal, the dataset includes the total count of active bakers, their individual stake weights, and their recorded voting choices. To ensure statistical reliability, every proposal was extensively simulated using 50 different connectivity settings, with 100 Monte Carlo runs performed for each parameter value to capture the random nature of the coordination process.

We first model the network using an Erdős–Rényi random graph. The analysis is defined by the number of nodes (N), which represents the total baker population, and the edge probability (p), which determines the chance of a random connection between any two agents. From the primary data, two complete datasets were produced for analysis: *all_phase_transition_enriched.csv* and *phase_transition_summary.csv*.

We used the same primary dataset for the Watts–Strogatz small-world model. The analysis is defined by the total number of bakers (N), the initial local neighborhood degree (k), and the rewiring probability (β). Using the primary data, a complete dataset was then generated: *smallworld_all_proposals.csv*.

4.2. Network Models and Phase Transition Detection

The simulation framework uses two different network models to examine validator interactions. In the random graph approach, nodes representing individual bakers (N) are connected using a random

probability $\mathcal{P}$, while the small-world model is defined by the number of bakers (N), the initial local neighborhood degree (k), and the rewiring probability (β). For each historical proposal, we build the network and simulate opinion clustering to calculate key structural metrics: the mean degree ($\langle k \rangle$), the largest connected component size (S), and the variance of that component size Var(S). To identify critical thresholds, we apply Phase Transition Detection. We define the empirical Giant Component Threshold as:

$$p_{gc} = \min\{p|S > 0.5\} \tag{4}$$

which shows the point where a dominant strategic cluster first appears. Additionally, we calculate the Variance Peak Threshold by finding the parameter values that produce the highest variance:

$$p_{var} = arg\ max_p(Var(S)) \tag{5}$$

These variance peaks act as a clear sign of critical transitions, showing the structural instability that appears as the governance network moves from a fragmented state to a unified coordination regime. Finally, results were averaged across all proposals, calculated as Mean(Results), to ensure the statistical reliability of the phase transitions across the historical cases.

5. Results

Across the historical Tezos proposals studied, the data show a clear match between theoretical percolation limits and real observations. We distinguish between the theoretical threshold $p_c \approx 0.005$ (the point where connectivity begins) and the empirical $p_{gc} \approx 0.008$ (the point where dominance appears). Our simulations indicate that while $1/N$ predicts when connectivity starts, about a 56% higher edge probability is needed for the giant component to include most of the network ($S > 0.5$).

The transition shows very high sensitivity near the critical point. Within a small range of ΔP = 0.004, the network changes from many small clusters (below 25% connectivity) to a mostly unified system (>75% connectivity). This confirms the "all-or-nothing" behavior of phase transitions in Erdős–Rényi graphs. In the random graph simulations, the observed giant component thresholds closely match the theoretical limit, with variance peaks appearing consistently near these predicted values. This confirms that decentralized governance systems follow classical percolation behavior. Even with differences in total stake distribution and the number of bakers, these phase transitions appear with structural universality, suggesting that coordination follows predictable tipping points. Fig. 1 shows the results of all random graph model simulations. We can see that the largest connected part of the network increases very fast after the edge probability $\mathcal{P}$ reaches a small critical value. This means the network quickly changes from mostly disconnected to highly connected. The sharp peaks in variance at the same point show that the system changes rapidly during this transition. Even though different proposals are used, the curves look similar, which means the overall network behavior remains consistent under different conditions.

The small-world simulations also show that network stability is highly sensitive to local connectivity (k) and rewiring randomness (β). In sparse networks (k = 2), the system breaks apart quickly as β increases; although fully connected at $\beta \approx 0$, connectivity drops below 10% after $\beta \approx 0.15$. In contrast, dense networks (k = 10) remain almost completely stable, keeping a normalized component size close to 1 across the full β range. A critical region appears at k =4, where variance rises sharply around $\beta \approx 0.20$. Statistical analysis supports these results, showing a strong positive link between neighborhood size and overall robustness (Pearson r = 0.7996, $p < 10^{-29}$), indicating degree centrality as the main driver of governance stability. Fig. 2 shows the simulation results, which demonstrate that β affects the network in different ways. For low connectivity (k = 2), the largest connected group becomes smaller very quickly and variance rises early, showing the network is unstable. For medium connectivity (k = 4), the change happens

more gradually, and variance peak is wider. For high connectivity (k = 10), the network stays mostly connected, and the variance remains low, which means the system is stable and strong.

