

A Transient Heat Conduction Simulation of An Anisotropic Material Applied in Wind Turbine Blade for the Reuse Context

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Abstract: The reusing process of wind turbine blades demands machining operations, commonly performed with laser tools due to the inherently anisotropic nature of the fiber reinforced polymer (FRP). Therefore, it is essential to understand the heat transfer in anisotropic media to control the heat-affected zone (HAZ), established when the blade is cutted. This work aims to use the Multi-Point Flux Approximation (MPFA) method in the numerical solution of transient heat conduction problems in anisotropic media. This poses as an alternative to the traditional Two-Point Flux Approximation method (TPFA), which has a wide range of applications however fails to generate accurate solutions to problems with certain anisotropic materials configurations. Notwithstanding that the MPFA method is well-known, the transient version developed here is still unexplored in literature. The results observed with classical benchmark problems show that MPFA can describe the solutions of transient anisotropic problems more accurately than the traditional TPFA.

Key words: Anisotropic media, heat transfer, reuse, wind turbine blade.

1. Introduction

The blade is a fundamental component of the wind turbine. It is made mostly of composite material, due to its good mechanical properties and its light weight, however, this material imposes difficulty to recycling owing to its nature [1]. Composite materials are responsible for over 90% of the blade's weight and it is estimated that until 2050 there will be 43 million tons of blade waste worldwide [2], that being said, it is urgent to find a sustainable destination for this refuse.

Due to the anisotropy and the inhomogeneous composition of the composite material, machining operations with ordinary machinery can be challenging to perform, to tackle this issue, laser operations are more suitable; since it is a thermal process, a laser-induced heat affected zone (HAZ) arises from the operation. In [3] the authors investigated the grooving of unidirectional Carbon/Epoxy in several directions related to the fiber axis, applying Finite Difference Method (FDM) to solve the heat conduction equation in anisotropic solids to numerically foresee the extent of the HAZ and compare its value to that obtained experimentally.

FDM and traditional Finite Volume Method (FVM) have a wide range of applications and pose as robust

framework for a wide class of problems. However, the flux in each discretized surface is computed by means of a difference between two unknown variable values. This classical approach is often called in literature as Two-Point Flux Approximation (TPFA) method [4] and can produce robust and convergent solutions, even for anisotropic problems, once the principal directions of the conductivity tensor are aligned to the grid orientation, i.e., when we have a k-orthogonal grid. However, it is well known that the TPFA method is unable to produce convergent solutions for those problems which are not k-orthogonal [4]. The MPFA methods take, by construction, more than two points to derive the flux in each control surface and a TPFA discretization is naturally obtained for k-orthogonal grids.

In this paper we adapt the well-known MPFA method, originally proposed to steady state problems, to discretization of the spatial term in transiente heat conduction. This approach ensures that an accurate and convergent numerical formulation, coming from a conservative numerical framework [4], can be used to simulate the temperature distribution and hence understand the extent of the HAZ during the laser cutting process in the wind turbine blade application. The numerical experiments point a promising result with the proposed adaptation

2. Mathematical Model

The time-dependent heat conduction in two-dimensional workspace Ω with boundary Γ is modelled in this paper by the following Partial Differential Equation:

$$\rho C \frac{\partial T}{\partial t} - \vec{\nabla} \cdot (\underline{K} \vec{\nabla} T) = Q \quad (1)$$

where ρ and C are, respectively, density and specific heat of the material. The thermal conductivity, $\underline{K}(\vec{x})$, is modelled as a symmetric ($k_{xy} = k_{yx}$), either diagonal or full tensor, which can be discontinuous in Ω . This denotes anisotropy in the material as it has been considered in literature for the blade wind turbine context [5]. In addition, even though the above mentioned parameters can be dependent on the temperature, T , we consider, as simplifying assumption and without loss of generality, a linear behaviour.

The thermal source term is denoted by Q , which can represent the laser beam used in the blade wind turbine cutting.

