Application of Squaring the Circle in Practice

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Abstract: The paper outlines a new approach to solving the classical Greek problem 'squaring the circle' in a constructive manner with a major focus on its application in practice. Though it is known that the problem of squaring the circle cannot be solved by means of Euclidean construction, this paper will provide an insight into innovative solutions to the problem using only a ruler and compass. The solution for squaring the circle is based on finding the original ratio of 11,000,000: 3,005,681. Squaring the circle leads to the introduction of a parameter and derived functional links which indicate the possibility of applying problems in practice by determining and calculating areas of related geometric figures on a flat plane and by calculating the volumes of related geometrical figures in space through specific examples such as turning the Pyramid of Cheops into the cone of the same volume, or a Leaning Tower of Pisa into the shape of a prism.

Keywords: Application, construction, geometry, squaring the circle

1. Introduction

The problems of squaring the circle, doubling the cube and angle trisection are the three most famous mathematical problems in history. They occupied the attention of the greatest minds of ancient Greece and up until modern times have interested mathematicians who studied them meticulously to prove their solvability using a simple method—Euclidian construction, i.e., a ruler and compass. Their attempts were linked to the use of other, less traditional means, which directly influenced important developments and advances in mathematics over the centuries [1].

With this in mind, I tried to prove that the above-mentioned problems can be solved in a constructive (geometric) and algebraic way. The development of the idea of the solvability of these problems in a constructive way took place gradually, starting from the assumption that the famous Greek problems can be solved using a ruler and compass. The approach to construction of the problems is based on geometric intuition, given that the core value of the paper lies in the original proportions for squaring the circle.

In this paper, the construction of squaring the circle will be thoroughly presented. The solution for squaring the circle is based on finding the original ratio 11,000,000: 3,005,681. Squaring the circle leads to the introduction of a parameter and derived functional links which indicate the possibility of applying problems in practice by determining and calculating areas of related geometric figures on a flat plane and by calculating the volumes of related geometrical figures in space through specific examples.

The original methods of solving squaring the circle used in the paper prove to have practical applications. The consistency between the research results and the practical application of squaring the circle is best seen in the following concrete examples: if we wanted to build another Wonder of the World—a twin of the

Pyramid of Cheops of the same volume in the shape of a cone, or a Leaning Tower of Pisa of the same volume in the shape of a prism.

Application of the "squaring the circle" problem in many different spheres of life, such as architecture, construction, electronics, robotics, mechanical engineering, astronomy, mechanics, etc. directly demonstrates the significance of this paper's results.

2. Squaring the Circle

Squaring the circle is a task that refers to the construction of a square whose area is equal to the area of a given circle [2]. The aim of this section is to point to the possibility of solving this problem using a ruler and compass and showcasing the application of squaring the circle in a few examples. Mathematical formulas containing the irrational number π are approximate formulas.

2.1. General Construction of Squaring the Circle Using a Ruler and Compass

When constructing the squaring of the circle, I used an experimental proportion that I came to intuitively, which proved to be precise and represents the key to solving this problem. The proportion $AC: CB = 11{,}000{,}000: 3{,}005{,}681$ is an original proportion.

On an arbitrary line p, determine the points A and B so that the line segment AB is equal to the diameter of the circle (Fig. 1). To determine the point C, which divides the radius AB by the same proportion, we construct an arbitrary ray Aq and, using Thales's theorem (T_1) starting from the point A, we determine the point M so that the line segment $AM = 11{,}000{,}000$ arbitrary unit segments, and then determine the point N so that the line segment $MN = 3{,}005{,}681$ arbitrary unit segments.

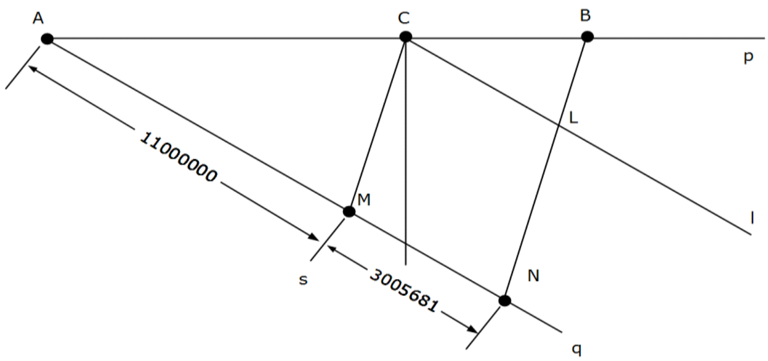

Fig. 1. The proportion for squaring the circle.

