Peter Chew Theorem and Application

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Abstract: Presenting Figures in surd form is relatively common in science and engineering especially where a calculator is either not allowed or unapproachable, and the computations to be undertaken involve irrational values. Every pupil who plans to take computation's at the advanced position in such a calculus-based or statistics should be suitable to manipulate and deal with surds. The purpose of Peter Chew's Theorem is to make solving the Quadratic Surds problem simple by converting any value of the Quadratic Surds $\sqrt{a\ +\ b\sqrt{c}}$ into the sum or difference of two real numbers. Peter Chew's theorem also converts square roots of complex numbers into complex numbers because square roots of complex numbers are also Quadratic Surds $\sqrt{a\ +\ b\,i}$ $=\sqrt{a\ +\ b\sqrt{-1}}$. In addition, Peter Chew's Theorem can also convert Quadratic Surds $\sqrt{a\ +\ b\sqrt{c}}$ into the sum or difference of two complex number [$\sqrt{z}\ +\sqrt{\bar{z}}$]. Technical tools have had a significant impact on advanced mathematics tutoring and mathematics literacy. However, today's online calculator only contains the knowledge that has been explained in the book, but the current method cannot or is difficult to solve some Quadratic Surds problems, this makes online calculators unable to solve Quadratic Surds problems. This can lead to reduced student interest and hinder the spread of technology tool use. In order to solve the above problems, my research is to create a new discovery for the Quadratic Surds problem, such as Peter Chew's theorem, so that all problems can be easily solved Apply Peter Chew's theorem to a AI Age calculator (Peter Chew Quadratic Surd Diagram calculator), allow the AI Age calculator to solve any problem in the topic of Quadratic Surds, which can make the AI Age calculator effectively help mathematics teaching, especially in the future when similar COVID-19 problems arise.

Keywords: Peter Chew theorem, quadratic surds, surds, Peter Chew

1. Introduction

Presenting Figures in surd form is relatively common in science and engineering especially where a calculator is either not allowed or unapproachable, and the computations to be undertaken involve irrational values [1]. Every pupil who plans to take computation's at the advanced position in such a calculus-based or statistics should be suitable to manipulate and deal with surds [2]. The purpose of Peter Chew's Theorem [3] is to make solving the Quadratic Surds problem simple by converting any value of the Quadratic Surds $\sqrt{a\ +\ b\sqrt{c}}$ into the sum or difference of two real numbers. Peter Chew's theorem also converts square roots

Because the aim of the Peter Chew's theorem is to facilitate the teaching and learning of the Topic "Quadratic Surds" easily especially during a pandemic such as COVID-19, the Peter Chew's theorem and application(preprint) has been published at the Europe PMC: https://europepmc.org/article/ppr/ppr300039.

of complex numbers into complex numbers because square roots of complex numbers are also Quadratic Surds $\sqrt{a+bi} = \sqrt{a+b\sqrt{-1}}$. In addition, Peter Chew's Theorem can also convert Quadratic Surds $\sqrt{a+b\sqrt{c}}$ into the sum or difference of two complex number $[\sqrt{z} + \sqrt{\bar{z}}]$.

The French mathematician Veda established the relationship between the equation root and the coefficient in 1615.

Veda's theorem states that if α and β are two roots of the quadratic equation $ax^2 + bx + c = 0$ and $a \neq 0$. Then, (i) the sum of the two roots, $\alpha+\beta = -\frac{b}{a}$; (ii) the product of the two roots, $\alpha\beta = \frac{c}{a}$.

Note: Peter Chew Method for quadratic equation [4] provided better method than Veda's theorem for solving some quadratic equation problem.

$$\text{Prove } \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}.$$

If α and β are two roots of the quadratic equation $ax^2 + bx + c = 0$ and $a \neq 0$.

$$\text{Then} \quad \alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \qquad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

$$\begin{aligned}
\text{So, } \alpha + \beta &= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a},\\
&= \frac{-2b}{2a}\\
&= -\frac{b}{a}
\end{aligned}$$

$$\begin{aligned}
\alpha\beta &= \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a}\\
&= \frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}\\
&= \frac{b^2-(b^2-4ac)}{4a^2}\\
&= \frac{4ac}{4a^2}\\
&= \frac{c}{a}.
\end{aligned}$$

2. Prove of Peter Chew Theorem

If roots of $x^2 - ax + b = 0$ are α and β,

Veda's theorem, $\alpha + \beta = -\frac{-a}{1} = a$ (i) , $\alpha\beta = \frac{b}{1} = b$ (ii).

$$\begin{aligned}
(\sqrt{\alpha} \pm \sqrt{\beta})^2 &= \alpha + \beta \pm 2\sqrt{\alpha}\sqrt{\beta}\\
&= a \pm 2\sqrt{b}
\end{aligned}$$

$$(\sqrt{\alpha} \pm \sqrt{\beta}) = \sqrt{a \pm 2\sqrt{b}},\ \alpha > \beta > 0$$

$\therefore \sqrt{\mathbf{a} \pm 2\sqrt{\mathbf{b}}} = \sqrt{\alpha} \pm \sqrt{\beta}$ where $\alpha > \beta > 0$. Proof [5–8].