For completeness, we define initial and boundary conditions as follows:

$$\begin{aligned} T(\vec{x}, 0) &= T_0(\vec{x}) \text{ in } \Omega; \quad T(\vec{x}_{\Gamma_D}, t) = g_D \text{ in } \Gamma_D \\ -\underline{K} \vec{\nabla} T \cdot \vec{n} &= g_N \text{ in } \Gamma_N; \quad -\underline{K} \vec{\nabla} T \cdot \vec{n} = h(T - T_\infty) \text{ in } \Gamma_R \end{aligned} \quad (2)$$

where $\Gamma = \Gamma_D \cup \Gamma_N \cup \Gamma_R$ as well as $\Gamma_D \cap \Gamma_N \cap \Gamma_R = \emptyset$. The initial condiction in the open domain Ω is represented by $T_0(\vec{x})$. Besides, g_D and g_N are, respectively, known parameters on the Dirichlet and Neumann boundaries. The parameters in Robin boundary, h and T_∞ are, respectively, the convection heat transfer coefficient and the environment temperature. At last, the unitary vector, outer to Γ , is denoted by \vec{n} .

3. Numerical Formulation

If We numerically resolve Eq. (1) by means of the cell-centered Finite Volume Method, an attractive alternative to solve problems governed by conservation laws [4]. Thus, after integrate Eq. (1) in the open domain Ω and through the time interval $[t_0, t]$, we subdivide Ω in N_{CV} non-overlapping control volumes $\Omega_{\hat{L}}$ with boundaries $\Gamma_{\hat{L}}$. We take the integral equation derived for a certain arbitrary control volume and we apply both Green-Gauss and Mean Value Theorems. In addition, we employ forward Euler approximation to discretize the time dependent term. This procedure leads to the discretized transient heat conduction

equation, for a certain arbitrary control volume \hat{L} , which is implicitly resolved in the present paper and is define by:

$$\rho C \Delta V_{\hat{L}} T_{\hat{L}}^{n+1} - \Delta t \sum_{IJ \in \Gamma_{\hat{L}}} F_{IJ}^{n+1} = \rho C \Delta V_{\hat{L}} T_{\hat{L}}^n + \bar{Q} \Delta t \Delta V_{\hat{L}} \quad (3)$$

where F_{IJ}^{n+1} is the numerical flux computed through the arbitrary surface IJ , belonging to the evaluated control volume \hat{L} , such as shown in Fig. 1. Each numerical strategy leads to a different way in its derivation. $\Delta V_{\hat{L}}$ and Δt are, respectively, the volume (area in 2-D) of the control volume and the discrete time step. Besides, since both methods are implicitly resolved in time dependent context, they are named here with a Full Implicitly acronym as prefix, i.e., FI-TPFA and FI-MPFA methods.

3.1. Spatial Discretization Technics

In this section we present the numerical strategies employed to resolve the time-dependent heat conduction in anisotropic material. Fig. 1 is adopted as reference for the nomenclature in the derived equations. The so-called Two-Point Flux Approximation (TPFA) denotes the traditional finite volume approximation for the energy rate F_{IJ}^{n+1} [4].

3.2. Spatial Discretization Technics

In this subsection we introduce some general concepts regarding to the Multi-Point Flux Approximation (MPFA) method. We therefore define an interaction region associated to an arbitrary vertex I (see Fig. 1). This dual region is bordered by the dashed line segment connecting the center of the N_{CV}^I surrounding cells \hat{L} , \hat{R} , \hat{M} , \hat{W} and the midpoint, in the control surfaces, which delimits the N_{HS}^I half-surfaces $\bar{E}I$, $\bar{F}I$, $\bar{O}I$, $\bar{Z}I$. The primal variable, which is the temperature, is approximated by a function defined through each sub-interaction region. In this paper we consider only the linear functions for that approximation, which lead to triangular support (triangle $\triangle \bar{L}\bar{E}\bar{Z}$, for example).