Triangles ACM and CBL are similar (Fig. 1).

The following proportion is valid: $AC: AM = CB: CL \Longrightarrow AC: CB = AM: CL$.

Given that $11{,}000{,}000 = 11 \cdot 10^6$ and $3{,}005{,}681 = 3.005681 \cdot 10^6$, the proportion

$$AM : MN = 11 \cdot 10^6 : 3.005681 \cdot 10^6,$$

After dividing through by 106, can be presented as

$$AM : MN = 11 : 3.005681 \ \ldots$$

By determining the point N, we construct the line segment BN. Using first a compass, and then a ruler, we construct the line S(M) parallel to the segment BN. Its intersections with the diameter AB are marked by point C. Point C is a unique point by which line segment AB is divided in the ratio $11 : 3.005681$, and according to Thales's theorem (T_1) the following proportion is true:

$$AC : CB = AM : MN$$

$$AC : 11 = CB : 3.005681$$

$$\text{Let } \frac{AC}{11} = \frac{CB}{3.005681} = t, t > 0, t \in \mathbb{R},$$

$$AC = 11t \text{ and } CB = 3.005681t \ldots \quad (1)$$

We construct a circle k whose diameter is the line segment AB. Through the point C we construct the normal n on the line segment AB and its intersections with the circle k are denoted by D_1 and D_2. We construct the line segment AD_1 and AD_2 and rays Bt_1 (to which the point D_1 belongs) and Bt_2 (to which the point D_2 belongs). We claim that the segments AD_1 or AD_2 are sides of the required square. The chords AD_1 and AD_2 of the circle k are equal to each other. We determine the point G_1 on Bt_1 such that $AD_1 = D_1G_1$ and the point G_2 on Bt_2 so that $AD_2 = D_2G_2$. The fourth vertex H_1 of the square $AD_1G_1H_1$ is in the intersection of the circular arcs $k_1(A, AD_1)$ and $k_2(G_1, AD_1)$, and the fourth vertex H_2 of the square $AD_2G_2H_2$ is in the intersection of the circular arcs $k_1(A, AD_1)$ and $k_3(G_2, AD_1)$.

We construct the line segments AH_1, G_1H_1, AH_2, G_2H_2 (Fig. 2).

The squares $AD_1G_1H_1$ and $AD_2G_2H_2$ are symmetrical about the line p.

This problem always has two solutions (Fig. 2).

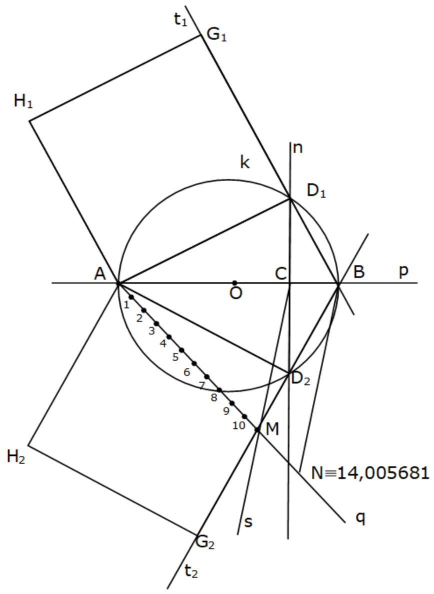

Fig. 2. Final construction of squaring the circle.