3. Current Method & Peter Chew Theorem

3.1. Current Method

At present, there are two methods [9] for solving the problem of the Quadratic Surds; they are the Quadratic Surds method and the Comparison method. If we can find two positive integers x and y such that the sum of x and y is equal to a, and the product of x and y is equal to b, then we usually use Quadratic Surds method. Therefore, $\sqrt{a \pm 2\sqrt{b}} = \sqrt{x} \pm \sqrt{y}, x > y > 0$.

However, if the values of a and b are large, it is not suitable to use Quadratic Surds method.

Quadratic surds method, for $\sqrt{a \pm 2\sqrt{b}}$, $x + y = a$, $xy = b$ $(a, b, x, y > 0)$.

$$\sqrt{a \pm 2\sqrt{b}}$$
$$= \sqrt{x + y \pm 2\sqrt{xy}}$$
$$= \sqrt{(\sqrt{x})^2 + (\sqrt{y})^2 \pm 2\sqrt{xy}}$$
$$= \sqrt{(\sqrt{x} \pm \sqrt{y})^2}$$
$$= \sqrt{x} \pm \sqrt{y}, (x > y > 0).$$

Note: For large value and complex root, this method not suitable.

Comparison method, let $\sqrt{a \pm 2\sqrt{b}} = \sqrt{x} \pm \sqrt{y}, x > y > 0$

Squaring gives, $a \pm 2\sqrt{b} = x + y \pm 2\sqrt{xy}$

By Comparison, $a = x + y$ (i), $b = x y$ (ii)

Solve (i) and (ii), get the value x and y.

3.2. Peter Chew Theorems

If roots of $x^2 - ax + b = 0$ are α and β, then $\sqrt{a \pm 2\sqrt{b}} = \sqrt{\alpha} \pm \sqrt{\beta}$ where $\alpha > \beta$.

4. Convert the Quadratic Surds

4.1. Convert the Quadratic Surds into the Sum or Difference of Two Real Numbers

4.1.1. For small value quadratic surds

Example 1: Find $\sqrt{11 + 2\sqrt{28}}$ in the form $\sqrt{x} + \sqrt{y}$, where $x > y > 0$.

Current Method,

Solution 1: $\sqrt{11 + 2\sqrt{28}} = \sqrt{7 + 4 + 2\sqrt{(7)(4)}}$

$$= \sqrt{(\sqrt{7})^2 + (\sqrt{4})^2 + 2\sqrt{(7)(4)}}$$
$$= \sqrt{\left(\sqrt{7} + \sqrt{4}\right)^2}$$
$$= \sqrt{7} + 2$$

Solution 2: Let $\sqrt{11 + 2\sqrt{28}}$ be $\sqrt{x} + \sqrt{y}$

$$11 + 2\sqrt{28} = (\sqrt{x} + \sqrt{y})^2$$
$$= x + y + 2\sqrt{xy}$$

Comparing the two sides of the above equation,

We have $x + y = 11$,

$y = 11 - x$ (i)

and $xy = 28$ (ii)

Substitute (i) in (ii), $x\,(11-x) = 28$

$$11x - x^2 = 28$$

$$x^2 - 11x + 28 = 0$$

$$(x-4)(x-7) = 0$$

$$x = 4, 7$$

From (i), When $x = 4,\ y = 11 - 4 = 7$

When $x = 7,\ y = 11 - 7 = 4$

$$\therefore \sqrt{11 + 2\sqrt{28}} = \sqrt{7} + \sqrt{4}$$

$$= \sqrt{7} + 2$$

Solution 3: Peter Chew Theorem. Cause $x^2 - 11x + 28 = 0$, then $x = 4, 7$

$$\therefore \sqrt{11 + 2\sqrt{28}} = \sqrt{7} + \sqrt{4}$$

$$= \sqrt{7} + 2$$

4.1.2. For large value quadratic surds

Example 2: Find $\sqrt{80235 + 2\sqrt{838102050}}$ in the form $\sqrt{x} + \sqrt{y}$, where $x > y > 0$.

Current Method.

