

# The Quantum Hall Effect: Interpretation of the Experimental Data

Keshav N. Shrivastava, *Senior Member, IACSIT*

**Abstract**—The theory of integers and fractions at which plateaus arise in the quantum Hall effect is explained. The experimental values are noted and explained by using the theory of angular momentum in quantum mechanics. The special treatment of spin introduced by this work eliminates the Lande's formula of g values and introduces a formula which is linear in the angular momentum variables, L, S and J. At high fields, the +S states are symmetric with the -S states so that when there is a plateau at + state there is also one at -S state. A lot of states require that the Landau levels are modified by this spin effect.

**Index Terms**—Quantum Hall effect, modified Lande's formula, landau levels, symmetric spin effect.

## I. INTRODUCTION

The preliminary experimental work on the detection of fractions and integers at which plateaus occur in the quantum Hall effect was done by von Klitzing, Dorda and Pepper [1] and by Tsui, Stormer and Gossard [2]. It was thought that the wave function of electrons should be two dimensional as the Laughlin's wave function is [3]. Anderson [4] and Schrieffer [5] thought that Laughlin's wave function will provide a prototype wave function which will form the basic theory of charge fractionalization. Whether the charge fractionalizes in the quantum Hall effect or not by electron correlations is a different question but we have found that all of the data can be explained on the basis of spin symmetry and the angular momentum and not by Laughlin's wave function. Hence, the charge fractionalization does not occur by Laughlin's correlations. Later work showed that Laughlin's wave function is a zero-energy ground state of a very unusual potential which is unlikely to occur in solids and hence will not be useful to interpret the experimental data. The Laughlin's potential cannot be transformed into a Coulomb potential. They never claimed that Laughlin's wave function is the ground state of Coulomb's potential. Simply, we find it convenient and have understanding of the Coulomb potential as a fundamental law of nature. It will be a great service to the physics community if we can prove the equivalence between our angular momentum theory and the Laughlin's theory. It seems that it will be very difficult to find such an equivalence if it exists at all. What is the Hall effect resistivity? We are able to explain the Hall effect without the need of a wave function [6]. That means that the wave functions are hydrogen type, made from Legendre's polynomials with suitable modifications. The Hall resistivity is a linear

function of magnetic field. In real systems it is found to have fractional values which require understanding. We find that the size of the devices is often only a few nm and temperatures of measurements are quite low such as mK. Our theory explains all of the data correctly [7]-[21].

## II. ELEMENTARY THEORY

We define the cyclotron frequency as,

$$g\mu_B B = \hbar\omega \quad (1)$$

The Bohr magneton is,

$$\mu_B = \frac{e\hbar}{2mc} \quad (2)$$

Hence,

$$\frac{g}{2} \frac{eB}{mc} = \omega_c \quad (3)$$

Since for  $L=0$ ,  $g=2$ , it is a common practice to define the cyclotron frequency as,

$$\omega_c = \frac{eB}{mc} \quad (4)$$

The electrons in a magnetic field behave like harmonic oscillators. Hence, the energy of a state is given by,

$$E_n = \hbar\omega_c \left(n + \frac{1}{2}\right) = \hbar \frac{eB}{mc} \left(n + \frac{1}{2}\right) \quad (5)$$

It is proper to include the factor of  $g/2$  so that the above energy is modified to,

$$E_n = \frac{1}{2} g\hbar \frac{eB}{mc} \left(n + \frac{1}{2}\right). \quad (6)$$

The same effect can be obtained by replacing  $e$  by  $e^* = (1/2)ge$ , so that the energy can be written as,

$$E_n = \hbar \frac{e^*B}{mc} \left(n + \frac{1}{2}\right). \quad (7)$$

The transition can occur from  $E_n$  to  $E_{n'}$ . The Hall effect resistivity is,

$$\rho_{xy} = \frac{B}{nec} = \frac{B}{ne^*c} \quad (8)$$

Hence the effective charge can be measured. The flux quantization is given by,

Manuscript received August 2, 2012; revised September 21, 2012  
The author is with the University of Hyderabad, India, also University of Malaya, Kuala Lumpur, Malaysia. (e-mail: keshav1001@yahoo.com).