2.2. General Formulas for the Area of a Circle or Area of a Square

Given that the line segment AB is the diameter of the circle:

$$AB = AC + CB \Rightarrow AB = 11t + 3.005681t$$

$$2r = 14.005681 \cdot t \Rightarrow r = 7.0028405 \cdot t \quad \ldots \quad (2)$$

The area of the circle is:

$$P = r^2\pi, \quad r = 7.0028405 \cdot t$$

$$P = (7.0028405 \cdot t)^2\pi, \quad \Rightarrow \quad P = 49.03977506844025 \cdot \pi \cdot t^2$$

$$P_\circ = 154.062t^2$$

The area of square $AD_1G_1H_1$ equals AD_1^2. Using Pythagoras' theorem in the right-angle triangle ACD_1, we have the equation

$$AD_1^2 = AC^2 + CD_1^2 \ \ldots \tag{3}$$

$$\text{From } AC = 11t \ \Rightarrow \ AC^2 = 121t^2 \ \ldots \tag{4}$$

From the similarity of triangles ACD_1 and D_1CB, the proportion is as follows:

$$AC : CD_1 = CD_1 : CB \ \Rightarrow CD_1^2 = AC \cdot CB$$

$$\Rightarrow CD_1^2 = 11t \cdot 3.005681 \cdot t \ \Rightarrow \ CD_1^2 = 33.062491t^2 \ \ldots \tag{5}$$

By substituting Eqs. (4) and (5) into (3) we obtain:

$$\mathrm{P}_{\square} = AD_1^2 = 121t^2 + 33.062491 \cdot t^2 \ \Rightarrow \ \mathrm{P}_{\square} = 154.062491\mathrm{t}^2$$

We notice that the first three decimals in the formula for the area of a square coincide with the area of a circle, i.e.,

$$\mathrm{P}_{\square} = \mathrm{P}_{\circ} = 154.062\mathrm{t}^2, \text{ accurate to three decimal places.}$$

Table 1 shows approximate formulae for the area of a circle depending on the number used for π. These formulas, starting from the value $\pi = 3.14159$, coincide with the formula for the area of a square with an accuracy of three decimal places, i.e. their areas are

$$P = 154.062 \cdot t^2, t > 0, t \in \mathbb{R}.$$

Table 1. Coefficients for squaring the circle by means of π

P_1 = 153.984 893 714 902 385 · t^2	π = 3.14
P_2 = 154.033 933 489 970 825 25 · t^2	π = 3.141
P_3 = 154.058 453 377 505 045 375 · t^2	π = 3.1415
***P*₄ = 154.062** 866 957 261 204 997 5 · t^2	π = 3.14159
P_5 = 154.062 965 036 811 341 878 · t^2	π = 3.141592
P_6 = 154.062 994 460 676 382 942 15 · t^2	π = 3.1415926
P_7 = 154.062 996 912 665 136 364 162 5 · t^2	π = 3.14159265
P_8 = 154.062 997 059 784 461 569 483 2 · t^2	π = 3.141592653
P_9 = 154.062 997 084 304 349 103 703 3 · t^2	π = 3.1415926535
P_{10} = 154.062 997 088 227 531 109 178 5 · t^2	π = 3.14159265358
P_{11} = 154.062 997 088 668 889 084 794 5 · t^2	π = 3.141592653589
P_{12} = 154.062 997 088 703.216 927 342 4 · t^2	π = 3.1415926535897
P_{13} = 154.062 997 088 707 630 507 098 6 · t^2	π = 3.14159265358979

From $P = r^2\pi$, for $r = 7.0028405 \cdot t$, $P = 49.03977506844025\,\pi \cdot t^2$.

3. Application of the Squaring the Circle in Practice

The concrete examples of determining and calculating the area and volumes of related geometric figures on a flat plane and in space are presented in this chapter. If we wanted to build another wonder of the world—a twin of the Pyramid of Cheops of the same volume in the shape of a cone, or a Leaning Tower of Pisa of the same volume in the shape of a prism, we could use the formulae below from Table 2 derived from the areas of the square and the area of the circle $P_{\square}(t) = P_{\circ}(t) = 154.062t^2$ accurate to three decimal places, where t is a non-negative real number.

Table 2. Rmuš's formulae

1.	$r = 7.0028405 \cdot t, \ t > 0, \ t \in \mathbb{R}$
2.	$P_{\square}(t) = P_{\circ}(t) = 154.062t^2$
3	$a(t) = 12.412191 \cdot t$, accurate to six decimal places
4.	$t(r) = 0.1427991 \cdot r$, accurate to seven decimal places
5.	$a(r) = 1.77245 \cdot r$, accurate to five decimal places
6.	$r(a) = 0.56419 \cdot a$, accurate to five decimal places

3.1. Example 1

Given a right cylinder whose radius (base) is $2r$ =10 cm, turn the volume of the cylinder into the volume of a prism so that its volume is equal to the volume of a right four-sided prism if both figures have equal heights (Fig. 3).