Solution 1: Not suitable

Solution 2: Let $\sqrt{80235 + 2\sqrt{838102050}}$ be $\sqrt{x} + \sqrt{y}$

$$80235 + 2\sqrt{838102050} = (\sqrt{x} + \sqrt{y})^2$$

$$= x + y + 2\sqrt{xy}$$

Comparing the two sides of the above equation,

We have $x + y = 80235$, $y = 80235 - x$ (i)

and $xy = 838102050$ (ii)

Substitute (i) in (ii), $x\,(80235- x) = 838102050$

$$x^2 - 80235x + 838102050 = 0$$

$$(x - 12345)(x - 67890) = 0$$

$$x = 12345, 67890$$

From (i), When $x = 12345,\ y = 80235 - 12345 = 47890$

When $x = 67890,\ y = 80235 - 67890 = 12345$

$$\therefore \sqrt{80235 + 2\sqrt{838102050}} = \sqrt{67890} + \sqrt{12345}$$

Peter Chew Theorems.

Cause $x^2 - 80235x + 838102050 = 0$, Then $x = 67890, 12345$

$$\therefore \sqrt{80235 + 2\sqrt{838102050}} = \sqrt{67890} + \sqrt{12345}$$

4.2. Convert the square root of a complex number into a complex number because the square root of the complex number is also the quadratic surd

Example: Find $\sqrt{-3 + 4i}$ in the form $\sqrt{x} + \sqrt{y}$ I, where $x,\ y > 0$.

Current Method.

Solution 1: Not suitable

Solution 2: let $\sqrt{-3 + 4i} = (\sqrt{x} + \sqrt{y}\ i\ ), (x, y > 0)$

$$-3 + 4i = x - y + 2\sqrt{xy}\ i$$

Comparing the two sides of the above equation, We have $x - y = -3$, $y = x+3$ (i) and $2\sqrt{xy} = 4$, $xy = 4$ (ii)

Substitute (i) in (ii), $x(x+3) = 4$

$$x^2 + 3x - 4 = 0$$

$$x = 1, \ -4 \ \text{(reject } x, y > 0)$$

From (i), When $x = 1$, $y = 1 + 3 = 4$

$$\therefore \quad z = \sqrt{1} + \sqrt{4}\ i$$
$$= 1 + 2\ i$$

Peter Chew Theorem. $\sqrt{-3 + 4i} = \sqrt{-3 + 2\sqrt{-4}}$

Cause $x^2 + 3x - 4 = 0$, then $x = 1, -4$

$$\therefore \quad \sqrt{-3 + 4i} = \sqrt{1} + \sqrt{-4}$$
$$= 1 + 2i$$

4.3. Convert the Quadratic Surds into the Sum or Difference of Two Complex Numbers

Example: Find $\sqrt{2 + 2\sqrt{5}}$ in the form $\sqrt{z} + \sqrt{\bar{z}}$.

Current Method.

Solution 1: Not suitable

Solution 2: Let $\sqrt{2 + 2\sqrt{5}}$ be $\sqrt{x} + \sqrt{y}$

$$2 + 2\sqrt{5} = (\sqrt{x} + \sqrt{y})^2$$
$$= x + y + 2\sqrt{xy}$$

Comparing the two sides of the above equation,

we have $x + y = 2$, $y = 2 - x$ (i) and $xy = 5$ (ii)

Substitute (i) in (ii), $x(2 - x) = 5$

$$2x - x^2 = 5$$
$$x^2 - 2x + 5 = 0$$
$$x = 1 + 2i, \quad 1 - 2i$$

From (i), When $x = 1 + 2i$, $y = 2 - (1 + 2i) = 1 - 2i$

When $x = 1 + 2i$, $y = 2 - (1 - 2i) = 1 + 2i$

$$\therefore \quad \sqrt{2 + 2\sqrt{15}} = \sqrt{1 + 2i} + \sqrt{1 - 2i}$$

Peter Chew Theorem

Cause $x^2 - 2x + 5 = 0$, then $x = 1 + 2i, 1 - 2i$

$$\therefore \quad \sqrt{2 + 2\sqrt{5}} = \sqrt{1 + 2i} + \sqrt{1 - 2i}$$

5. The Application of Peter Chew's Theorem

Application of Peter Chew theorem in Peter Chew Quadratic Surd Diagram (PCQSD) calculator allows PCQS calculator to convert all Quadratic Surd, including decimal value Quadratic Surd to sum or difference of two real numbers or sum of two real numbers Complex numbers that cannot be converted by current online calculators, such as Wolfram Alfa, Symbolab, Mathphoto, and Geogebra.

Since current online calculators, such as Wolfram Alfa, cannot convert the decimal value Quadratic Surd to the sum or difference of two real numbers or the sum or sum of two real numbers and complex numbers, it is difficult to help mathematics teaching, because teachers or students just prefer to use Online calculator that can convert all Quadratics Surd problem. Therefore, the PCQSD calculator can effectively help mathematics teaching and can help overcome the limitations of existing online calculators. Please refer to the article Peter Chew Quadratic Surd Diagram [10] for details.