$$B.A = n' \frac{hc}{e} \quad (9)$$

where  $A$  is the area in which flux is quantized. Substituting this field in the Hall effect formula gives,

$$\rho_{xy} = \frac{n'hc}{ne^*ceA} = \frac{n'}{nA} \frac{h}{ee^*} = n'' \frac{h}{\frac{1}{2}ge^2} \quad (10)$$

Which gives the quantum Hall effect. We do not use the Lande's formula but suggest a formula linear in the angular momentum. This formula gives,

$$g = \frac{2j+1}{2l+1} \quad (11)$$

Here,  $j = l \pm s$  is the total angular momentum quantum number. The effective charge is thus,

$$v_{\pm} = \frac{1}{2} g_{\pm} e = \frac{l + \frac{1}{2} \pm s}{2l+1} \quad (12)$$

For positive sign and  $s=1/2$ ,

$$v_{+} = \frac{l+1}{2l+1} \quad (13)$$

and for the negative sign and  $s=1/2$ ,

$$v_{-} = \frac{l}{2l+1} \quad (14)$$

Here ,

$$v_{\pm} = (1/2)ge \quad (15)$$

gives the correct fractional charges such as,

For  $l=1$ ,  $v_{-} = 1/3$  and  $v_{+} = 2/3$ , etc. These values are the same as tabulated in 1985 which agree with experimental data. We call these as "principal fractions". Instead of the principal values, we can also generate the resonances,  $V_1 - V_2$ . This process produces some more fractions not already present in the "principal fractions", except in the case when we consider the energy level difference  $E_1 - E_2$  with  $E_2 = 0$ . Indeed, there is a zero energy state for  $l=0$ , with negative sign,  $s=1/2$ ,  $v_{-} = 0$ . Many resonances are in fact present in the experimental data and indeed predicted from the linear theory. At low temperatures, the excitation populations are small so that interactions are minimized. Hence we predict the "resonances". We are thus able to produce a large number of principal fractions and resonances so that "two-particle states" occur. For these particles we have  $v_1 + v_2$  so that these processes produce more fractions than are found in resonances. Hence we have (i) principal fractions, (ii) resonances and the (iii) two-particle states. The real material is often having electron clusters so that the spin need not be  $1/2$ .

### III. CLUSTERS

In real materials aluminium is mixed in gallium arsenide so that the film is not a single crystal and clusters of atoms are formed. The electron clusters are formed in between spaces of the atomic clusters. In these clusters spin becomes

$$NS = (N_{\uparrow} - N_{\downarrow})S \quad (16)$$

Hence the spin may be  $3/2$ ,  $5/2$ ,  $2$  or  $3$ . For example for,

$$l=0 \quad (17)$$

The fractional charge can occur at,

$$e^* = \frac{1}{2} \pm s \quad (18)$$

For finite  $l$ ,

$$e^*/e = \frac{l + \frac{1}{2} \pm s}{2l+1} \quad (19)$$

For

$$N_{\uparrow} - N_{\downarrow} = 0, \quad (20)$$

$S=0$ , so that the effective charge becomes,

$$e^*/e = \frac{1}{2} \frac{2l+1}{2l+1} = \frac{1}{2} \quad (21)$$

Thus only half of the particle is seen in the resistivity which has even denominator. For  $S=1$ ,

$$e^*/e = \frac{l + \frac{1}{2} \pm 1}{2l+1} = \frac{l + \frac{3}{2}}{2l+1}, \frac{l - \frac{1}{2}}{2l+1} \quad (22)$$

Which has even denominators and occurs in pairs. We have explained 101 plateaus correctly by this theory. The fractions of charge which are measured by using  $\rho_{xy}$  are usually the same as those measured by using  $\rho_{xx}$ . The fractions occur in the form of (i) principal fractions, (ii) resonances, (iii) sum process and those of the (iv) electron clusters. In the case of electron clusters, the fractions derived from  $xx$  direction are slightly different from those found from  $xy$  values. The effective charge of the electron becomes anisotropic due to the spin wave propagation in the micro-cluster of electrons. The wave vector of the spin waves appears in the effective value of the spin

$$S_{\text{effective}} = S - \delta S \quad (23)$$

There is an explicit dependence of the charge on the spin which modifies the condition of the flux quantization and leads to an anisotropic charge. In the Hall effect, the resistivity is a linear function of magnetic field. When field is quantized there occur plateaus in the  $\rho_{xy}$  and minima in  $\rho_{xx}$ .