Solution:

$$\mathrm{V}_{(\text{cylinder})} = B \cdot H, \quad B = r^2\pi, \quad r = 5 \text{ cm}, \quad H = 10 \text{ cm}, \quad \pi \approx 3.14159$$

$$\mathrm{V}_{(\text{cylinder})} = r^2\pi \cdot H \Longrightarrow \mathrm{V}_{(\text{cylinder})} = 5^2 \cdot 3.14159 \cdot 10$$

$$\Longrightarrow \mathrm{V}_{(\text{cylinder})} = 25 \cdot 10 \cdot 3.14159$$

$$\Longrightarrow \mathrm{V}_{(\text{cylinder})} = 785.3975 \text{ cm}^3 \cdots \tag{6}$$

Rmuš's formula $a(r) = 1.77245 \cdot r$

According to Rmuš's formula $a(5) = 1.77245 \cdot 5$ cm

$$\Longrightarrow a = 8.86225 \text{ cm}$$

$$V_{(\text{right four-sided prism})} = B \cdot H, \quad B = a^2, \quad H = 10 \text{ cm}$$

$$V_{(\text{right four-sided prism})} = a^2 \cdot 10$$

$$\Longrightarrow V_{(\text{right four-sided prism})} = 8.86225^2 \cdot 10$$

$$\Longrightarrow V_{(\text{right four-sided prism})} \approx 785.395 \text{ cm}^3 \cdots \tag{7}$$

The absolute error of Eqs. (7) and (6) is

$$|785.3975 \text{ cm}^3 - 785.395 \text{ cm}^3| = 0.0025 \text{ cm}^3$$

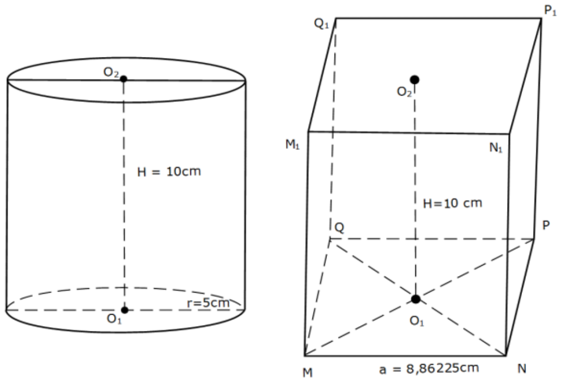

Fig. 3. Similarity of a cylinder and four-sided right prism.

3.2. Example 2

The base of the Pyramid of Cheops is square in shape with sides a = 230.34 m and height 146.7 m (Fig. 4) [3]. Turn that pyramid into a cone of the same height and calculate how much their volumes differ (Fig. 5). $a = 230.34$ m, $H = 146.7$ m.

$$\mathrm{V}_{(\text{right four-sided pyramid})} = \frac{B \cdot H}{3}, \quad B = a^2, \quad H = 146.7 \text{ m}, \quad a^2 = (230.34 \text{ m})^2$$

$$\Longrightarrow a^2 = 53056.5156 \text{ m}^2$$

$$\mathrm{V}_{(\text{right four-sided pyramid})} = \frac{53056.5156 \cdot 146.7}{3}$$

$$\Longrightarrow \mathrm{V}_{(\text{right four-sided pyramid})} = \frac{7783390.83852}{3}$$

$$\Longrightarrow \ \mathrm{V}_{(\text{right four-sided pyramid})} \approx 2594463.61284 \ \mathrm{m}^3 \ \ldots \tag{8}$$