As we know, many AI apps such as Chat GPT are using current method for help solve maths problem, those AI apps such as Chat GPT will face problem as same as Wolfram Alfa which unable to convert decimal value Quadratic Surd to sum or difference of two real numbers or sum or sum of two real numbers Complex numbers. Therefore, AI applications such as Chat GPT must program Peter Chew Theorem into their AI applications to help convert all Quadratic Surds. Therefore, it is imperative to develop Peter Chew's theorem in AI Applications. In addition, there will definitely be new AI Application development in the future, so it is imperative to use Peter Chew Theorem in their programming. Therefore, Peter Chew's theorem will be greatly developed in the future.

While the PCQSD calculator relies on technology, not all students have access to it, especially those who do not have access to a computer or the Internet. On the original learning method, without technology, those students can still use the simple and easy Peter Chew's theorem to help convert all Quadratic Surd in their learning. Main page Peter Chew Quadratic Surd Diagram (PCQS) calculator is shown in Fig. 1.

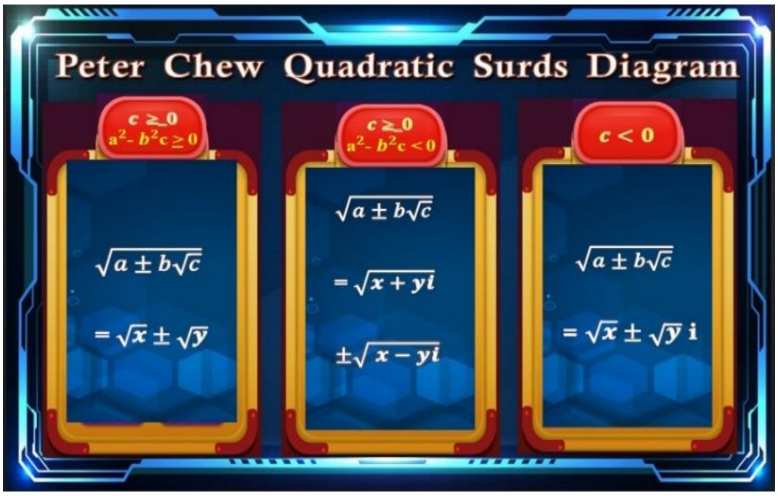

Fig. 1. Main page Peter Chew Quadratic Surd Diagram (PCQS) calculator.

In addition to applying Peter Chew Theorem in the Peter Chew Quadratic Surd Diagram (PCQS) calculator , Peter Chew Theorem are also applicable to many different Engineering fields, such as Civil Engineering [11], Mechanical Engineering [12, 13], Electrical Engineering [14], Aerospace Engineering [15], Marine Engineering [16] and Astronomical Engineering [17]. Furthermore, Peter Chew Theorem are also applicable to Pool Game [18] and Criminology [19].

6. Conclusion

Presenting numbers in surd form is relatively common in science and engineering especially where a calculator is either not allowed or unapproachable, and the computations to be undertaken involve irrational values. Every pupil who plans to take computation's at the advanced position in such a calculus-based or statistics should be suitable to manipulate and deal with surds. Technical tools have had a significant impact on advanced mathematics tutoring and mathematics literacy. However, today's online calculator only contains the knowledge that has been explained in the book, but the current method cannot or is difficult to solve some Quadratic Surds problems, this makes online calculators unable to solve quadratic surds problems. This can lead to reduced student interest and hinder the spread of technology tool use.

In order to solve the above problems, my research is to create a new discovery for the quadratic surds problem, such as Peter Chew's theorem, so that all problems can be easily solved Apply Peter Chew's theorem to a AI Age calculator (Peter Chew Quadratic Surd Diagram calculator), making the AI age calculator able to solve any problem in the quadratic remainder problem can make the AI age calculator effective in helping mathematics teaching, especially when similar COVID-19 problems arise in the future.

The main goal of Peter Chew's theorem is to simplify the solution, which is in line with Albert Einstein's quotation: Everything should be made as simple as possible. In addition, Albert Einstein's also quote:

i) We cannot solve our problems with the same thinking we used when we created them.

ii) If you can't explain it simply you don't understand it well enough,

iii) "Genius is making complex ideas simple, not making simple ideas complex."

iv) "Any intelligent fool can make things bigger and more complex. It takes a touch of genius - and a lot of courage - to move in the opposite direction."

v) God always takes the simplest way.

vi) When the solution is simple, God is answering.

Isaac Newton quote Nature is pleased with simplicity. And nature is no dummy.

From the Albert Einstein's and Isaac Newton quote above, it can be seen that simplifying knowledge is very important.

Conflict of Interest

The author declares no conflict of interest.

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