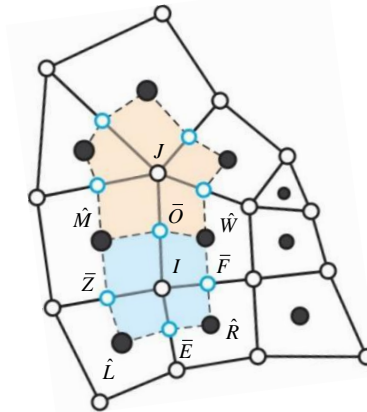


Fig. 1. Mesh fragment with two interaction regions and indication of half-surfaces.

Besides, since our approach is essentially cell-centered, we must discretize the energy rates through the N_{HS}^I half-surfaces only as a function of the temperatures on the N_{CV}^I control volumes. Hence, the temperature on the auxiliary points \bar{E} , \bar{F} , \bar{O} , \bar{Z} must be written as a function of those temperatures on the cells surrounding the arbitrary vertex I . For the sake of simplicity, the superindex $n + 1$ is suppressed in further equations. Since $F_{IJ} = F_{I\bar{O}} + F_{\bar{O}J}$, we can write, for instance, the energy rate derived through the arbitrary half-surface $\bar{E}I$, which can be computed regarding the cell \hat{L} by:

$$F_{\bar{E}I}^{\hat{L}} = \vec{q}_{\bar{E}I}^{\hat{L}} \cdot \vec{N}_{\bar{E}I} = -K_{\hat{L}} \vec{\nabla} T_{\hat{L}, \bar{E}, \bar{Z}} \cdot \vec{N}_{\bar{E}I} \quad (4)$$

Analogously, we can compute all energy rates in the interaction region I , on both sides of the N_{HS}^I half-surfaces. Fig. 1 must be used as reference. Those energy rates are given, respectively, by

$$\mathbf{f}_{left} = \{F_{EI}^L, F_{FI}^R, F_{OI}^W, F_{ZI}^M\}^T; \mathbf{f}_{right} = \{F_{IE}^R, F_{IF}^W, F_{IO}^M, F_{IZ}^L\}^T \quad (5)$$

Since the N_{HS}^I equations on the left and on the right are function of the temperature on N_{CV}^I colocation points and their adjacent auxiliary points, the set of equations can be written in a compact matrix form, for both sides by $\mathbf{f}_j = A_j \hat{\mathbf{T}} + B_j \bar{\mathbf{T}}$, with $j = \text{left, right}$, where \mathbf{f}_j , $\hat{\mathbf{T}}$ and $\bar{\mathbf{T}}$ are arrays with $N_{HS}^I \times 1$ and represent the set of energy rates, on both sides, the unknown and the auxiliary temperatures, respectively. The matrices A_j and B_j , in general, have size $N_{HS}^I \times N_{HS}^I$ and represent the physical and geometric parameters used to compute the energy rates vectors \mathbf{f}_j on the N_{HS}^I half-surfaces.

The procedure originally adopted to ensure a purely cell-centered discretization consists in imposing flux continuity in the N_{HS}^I half-surfaces, i.e., $\mathbf{f}_{left} = -\mathbf{f}_{right}$. After some manipulation, a local algebraic system is obtained as $\mathbf{G}\hat{\mathbf{T}} = \mathbf{H}\bar{\mathbf{T}}$, where $\mathbf{G} = (A_{left} + A_{right})$ and $\mathbf{H} = -(B_{right} + B_{left})$. Thus, by writing $\bar{\mathbf{T}}$ as a function of $\hat{\mathbf{T}}$ by means of inverting \mathbf{H} , we can write the energy rate vectors on both sides of the N_{HS}^I half-surfaces, depending only of the temperatures in the colocation points, which is the actual unknown variables. The vector of fluxes can therefore be written as:

$$\mathbf{f}_j = [A_j - B_j \mathbf{H}^{-1} \mathbf{G}] \hat{\mathbf{T}} \quad \text{with } j = \text{left, right} \quad (6)$$

The energy rates are accordingly assembled in the global algebraic system in order to calculate the temperature field in the whole domain. Further details concerning the MPFA derivation can be found in [4], [6].