According to Rmuš's formula $r(a) = 0.56419 \cdot a$, for $a = 230.34$ m gives $r = 129.9555246$ m

$$\mathrm{V}_{(\text{cone})} = \frac{B \cdot H}{3}, \quad B = r^2\pi, \quad r = 129.95552 \ \mathrm{m}, \quad \pi \approx 3.14159, \qquad H = 146.7 \ \mathrm{m}$$

$$\mathrm{V}_{(\text{cone})} = \frac{r^2\pi \cdot H}{3}$$

$$\Longrightarrow \mathrm{V}_{(\text{cone})} = \frac{(129.95552 \ \mathrm{m})^2 \ \cdot \ 3.14159 \ \cdot \ 146.7 \ \mathrm{m}}{3}$$

$$\Longrightarrow \mathrm{V}_{(\text{cone})} = \frac{16888.4371784704 \ \cdot \ 146{,}7 \ \cdot \ 3.14159}{3}$$

$$\Longrightarrow \mathrm{V}_{(\text{cone})} = \frac{7783395.2036534378714112}{3}$$

$$\Longrightarrow \mathrm{V}_{(\text{cone})} \approx 2594465.0678844792904704 \ \mathrm{m}^3 \cdots \tag{9}$$

The absolute error of Eqs. (9) and (8) is

$$|2594465.0678844 \ \mathrm{m}^3 - 2594463.61284 \ \mathrm{m}^3| = 1.455 \ \mathrm{m}^3$$

Fig. 4. Photo of the Pyramid of Cheops [4].

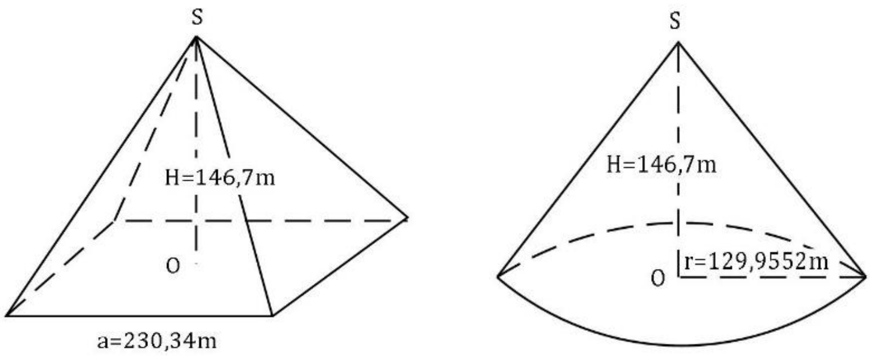

Fig. 5. Similarity of the Pyramid of Cheops and the cone of the same height.

3.3. Example 3

Turn a truncated cone, whose radius r_1 of the lower base (base) is equal to 6 cm and whose radius r_2 of the upper base is 2 cm and height H = 4 cm, into a regular four-sided truncated pyramid whose height is equal to the height of the cone (Fig. 6). Prove.

Solution:

The form for calculating the volume of a truncated cone is:

$$V = \frac{H\pi}{3}(r_1^2 + r_1 \cdot r_2 + r_2^2)$$

Data: $H = 4$ cm, $r_1 = 6$ cm, $r_2 = 2$ cm, $\pi \approx 3.14159$

$$\mathrm{V}_{(\text{truncated cone})} = \frac{4}{3} \cdot 3.14159 \cdot (6^2 + 6 \cdot 2 + 2^2)$$

$$\Longrightarrow \mathrm{V}_{(\text{truncated cone})} = \frac{4}{3} \cdot 3.14159 \cdot 52$$

$$\Longrightarrow \mathrm{V}_{(\text{truncated cone})} = \frac{653.45072}{3}$$

$$\mathrm{V}_{(\text{truncated cone})} \approx 217.8169\ \text{cm}^3\ \ldots \tag{10}$$

According to Rmuš's formula $a_1(r) = 1.77245 \cdot r$
for $r = 6$ cm, we obtain $a_1 = 10.6347$ cm, and
for $r = 2$ cm, we obtain $a_2 = 3.5449$ cm.

The form for calculating the volume of a truncated pyramid is

$$\mathrm{V}_{(\text{truncated pyramid})} = \frac{H}{3}(a_1^2 + a_1 a_2 + a_2^2)$$

$$\mathrm{V}_{(\text{truncated pyramid})} = \frac{4}{3} \cdot (10.6347^2 + 10.6347 \cdot 3.5449 + 3.5449^2)$$

$$\Longrightarrow \mathrm{V}_{(\text{truncated pyramid})} = \frac{4}{3} \cdot (113.09684 + 37.698948 + 12.566316)$$

$$\mathrm{V}_{(\text{truncated pyramid})} = \frac{4}{3} \cdot 163.362$$

$$\Longrightarrow \mathrm{V}_{(\text{truncated pyramid})} = \frac{653.448416}{3}$$

$$\Longrightarrow \mathrm{V}_{(\text{truncated pyramid})} \approx 217.816138\ \text{cm}^3 \cdots \tag{11}$$

The absolute error of Eqs. (11) and (10) is $0.000762\ \text{cm}^3$.

