Determination of the Axial-Vector Coupling Constant from the Extended Linear Sigma Model

Tarek Sayed Taha Ali

***Abstract*—We reexamine the work of Rashdan et al., who considered a chiral model for the nucleon based on the linear sigma model with scalar-isoscalar scalar-isovector mesons coupled to quarks. The dependence of the axial-vector coupling constant g_A and the pion nucleon coupling constant $g_{\pi NN}$ on the quark masses and sigma masses have been investigated in the frame work of the extended linear sigma model. In this work we calculate both of g_A and $g_{\pi NN}$ to investigate the effect of the quark masses on the g_A in the framework of the extended linear sigma model, which is proposed by Rashdan et al. and compare it with the free Skyrmion model, extended Skyrmion model and finally with Birse and Banerjee model. The field equations have been solved in the mean-field approximation by Goldflam and Wilets. Our study shows a better fitting to the experimental data compared with the existing models.**

***Index Terms*—Extended linear sigma model, axial vector coupling constant, quark mass and mean field approximation.**

I. Introduction

The axial-vector coupling constant g_A is important to understand Quantum Chromodynamics (QCD). In recent years there has been a growing interest in studying g_A. A lot of groups have made significant progress towards understanding g_A using several models (see Cloet *et al.* [1] and Ali *et al*. [2]). We study the extended linear sigma model as one of these models to describe the interactions of quarks and $\overleftarrow{\pi}$ meson in a mean field approximation which has the hedgehog property. A similar model has been considered by Kalbermann and Eisenberg [3], Birse and Banerjee [4], while the higher order of the mesonic inteactions in the linear sigma model was considered by Sahu and Ohnishi [5], [6] and M.Rashdan *et al*. [7]-[9], who used the mean field approximation to get a better description of the g_A. In our study [10], we used the coherent pair approximation to study the g_A. Few solutions for the lagrangian of chiral linear soliton models applied to the nucleon have already been suggested. The mean-field equations are a straightforward extension of the finding by Goldflam and Wilets [11]. In this work, we consider a model based on the idea of strong QCD forces. The aim is to investigate the effect of the quark masses on the g_A in the framework of the extended linear sigma model, which is proposed by Rashdan *et al*. [7] with prameters like the pion decay constant $f_\pi = 91.9$ MeV and the pion mass $m_\pi = 138.04$ MeV fixed in similar way as Struber and Rischke [12]. The paper is organized as follows; first, the explaination of extended linear sigma model in Section II, the numerical results and the discussion in Section III , and finally, the conclusion presented in Section IV.

Manuscript received January 10, 2014; revised April 4, 2014.

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II. The Extended Linear Sigma Model

The extended linear sigma model is described in details in [7]. We describe the interactions of quarks with $\sigma-$ mesons and pions by Birse and Banerjee [4]. The Lagrangian density is,

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}\left((\sigma^2+\vec{\pi}^2)^2-\nu^2\right)^2 + m_\pi^2 f_\pi \sigma, \quad (1)$$

It is clear that this potential also satisfies chiral symmetry. Applying the PCAC we get,

$$\nu^2 = f_\pi^4 - \frac{m_\pi^2}{2\lambda^2 f_\pi^2}, \quad (2)$$

and

$$\lambda^2 = \frac{m_\sigma^2 - 3m_\pi^2}{8 f_\pi^6}. \quad (3)$$

Now, we expand the extremum, with the shifted field defined as

$$\sigma = \sigma' - f_\pi, \quad (4)$$

inserting equ.(4) into equ.(1), we obtain

$$L(r) = i\overline{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}\left(\partial_\mu\sigma'\partial^\mu\sigma' + \partial_\mu\overleftarrow{\pi}.\partial^\mu\overleftarrow{\pi}\right) - g\overline{\Psi}f_\pi\Psi + g\overline{\Psi}\sigma'\Psi + ig\overline{\Psi}\gamma_5\vec{\tau}.\overleftarrow{\pi}\Psi - U(\sigma',\pi), \quad (5)$$

with

$$U(\sigma',\vec{\pi}) = \frac{\lambda^2}{4}\left(\left((\sigma'-f_\pi)^2+\vec{\pi}^2\right)^2-\nu^2\right)^2 + m_\pi^2 f_\pi\sigma' - m_\pi^2 f_\pi^2. \quad (6)$$