4. Numerical Experiments

In this section we present some benchmark problems to illustrate the incapability of the traditional TPFA to perform convergence in non-k-orthogonal grids.

4.1. Transient Heat Conduction in Homogeneous and Anisotropic Material

This test case is adapted from [7] and consists in a simple transient heat conduction with source term in a square domain with $[0, 3.14]^2$. We consider an anisotropic diagonal thermal conductivity tensor, where its principal directions (transversal and longitudinal components) are $k_{xx} = 10$ and $k_{yy} = 1$ (both in $\text{W/m}^\circ\text{C}$). The exact solution which satisfies Eq. (1) is given by:

$$T(x, y, t) = e^{-t} \cos(x + y) \quad (7)$$

Which is also used as Dirichlet boundary condition, applied in all boundary faces of the domain. We use both Cartesian and distorted meshes, such as depicted in Fig. 2. Several refinements are considered (8, 16, 32, 64 and 128 subdivisions) to evaluate the performance of the methods.

First we evaluate how each method individually perform the convergence for a series of time steps (Δt) refinement. We adopt $\Delta t = 0.001; 0.01; 0.1$ and 1 second. The convergence performances for each method in both meshes are shown in Fig. 3. As we can observe, both methods suffer degradation in convergence when bigger Δt values are considered, regardless of the considered mesh. This occurs since forward Euler approximation has truncation error of order $O(\Delta t)$, while the spatial discretization for the referred methods present $O(\Delta x^2)$. To evaluate capability of each formulation, we compute the error between the numerical and the exact solutions by means of L_2 norm [4].



Fig. 2. Meshes adopted in classical test case. Both with 16x16 subdivisions: (a) Cartesian mesh; (b) distorted mesh.

Regarding to performance of both methods for each considered mesh, we observe that, for a sufficiently refined Δt , when the Cartesian mesh is adopted, both FI-TPFA and FI-MPFA present satisfactory results.

That equivalent performance occurs due to the principal directions of the thermal conductivity tensor are aligned with the grid orientation (k-orthogonal grid) in this test case. Thus, FI-MPFA recovery FI-TPFA [4]. However, when a distorted mesh is adopted we observe the incapability of the FI-TPFA in reduce the error, even for a more refined spatial and temporal meshes, as we can see in Fig. 3a (curve with painted triangle marker). On the other hand, we observe, in Fig. 3b, that the convergence performance reached with FI-MPFA for the distorted mesh is equivalent to that obtained with the Cartesian one.

Naturally and as expected the absolute values of the error in the simulation with the distorted mesh are bigger than those obtained with the Cartesian grid. Note that there is a superimposition of the convergence curves when the Cartesian mesh is adopted.