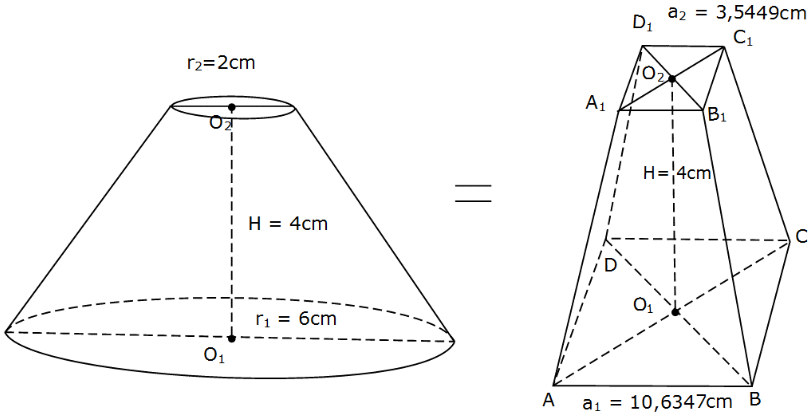

Fig. 6. Similarity of a truncated cone and truncated pyramid.

3.4. Example 4

Turn the volume of the cylinder—Tower of Pisa (Fig. 7) —into two regular four-sided prisms of the same height and calculate how much their volumes differ. The base of the Leaning Tower of Pisa is circular in shape with outer diameter 15.484 m, i.e., $r_1 = 7.74$ m. The height of the tower is 55.86 m from the ground on the low side and 56.7 m on the high side. The thickness of the walls at the base is 4.09 m, and at the top 2.48 m. The difference between 4.09 m and 2.48 m is 1.61 m, which should be subtracted from radius r_1 = 7.74 m to obtain r_2 = 6.13 m (Fig. 8). The height to the seventh floor is 47.27 m on the low side and 48.55 m on the high side [5].

Fig. 7. Photo of the Leaning Tower of Pisa [6].

$$V_1 = r_1{}^2 \cdot \pi \cdot H_1 \quad \text{for } r_1 = 7.74 \text{ m}, \; H_1 = 47.27 \text{ m}, \; \pi \approx 3.14159$$

$$\Longrightarrow V_1 = (7.74m)^2 \cdot 47.27 \text{ m} \cdot 3.14159$$

$$\text{Finally, } V_1 = 8896.4558 \; ..... \tag{12}$$

$$V_2 = r_2{}^2 \cdot \pi \cdot H_2, \; \text{for } r_2 = 6.132 \text{ m}, \; H_2 = 8.59 \, m$$

$$\Longrightarrow V_2 = (6.13 \text{ m})^2 \cdot 8.59 \text{ m} \cdot 3.14159$$

$$\Longrightarrow V_2 = 1014.0599 \text{ m}^3 \; ... \tag{13}$$

$$\text{V}_{(\text{tower})} = V_1 + V_2$$

$$\text{V}_{(\text{tower})} = 8896.4558 \text{ m}^3 + 1014.0599 \text{ m}^3$$

$$\Longrightarrow \text{V}_{(\text{tower})} = 9910.5157 \text{ m}^3 \qquad ... \tag{14}$$

Now we will calculate the volumes of two regular four-sided prisms.
The side of a regular four-sided prism (the base is square) is
According to Rmuš's formula, $a(r) = 1.77245 \cdot r$, with accuracy to six decimal places.