The time-independent fields $\sigma'(r)$ and $\vec{\pi}(r)$ are to satisfy

DOI: 10.7763/IJAPM.2014.V4.280

the Euler-Lagrangian equations, and the quark wave function satisfies the Dirac eigenvalue equation. The meson field equations are written as:

$$\Delta\sigma' = g\overline{\Psi}\Psi - 2\lambda^2(\sigma' - f_\pi)\left((\sigma' - f_\pi)^2 + \vec{\pi}^2\right)\left(\left((\sigma' - f_\pi)^2 + \vec{\pi}^2\right)^2 - \nu^2\right) - m_\pi^2 f_\pi, \quad (7)$$

$$\Delta\vec{\pi} = ig\overline{\Psi}\gamma_5 \cdot \vec{\tau}\,\Psi - 2\lambda^2\vec{\pi}\left((\sigma' - f_\pi)^2 + \vec{\pi}^2\right)\left(\left((\sigma' - f_\pi)^2 + \vec{\pi}^2\right)^2 - \nu^2\right), \quad (8)$$

where $\vec{\tau}$ refers to Pauli isospin matrices,

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

We used Hedgehog Ansatz in the pion field [4] where

$$\vec{\pi}(r) = \hat{r}\,\pi(r). \quad (9)$$

Now, the pion isospin and space angular momentum are correlated because the quark source terms are themselves correlated corresponding to $SU_{spin}(2) X\, SU_{isospin}(2)$ wave functions. This will be established using hedgehog ansatz, which breaks $\vec{I}$ symmetry and breaks $\vec{J}$ symmetry, but conserves the Grand spin $\vec{G}$

$$\vec{G} = \vec{J} + \vec{I}. \quad (10)$$

The Dirac equation for the quarks are

$$\frac{du}{dr} = -p(r)u + \left(E - m_q + S(r)\right)w, \quad (11)$$

$$\frac{dw}{dr} = -\left(E - m_q + S(r)\right)u + \left(\frac{2}{r} - p(r)\right)w, \quad (12)$$

where $S(r) = g\langle\sigma'\rangle$, $p(r) = g\langle\vec{\pi}.\hat{r}\rangle$, E are the scalar potential, the pseudoscalar potential and the eigenvalue of the quarks spinor Ψ, respectively. Including the color degrees of freedom, one has $g\overline{\Psi}\Psi \to N_c g\overline{\Psi}\Psi$ where $N_c = 3$ colors, g is *the coupling constant*. The Dirac wave functions $\Psi(r)$ and $\overline{\Psi}(r)$ are given by

$$\Psi(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) \\ iw(r) \end{bmatrix}$$

and (13)

$$\overline{\Psi}(r) = \frac{1}{\sqrt{4\pi}}\left[u(r) \quad iw(r)\right],$$

and the sigma, pion and vector densities are given by

$$\rho_s = N_c g\overline{\Psi}\Psi = \frac{3g}{4\pi}\left(u^2 - w^2\right), \quad (14)$$

$$\rho_p = iN_c g\overline{\Psi}\gamma_5.\vec{\tau}\Psi = \frac{3}{4\pi}g(-2uw), \quad (15)$$

$$\rho_v = \frac{3g}{4\pi}\left(u^2 + w^2\right). \quad (16)$$

These equations are subject to the boundary conditions that asymptotically the fields approach their vacuum values,

$$\sigma(r) \approx -f_\pi \text{ MeV} \quad \pi(r) \approx 0 \quad \text{at} \quad r \to \infty. \quad (17)$$