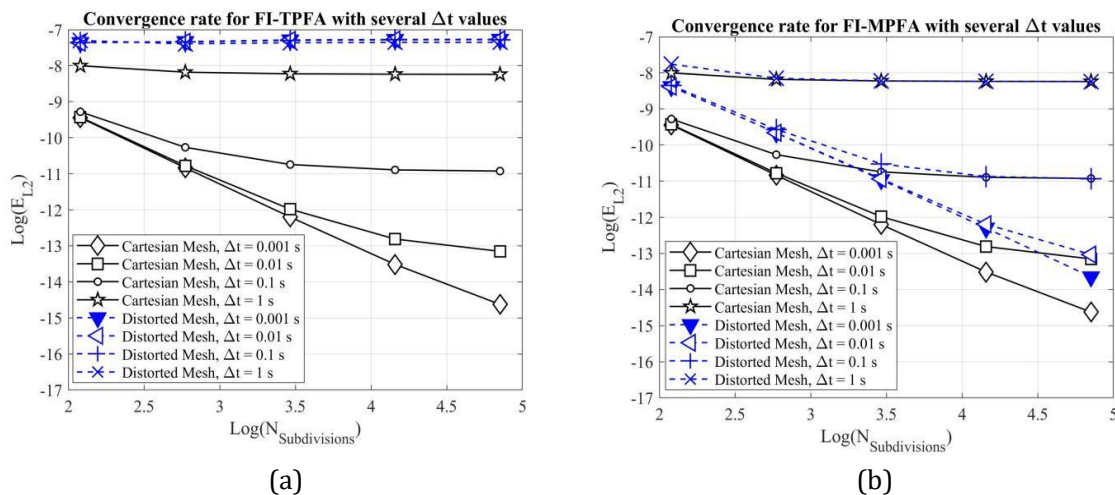


Fig. 3. Convergence rate curves obtained with the meshes depicted in Fig. 2 for different time steps: (a) adopting FI-TPFA; (b) adopting FI-MPFA.

Nevertheless, as already mentioned FI-MPFA converges for the simulation with the distorted mesh while FI-TPFA does not. This show how necessary is an accurate discretization when problems with anisotropic material and non k-orthogonal meshes are considered.

4.2. Transient Heat Conduction in Anisotropic Wind Turbine Blade Due to Laser Cutting Process

This problem poses as an applied employment of the FI-MPFA. We consider a wind turbine blade with geometry adapted from [3], as shown in Fig. 4.

The blade section dimensions and material properties are obtained from [8]. Even though we can have different material features for a wind turbine blade, as that depicted in Fig. 4, we consider as computational domain, only the piece of the section which can be thermally affected by a laser cutting procedure (see Fig. 4c). In this case, we consider that the piece is homogeneous and it has, for each point of the domain, different values of thermal conductivity in longitudinal and transverse reinforced fiber orientation. Thus, by following [8], we assume a thermal conductivity tensor with principal directions $k_{xx} = 8$ and $k_{yy} = 0.67$ (W/m°C). This tensor is rotated by 45° , which leads to a full thermal conductivity tensor [9].

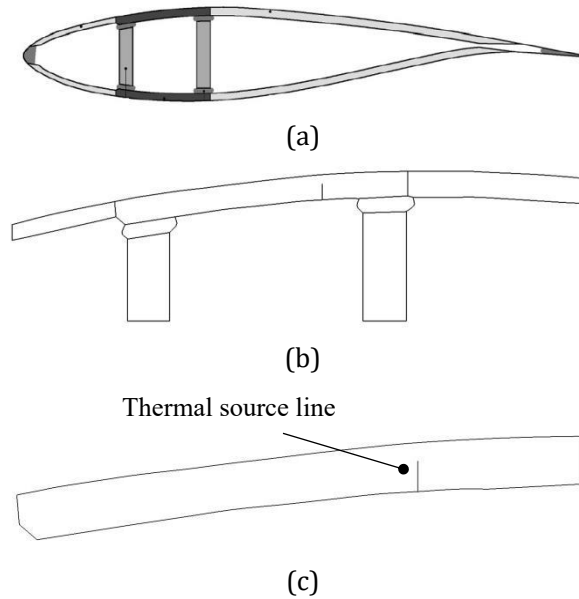


Fig. 4. Sketch of the wind turbine blade section submitted to laser cutting: (a) general view of the section; (b) slice with indication of the cut region; (c) computational domain with indication of thermal source line.

In addition, we also adopt $\rho = 1520$ kg/m³, $C = 1065$ J/kg°C [8] and an average thermal source term, applied only in those control volumes adjacent to the cutting line (see Fig. 4c), given by $\bar{Q} = 35000/V_{CVAdj}$ [8], where V_{CVAdj} is the volume (area in 2-D) of all control volumes adjacent to the laser cutting line.

We apply as boundary condition a Robin convection condition on those surfaces free of material contact. In this case, the convection heat transfer coefficient is $h = 10$ W/m²°C [9] and the environment temperature $T_\infty = 25$ °C, since we consider that the blade is in an open place. In those surfaces which have interface with other material we adopt null Neumann boundary condition.