$$a_1(r_1) = 1.77245 \cdot 7.74 \text{ m} \Longrightarrow a_1(r_1) = 13.72 \text{ m}$$

$$V_{1\,(\text{prism})} = a_1{}^2 \cdot H_1, \; a_1 = 13.72, \; H_1 = 47.27 \text{ m}$$

$$V_{1\,(\text{prism})} = (13.72 \text{ m})^2 \cdot 47.27 \text{ m}$$

$$\Longrightarrow V_{1\,(\text{prism})} = 8898.168 \text{ m}^3 \; ... \tag{15}$$

On the other hand, $a_2(r_2)$ = 1.77245 $\cdot$ 6. 13 m

$$\Longrightarrow a_2(r_2) = 10.86 \text{ m} \Longrightarrow$$

$$V_{2\,(\text{prism})} = a_2{}^2 \cdot H_2, \; a_2 = 10.86, \; H_2 = 8.59 \text{ m}$$

$$V_{2\,(\text{prism})} = (10.86 \text{ m})^2 \cdot 8.59 \text{ m}$$

$$\Longrightarrow\ V_{2\,(\text{prism})} = \ 1013.101\ \text{m}^3\ \ldots \tag{16}$$

$$\text{V}_{(\text{prism})} = V_{1(\text{prism})} + V_{2(\text{prism})}$$

$$\text{V}_{(\text{prism})} = 8898.029\ \text{m}^3 + 1013.101\ \text{m}^3$$

$$\Longrightarrow\ \text{V}_{(\text{prism})} = 9911.13\ \text{m}^3 \tag{17}$$

The absolute error of Eqs. (17) and (14) is

$$|9911.13\ \text{m}^3 - 9908.5157\ \text{m}^3| = 2{,}61\ \text{m}^3$$

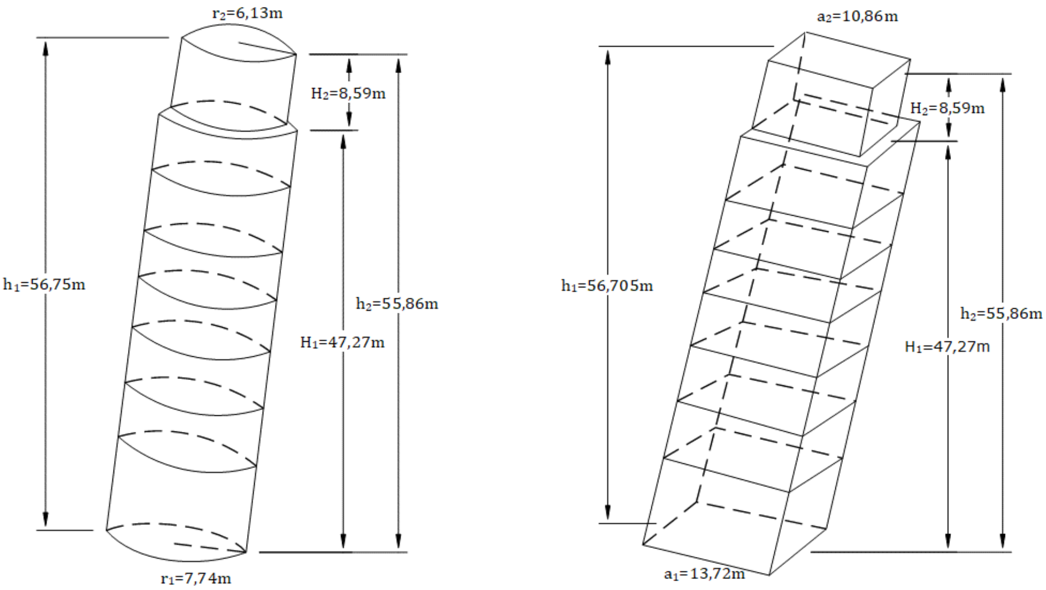

Fig. 8. Similarity of the Leaning Tower of Pisa and two right four-sided prisms.

3.5. Example 5

Turn an oblique regular four-sided prism, whose base is a square of edge a = 5 cm and height H = 12 cm, into an oblique cylinder (Fig. 9). Prove.

$$\text{V}_{(\text{oblique regular four-sided prism})} = B \cdot H,\ \ B = a^2, \qquad H = 12\ \text{cm}$$

$$\text{V}_{(\text{oblique regular four-sided prism})} = a^2 \cdot 12 \Longrightarrow$$

$$\text{V}_{(\text{oblique regular four-sided prism})} = (5\ \text{cm})^2 \cdot 12\ \text{cm} \Longrightarrow$$

$$\text{V}_{(\text{oblique regular four-sided prism})} = 300\ \text{cm}^3\ \ldots \tag{18}$$