III. Numerical Calculations and Discussion

A. *The Scalar Field* σ'

To solve equ.(7), we integrate a suitable Green's function over the source fields [13]-[17]. Thus

$$\sigma'(\vec{r}) = \int d^3\vec{r}'\,D_\sigma(\vec{r} - \vec{r}')(g\rho_s(\vec{r}') - 2\lambda^2(\sigma' - f_\pi)\times ((\sigma' - f_\pi)^2 + \vec{\pi}^2)\times (((\sigma' - f_\pi)^2 + \vec{\pi}^2)^2 - \nu^2) - m_\pi^2 f_\pi), \quad (18)$$

where

$$D_\sigma(\vec{r} - \vec{r}') = \frac{1}{4\pi|\vec{r} - \vec{r}'|}\exp\left(-m_\sigma|\vec{r} - \vec{r}'|\right). \quad (19)$$

The scalar field is spherical in this model as we only need the $l = 0$, therefore

$$\sigma'(\vec{r}) = \int d^3\vec{r}'\,D_\sigma(\vec{r} - \vec{r}'))(g\rho_s(\vec{r}')) + \lambda^2 f_\pi\left(3\sigma'^2 + \pi^2(\vec{r}')\right) - \lambda^2\left(\sigma'(r'^2) + \pi^2(\vec{r}')\right)\sigma'(r'), \quad (20)$$

Note that this form is implicit in that the solution of σ' involves integrals over the unknown σ' itself. We will solve these implicit integral equation by iterating to self consistency.

B. *The Pion Field* π

To solve equ.(8), we integrate a suitable Green's function over the source fields [16]-[18]. We use $l = 1$ component of the pion Green's function. thus

$$\vec{\pi}(\vec{r}) = m_\pi\int_0^\infty r'^2 dr' \times \frac{(-\sinh(m_\pi r_<) + m_\pi r_< \cosh(m_\pi r_<))}{(m_\pi r_>)^2} \times (1 + \frac{1}{m_\pi r_>})\frac{\exp(-m_\pi r_>)}{m_\pi r_>} \times (g\rho_p - 2\lambda_1^2\vec{\pi}((\sigma' - f_\pi)^2 + \vec{\pi}^2) \times (((\sigma' - f_\pi)^2 + \vec{\pi}^2)^2 - \nu^2))), \quad (21)$$

We have solved the equations of (11, 12) fields using fourth order Rung-Kutta. Due to the nonlinearly of these equations it is necessary to iterate the solution until self-consistency is achieved. To start this iteration process, we use the chiral circle form for the meson fields

$$S(r) = m_q(1-\cos\theta), \tag{22}$$

$$p(r) = -m_q \sin\theta, \tag{23}$$

where $\theta = \pi \tanh r$, and m_q is quark mass

C. Nucleon Axial Vector

The nucleon axial-vector coupling constant is found from

$$\frac{1}{2} g_A(0) = < P\uparrow | \int d\vec{r}\, A^z{}_3(\vec{r}) \,|\, P\uparrow >, \tag{24}$$

where the z-component of the axial vector current is given by

$$A^z{}_3(\overleftarrow{r}) = \overline{\Psi}(\vec{r}) \frac{1}{2}\gamma_5\gamma^3\tau_3\Psi(\vec{r}) - \sigma(\vec{r})\frac{\partial}{\partial z}\pi_3(\vec{r}) + \pi_3(\vec{r})\frac{\partial}{\partial z}\sigma(\vec{r}), \tag{25}$$

$$g_A^{quark} = \frac{5}{3}\int_0^\infty r^2 dr(u(r)^2 - \frac{1}{3}w(r)^2) \tag{26}$$

$$g_A^{meson} = -\frac{8\pi}{9}\int_0^\infty r^2 dr[\pi(r)\frac{d}{dr}\sigma(r) - \sigma(r)(\frac{d}{dr}+\frac{2}{r})\pi(r)]. \tag{27}$$