Finally we establish a cutting procedure duration of 5 seconds and 5000 time steps in order to obtain a refined Δt value, which therefore leads to an accurate solution, such as we have observed in the previews test case. Furthermore, in order to honor the geometry curves we adopt a Delaunay triangular unstructured mesh with 9.459 control volumes. A more refined region is defined around the thermal source line, which is where the thermal effects are relevant. In the present problem the unstructured mesh does not have a defined orientation, such as occurs in the meshes adopted in the previews test case, we still can observe some difference in the temperature field when we simulate the thermal effects of the laser cutting with FI-TPFA and FI-MPFA.

Even though we do not have an exact solution for the present problem, we can suppose that the temperature field obtained with FI-MPFA represent better the simulated procedure. We have a full thermal conductivity tensor and an unstructured mesh, which produces a non k-orthogonal condition. Hence, FI-TPFA does not converge to the adopted mathematical model.

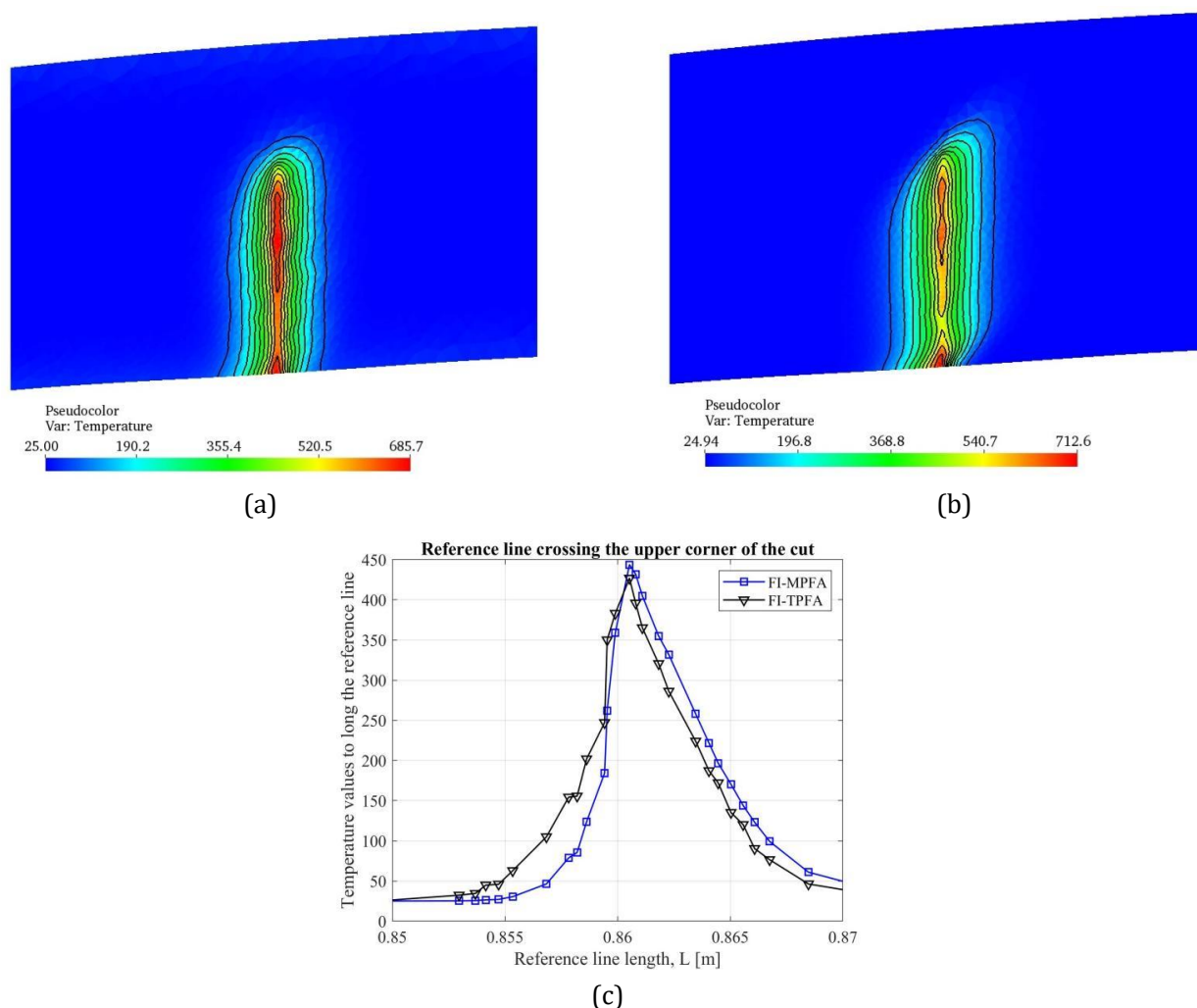


Fig. 5. Temperature field due to laser cutting procedure in a wind turbine blade: (a) solution obtained with FI-TPFA; (b) solution obtained with FI-MPFA; (c) temperature distribution in a line crossing the upper border of the cut.

In Fig. 5 we can see the temperature field for the simulation with both methods as well as the distribution of temperature for an imaginary line crossing the upper border of the cut. It is clear that both methods produce different numerical solutions, even though we also observe that, for this configuration, the difference is not so pronounced in terms of HAZ delimitation.

5. Conclusions

In this paper we compare the FI-MPFA with a conventional FI-TPFA. In a theoretical problem, even though we have observed equivalent behaviour in terms of convergence rate for Cartesian mesh, we also observed an incapability of the FI-TPFA to obtain convergent solutions when a distorted mesh was adopted in the domain discretization. On the other hand, FI-MPFA satisfactorily converged, except when bigger time step were used. Regarding an applied problem, such as expected, both methods produced reasonable difference in terms of absolute values of temperature, but with a similar HAZ delimitation.