### IV. SPIN WAVES

The Hamiltonian of the ferromagnetic spin waves with exchange interaction is given by,

$$H = -J\sum_{j\delta} S_j \cdot S_{j+\delta} - g\mu_B H \sum_j S_{jz} \quad (24)$$

where  $J$  is the exchange interaction and  $H$  is the magnetic field.  $S_j$  are the spin operators at the  $j$ th site and the summation can be carried out to nearest neighbors,  $S_{j+\delta}$ . The number of nearest neighbors is  $z$  so that the Fourier transforms of spin operators requires,

$$\gamma_k = \frac{1}{z} \sum_{\delta} \exp(ik \cdot \delta) \quad (25)$$

Leaving out the magnetic field dependent term, the unperturbed frequency of a magnon is,

$$\hbar\omega_k = 2JzS(1 - \gamma_k) \quad (26)$$

Hence, the spin has been changed from  $S$  to  $S(1 - \gamma_k)$  in going from the site variables to spin wave variables. This is called the spin deviation and it amounts to a few per cent in real materials. For  $k \cdot \delta \ll 1$ ,  $\exp(ik \cdot \delta)$  has only the sine term and the cos term is zero so that for small wave vectors,

$$z(1 - \gamma_k) \approx \frac{1}{2} \sum_k (k \cdot \delta)^2 \quad (27)$$

so that

$$\hbar\omega_k \approx JS \sum_k (k \cdot \delta)^2 \quad (28)$$

Which for cubic lattices is  $2JS(k.a)^2$  where  $a$  is the lattice constant. Hence, the spin is changed to  $S(k.a)^2$ . In the case of two-sublattice antiferromagnets,  $\hbar\omega_k \approx 4\sqrt{3}JSka$ . In a cluster, in the  $a$ - $b$  plane, the frequency of a magnon is given by the expression  $2JS[(k.a)^2 + (k.b)^2]$  whereas along the  $x$  direction it appears as  $2JS(k.a)^2$ . Hence, the spin value becomes anisotropic. The Hall effect resistivity is,

$$\rho_{xy} = \frac{h}{2ge^2} = \frac{h}{\frac{l + \frac{1}{2} \pm s}{2} e^2} \quad (29)$$

In the case of an electron cluster, the value of  $S$  along  $xx$  direction is  $NS$  which due to spin waves becomes  $S(k.a)^2 N$  whereas along the  $xy$  direction it will be  $\frac{1}{2} S[(k.a)^2 + (k.b)^2] N$ . Hence the value of  $(1/2)g$  depends of the direction. The value of the filling factor  $\nu = (1/2)g$  also depends on the direction. Hence  $\nu_{xx}$  need not be equal to  $\nu_{xy}$  where  $\nu_{xx}$  is deduced from  $\rho_{xx}$  and  $\nu_{xy}$  from  $\rho_{xy}$ .

Kumar *et al.* [22] have found that in the  $xx$  direction  $\nu = 2.463 \pm 0.002$  whereas in the  $xy$  direction  $\nu = 2.461$ . Hence, the value deduced from the  $xy$  is slightly different from that deduced from the  $xx$  which we assign to the value of the spin due to spin waves. For  $L=0$ , the filling fraction is

$$\frac{1}{2} \pm s = \frac{2463}{1000} \quad (30)$$

Hence  $\pm s = 1.963$  which shows 1.8 % spin deviation compared with  $S=2$ . The Hall effect experiment is performed on a polycrystalline heterostructure film so that the crystallographic directions are not the same as those used in the Hall effect. Thus spin deviation occurs in the Hall effect of clusters. The effective spin is found to be slightly different from the integer value,