To calculate *the pion-nucleon coupling constant*, we consider the limit $\overleftarrow{q} \to 0$ of

$$\frac{g_{\pi NN(0)}}{M_B} = < P\uparrow | \int d\vec{r}\, e^{i\vec{q}\bullet\vec{r}} j_{\pi 3}(\vec{r}) \,|\, P\uparrow > \tag{28}$$

where the pion source current is defined by [4]-[21]

$$(\Delta + m_\pi^2)\pi_3(\vec{r}) = j_{\pi 3}(\vec{r}) \tag{29}$$

Therefore (For details see Birse and Banerjee [4] and M. Rashdan *et al.* [7]

$$(\frac{g_{\pi NN(0)}}{M_B})^{quark} = -\frac{10}{9}\int d\vec{r} u(\vec{r})\, w(\vec{r}) \tag{30}$$

$$(\frac{g_{\pi NN(0)}}{M_B})^{meson} = \frac{4\pi}{3}\lambda^2 \int d\vec{r}\pi(\vec{r})(2 f_\pi\sigma - \sigma^2 - \pi^2) \tag{31}$$

IV. Conclusion

TABLE I: Values of $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_B}$. At $m_q = 460$ MeV. All Quantities in MeV

m_σ (MeV)	700	900	1100	1200	Exp.
$g_{\pi NN}(0)\frac{m_\pi}{2M_B}$	1.19	1.14	1.1	1.09	1.00
$g_A(0)$	1.76	1.79	1.82	1.80	1.25

TABLE II: Values of $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_B}$. At $m_\sigma = 1000$ MeV. All quantities in MeV

m_q(MeV)	440	420	400	380	Exp.
$g_{\pi NN}(0)\frac{m_\pi}{2M_B}$	1.44	1.40	1.35	1.29	1.00
$g_A(0)$	1.82	1.80	1.78	1.74	1.25

TABLE III: Values of $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_B}$ Calculated for the Free SKYRMION Model [22], Extended Skyrmion Model [23], Birse and Banerjee [4] Which Are Compared with Our Calculations

Observable	[24]	[22]	[23]	Our work
$g_{\pi NN}(0)\frac{m_\pi}{2M_B}$	0.76	0.61	1.29	1.09
$g_A(0)$	1.01	0.78	1.85	1.80

The field equations (7, 8, 11, 12) have been solved by iteration for different values of quark and sigma masses [19], [20]. Table I shows the values of $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_B}$ calculated for $m_q = 460$ MeV and for different values of m_σ [19]. As shown in Table I below, the increase in the order of the mesonic interaction greatly modifiey the values of $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_B}$. In comparison with the linear sigma model of Gell-Mann and Levy [15] and Birse and Banerjee [4], the quark and sigma masses are taken as 500 MeV and 1200 MeV, respectively; in which m_q is larger than that commonly quoted [13]-[15]. This is due to the fact that in [16], [17], bound solutions have only been obtained for $3.9 < g < 4.55$ and to the less degree of the mesonic interaction which has been taken in the lowest order. Increasing the order of this interaction has resulted in the erradication of our problem, where we obtained good results for more reasonable values for the quark and sigma masses. The results obtained for $m_\sigma = 1000$ MeV and for different values of m_q are presented in Table II. This value of the quark mass is consistent with the value deduced from NJL soliton models similar to [22]. Furthermore, the values of $g_A(0)$ and $g_{\pi NN}(0)\frac{m_\pi}{2M_B}$ are more adequately reproduced than that of Birse and Banarjee [4] who used standard potential and Skyrme model [22], [23] as seen from Table III. In conclusion, the present calculations show the importantance of mesonic correlations which, may be of higher-order than that normally used in most soliton models.

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