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Larissa Mendes H. Rocha and Márcio R. A. Souza conducted the research and wrote the paper; Tiago Fonseca Costa reviewed the literature; Synara B. Pereira wrote the paper; Erlon Rabelo Cordeiro conducted the data analysis; all authors had approved the final version.

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References

- [1] Bank, L. C., Arias, F. R., Yazdanbakhsh, A., Gentry, T. R., Al-Haddad, T., Chen, J. F., & Morrow, R. (2018). Concepts for reusing composite materials from decommissioned wind turbine blades in affordable housing. *Recycling*, 3(1), 11.
- [2] Kalkanis, K., Psomopoulos, C. S., Kaminaris, S., Ioannidis, G., & Pachos, P. (2019). Wind turbine blade composite materials-End of life treatment methods. *Energy Procedia*, 157, 8.
- [3] Pan, C. T., & Hocheng, H. (2001). Evaluation of anisotropic thermal conductivity for unidirectional FRP in laser machining. *Compos. - A: Appl. Sci. Manuf.*, 32(11), 11.
- [4] Aavatsmark, I. (2002). An introduction to multipoint flux approximations for quadrilateral grids. *Comput. Geosci.*, 6(3), 27.
- [5] Liu, P., & Barlow, C. Y. (2017). Wind turbine blade waste in 2050. *J. Waste Manag.*, 62, 11.
- [6] Eigestad, G. T., & Klausen, R. A. (2005). On the convergence of the multi-point flux approximation O-method: Numerical experiments for discontinuous permeability. *Numer. Methods. Partial. Differ. Equ.*, 21(6), 19.
- [7] Yao, W., Yu, B., Gao, X., & Gao, Q. (2014). A precise integration boundary element method for solving transient heat conduction problems. *Int. J. of Heat and Mass Transf.*, 78, 8.
- [8] Glasfiber, L. M. (2009). Technical blade specification. Project LM 48.8 P.
- [9] Bergman, T. L., Incropera, F. P., DeWitt, D. P., & Lavine, A. S. (2011). *Fundamentals of Heat and Mass Transfer*. John Wiley & Sons.

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