$$S - \delta S = 2 - 0.04 = 1.96 \quad (31)$$

## V. DATA INTERPRETATION

Usually, the Lande's formula for the  $g$  value uses the squares of the angular momenta operators and spin is positive only. In 1985, we decided to use linear expression and allow both signs for the spin. Therefore, we obtained an expression in the form of angular momenta variables which multiply the Bohr magneton so that the charge can be replaced by an effective value. In this way, we obtained the fractional charges. All of the calculated values are in agreement with the experimental data. Because of the two signs of the spin, there are symmetries in the data which are well explained by the theory [6]. For the present purpose, we mention that the  $g$  values obtained in the previous paper [6] are given by,

$$\frac{1}{2} g = \frac{l + \frac{1}{2} \pm s}{2l + 1} \quad (32)$$

Which are just the  $g = (2j+1)/(2l+1)$  for  $j = l \pm s$ . Accordingly, for  $s=1/2$ ,

$$\frac{1}{2} g_- = \frac{l}{2l + 1} \quad (33)$$

$$\frac{1}{2} g_+ = \frac{l + 1}{2l + 1} \quad (34)$$

*Type 1.* For  $l = 0$ ,  $(1/2)g_- = 0$  and  $(1/2)g_+ = 1$ . These values are very important which we will need at some time. In particular, the zero value of  $(1/2)g_-$  will be useful. For finite values of  $l$  the two series are given in Table I. All of the 19 values are the same as those reported by Pan *et al.* [23]. We have included three more values which are the predicted values, perhaps not yet reported but we believe that these values are correct. It may also be noted that the values given in Table I are pairwise and they are Kramers conjugates of each other and may appear as particle-hole symmetric states. According to our scheme,

$$g\mu_B H = \hbar\omega \quad (35)$$

so that substituting the value of Bohr magneton, we find,

$$g \frac{e\hbar}{2mc} H = \hbar\omega \quad (36)$$

so that the cyclotron frequency becomes,

$$\omega = \frac{g eH}{2 mc} \quad (37)$$

Therefore,  $\hbar\omega$  should be multiplied by  $(1/2)g$ . The values of  $(1/2)g$  are given in Table I for both signs of spin. The effective charge is  $e^*=(1/2)ge$ . Hence, the value of  $(1/2)g$  appears in the resistivity as the effective charge. When the energy of a state is given by  $(1/2)g\hbar\omega$ , then the energy level difference,  $E= (1/2)g_1 - (1/2)g_2$  also becomes an allowed transition. We consider the Landau levels at,

$$E=(n+\frac{1}{2})\frac{1}{2}g\hbar\omega \quad (38)$$

In the case of two oscillators with equal  $g$  values,

$$E=(n_1+\frac{1}{2})\frac{1}{2}g\hbar\omega-(n_2+\frac{1}{2})\frac{1}{2}g\hbar\omega=(n_1-n_2)\frac{1}{2}g\hbar\omega \quad (39)$$

which is an integer multiple of  $(1/2)g$ . So when a state occurs at  $(1/2)g$ , then for  $n_1 - n_2 = 2$ ,  $g\hbar\omega$  also becomes allowed and it is due to the Landau level type effect.