According to Rmuš's formula $r(a) = 0.56419 \cdot 5\ \text{cm}$

$$\Longrightarrow r = 2.82095\ \text{cm}.$$

$$\text{V}_{(\text{cylinder})} = B \cdot H,\ \ B = r^2\pi, \qquad H = 12\ \text{cm}$$

$$\text{V}_{(\text{cylinder})} = r^2 \cdot \pi \cdot 12,\ \ r = 2.82095\ \text{cm}, \quad \pi \approx 3.14159$$

$$\text{V}_{(\text{cylinder})} = 2.82095^2\ \text{cm}^2 \cdot 3.14159 \cdot 12\ \text{cm}$$

$$\Longrightarrow \text{V}_{(\text{cylinder})} \approx 7.9577589\ \text{cm} \cdot 37.69908\ \text{cm}$$

$$\Longrightarrow \text{V}_{(\text{oblique regular four-sided prism})} \approx 300.00018\ \text{cm}^3\ \ldots \tag{19}$$

The absolute error of Eqs. (19) and (18) is $0.00018\ \text{cm}^3$.

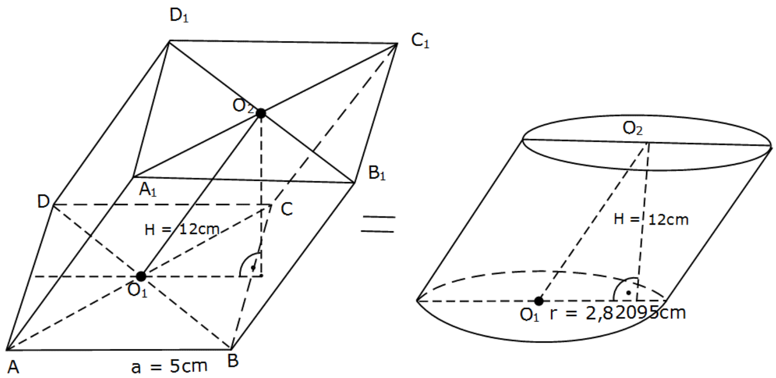

Fig. 9. Similarity of an oblique regular four-sided prism and an oblique cylinder.

4. Conclusion

The manuscript demonstrates an original approach to the solution of the squaring the circle. The contribution of the manuscript lies in the fact that the constructions of this ancient problem has been proven to be solvable by both geometric (constructive) and algebraic means.

Formulae derived from the original ratio 11,000,000: 3,005,681 for squaring the circle led to the determination and calculation of the area and volume of related geometric figures, i.e., their areas and volumes are the same if the side of the square and the radius of the circle are in a functional relation in accordance with Rmuš's formulae.

It is important to emphasize, when thinking about the number π and its approximation, that a large number of constants have been created by the development of natural and engineering sciences, whose exact values we cannot know by mathematics, but only by rounding to some degree of accuracy (decimals).

The efforts to solve the problems lie in an eagerness and intrinsic motivation to make a significant contribution to mathematics and science in general. The problem-solving work was based on geometric intuition, starting from the premise that the ideal proportion could be found, and which was gradually improved on with time. In-depth analysis of the results led to the construction of the problem, which verified that the application of the squaring the circle problem is possible in practice using straightforward and concrete examples. The comprehensive results obtained in the manuscript represent a novelty in mathematics and could eventually lead to the use of all these applications in different spheres of life, such as architecture, construction, mechanical engineering, astronomy, etc.

Conflict of Interest

The author declares no conflict of interest.

Acknowledgment

The research has been based on intuition in coming to an ideal proportion for squaring the circle and doubling the cube, respectively, which represent the essence of the solutions to the problems. It did not happen by accident, but unfortunately during the tremendous grief and pain the author underwent when his son Anto died in 2008. During the first months after his death, the author found an escape from reality in working on these ancient problems, trying to solve a mathematical mystery that is as old as two and a half millennia. The paper is dedicated to his late son.

The author would like to express his sincere gratitude to his family, friends and relatives for their support and appreciation of his discovery. The acknowledgment of true encouragement goes to his youngest daughter Marina-Nina Rmuš who wholeheartedly believed in his work and has been persistent to find a way for it to

be visible and widely recognized. Marina translated this paper and has been his voice in communication with all the relevant stakeholders in previous years.

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