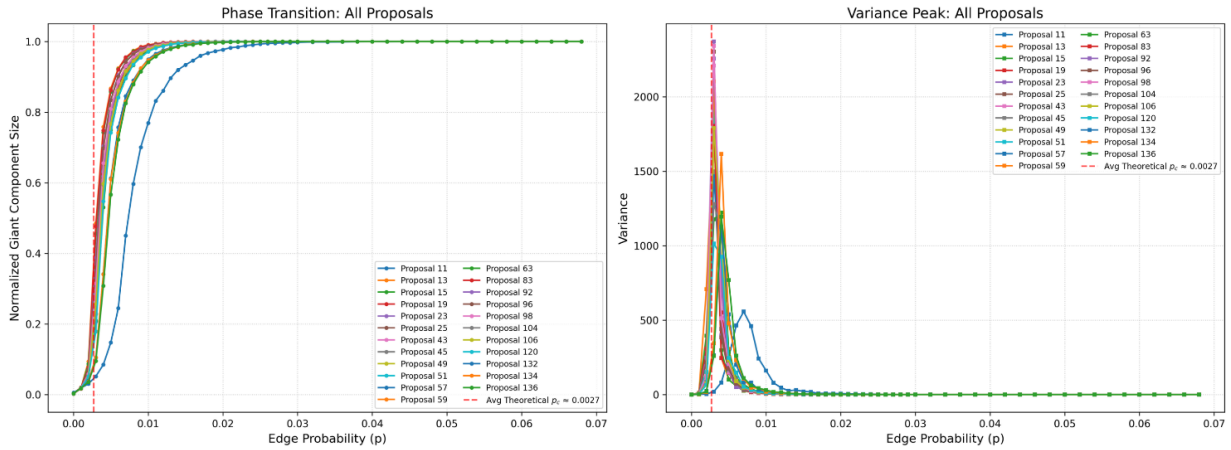

Fig. 1. Random graph simulation results.

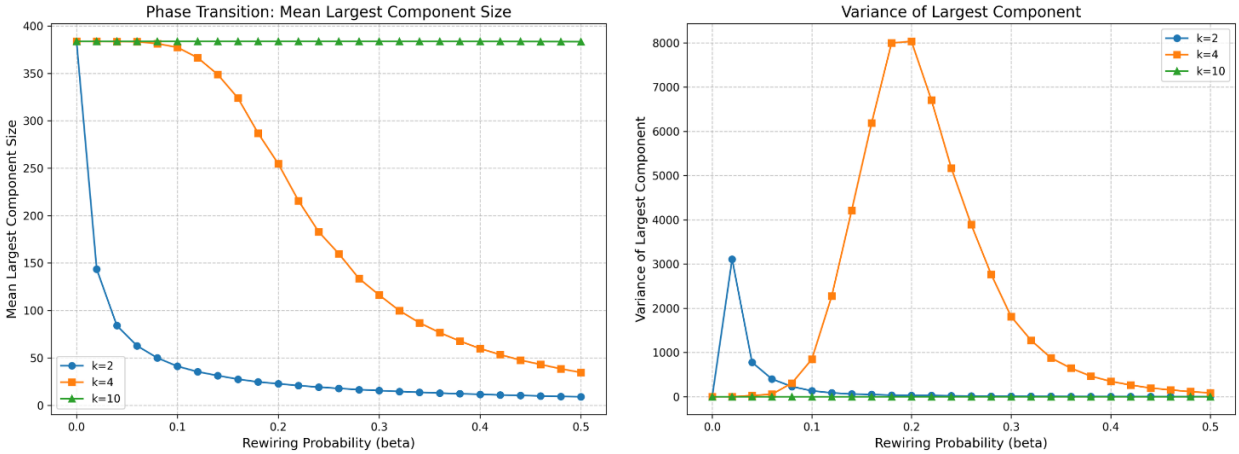

Fig. 2. Small-world network simulation results.

Unlike the random graph, which must gradually build a giant component from the beginning, the Small-World model begins already fully connected. Our data shows that rewiring edges ($0 < \beta < 0.1$) keeps the S value high, demonstrating that Small-World properties support global connectivity even with much fewer total edges compared to what the random model requires. Table 1 presents the results of both models for Tezos Proposal 11.

Table 1. Comparison of Results for Proposal 11

Metric	Value (Random Graph)	Value (Small World)
Nodes N	194	194
Theoretical $1/N$	0.0051	NA (Structure based)
Empirical $p_{gc} = S > 0.5$	0.008	Always > 0.5 (for k = 2)
Transition Window	0.006–0.010	Stable across β range

Taken together, the results show that governance interaction networks follow classical random graph scaling laws, small-world robustness is mainly controlled by degree, variance acts as an early warning sign of structural instability, and the percolation threshold serves as a large-scale boundary for connectivity feasibility. Therefore, decentralized governance networks can be described as structurally critical systems

whose stability is determined by universal connectivity thresholds.

6. Discussion

The results of this study suggest that blockchain governance works like a complex coordination process on a network, where stable strategic alignment becomes possible only after connectivity passes certain critical levels. Near these levels, the system becomes very sensitive to small disturbances, like global cascades and contagion thresholds described in evolutionary game theory and strategic complementarity models. The observed variance spike at $k = 4$ resembles classic percolation behavior: a shift from a fragmented state below the threshold to one where a giant connected component dominates. This indicates that the robustness of on-chain governance follows basic principles from statistical physics.

These findings have important implications for protocol design and for understanding complex adaptive systems. Our analysis shows that validator environments with low connectivity are highly vulnerable to random structural disturbances, while high connectivity provides resilience regardless of rewiring randomness. This suggests that network structure is just as important for governance stability as quorum rules. To improve robustness, protocols should promote more cross-validator interactions, since sparse participation networks may unintentionally increase governance volatility. Theoretically, the connectedness of small-world governance networks can be expressed using Eq. (3), where local degree (k) mainly drives growth and rewiring (β) becomes dominant only when (k) is small. This dual dependence is consistent with random graph theory and offers a framework for modeling blockchain governance as a resilient, complex adaptive system.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Qurratulain conducted the research, analyzed the data; Qurratulain and Ayesha Maqbool wrote the paper; Ayesha Maqbool supervised the research. All authors had approved the final version.

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