Type 2. If one particle has energy  $\hbar\omega$ , then two particle state will occur at  $\hbar(\omega_1 + \omega_2) = 2\hbar\omega$ . When a particle has energy  $2/3$ , then two particle state will be at  $(2/3)+(2/3)=4/3$ . Now there is a particle at  $4/3$  and there was already at  $1/3$ , then two particle state occurs at  $(4/3)+(1/3)=5/3$ . Similarly,  $(5/3)+(2/3)=7/3$  and  $(1/3)+(7/3)=8/3$ . As we keep adding particles, the states become weak due to low probability of occurring. This explains  $4/3$ ,  $5/3$ ,  $7/3$  and  $8/3$  and predicts two weak states at  $10/3$  and  $14/3$ . In Table I,  $\nu_1 + \nu_2 = 1$  exists but now these states have much higher filling factors than 1 so they are two particle states but not particle-hole symmetric states. The two-particle state of  $2/5$  is at  $4/5$ , and the two particle state of  $2/5$  and  $4/5$  is at  $(2/5)+(4/5) = 6/5$ , which can be taken as a three particle state. Similarly,  $(3/5)+(4/5) = 7/5$ ,  $(4/5)+(4/5) = 8/5$ ,  $(3/5)+(6/5) = 9/5$ ,  $(2/5)+(9/5) = 11/5$ ,  $(3/5)+(9/5) = 12/5$ ,  $(1/5)+(12/5) = 13/5$ ,  $(13/5)+(1/5) = 14/5$ ,  $(4/5)+(12/5) = 16/5$ ,  $(6/5) + (13/5) = 19/5$ ,  $(9/5)+(12/5)=21/5$ ,  $(12/5)+(12/5) = 24/5$ . This explains the two particle states at  $4/5$ ,  $6/5$ ,  $7/5$ ,  $8/5$ ,  $9/5$ ,  $11/5$ ,  $12/5$ ,  $13/5$ ,  $14/5$ ,  $16/5$ ,  $19/5$ ,  $21/5$ ,  $24/5$  and predicts  $17/5 = 6/5+11/5$ ,  $18/5=6/5+12/5$ ,  $22/5= 11/5+11/5$ . and  $23/5=11/5+12/5$ . As the number of particles increases, the states become weaker. For example, 2 particle states are stronger than 3 particle states.

The difference states. In eq.(39),  $n_1 - n_2 = 1$  leads to states at  $(1/2)g\hbar\omega$  and  $n_1 - n_2 = 2$  gives states at  $g\hbar\omega$ . We can get  $E_1 - E_2$  type states such as of the form  $\frac{1}{2}g_1 - \frac{1}{2}g_2$ . We already have  $3/7$  and  $4/7$  in Table I. the difference between these two energies is  $4/7-3/7=1/7$ ,  $3/7-1/7 =2/7$ . The sum of the states are at  $\omega = \omega_1 + \omega_2$ . Hence  $2/7+3/7 =5/7$ ,  $4/7+5/7= 9/7$ ,  $5/7+5/7=10/7$ ,  $2/7 +9/7=11/7$ ,  $3/7+9/7=12/7$ ,  $5/7+11/7= 16/7$ ,  $9/7+10/7= 19/7$ . Some of the states are given as follows:

$$\begin{aligned} 10/7-9/7=1/7, \\ 2/7+9/7=11/7, \\ 3/7+9/7=12/7, \end{aligned}$$

$$\begin{aligned} 9/7+2/7=11/7, \\ 5/7+11/7= 16/7, \\ 9/7+10/7=19/7. \end{aligned}$$

Which are given in Table II. The  $3/7$  and  $4/7$  given in Table I, can generate all of these fractions by consideration of differences (transitions) and sums (two-particle states) of the two energies at a time. We see that,  $5/9-4/9=1/9$  which is a transition and  $1/9+1/9=2/9$  is a two particle state. The particles obtained by confluence are  $2/9+5/9=7/9$ ,  $7/9+4/9=11/9$ ,  $11/9+2/9=13/9$ ,  $7/9+7/9=14/9$  and  $11/9+14/9=25/9$ , which are two particle states. The transition state also produces  $6/11-5/11=1/11$ ,  $1/11+1/11= 2/11$ ,  $1/11+2/11=3/11$ ,  $2/11+2/11=4/11$ ,  $3/11+4/11=7/11$ ,  $4/11+4/11=8/11$ ,  $6/11+8/11=14/11$ ,  $8/11+8/11=16/11$ ,  $14/11+3/11=17/11$ . These predicted fractions are given in Table III. The transition energy between  $7/13$  and  $6/13$  is  $1/13$ . The two particle state of  $1/13$  is at  $2/13$ . Further, particle states are  $1/13+2/13=3/13$ ,  $3/13+1/13=4/13$ ,  $4/13+1/13=5/13$ ,  $5/13+5/13=10/13$  and  $10/13+10/13 =20/13$ . The  $19/13$  may be a three particle state  $10/13+7/13+2/13=19/13$  which will be weak. The transition state between  $8/15$  and  $7/15$  is at  $1/15$ . A two particle state occurs at  $2/15$ . The two particle states occur at  $2/15+2/15=4/15$ ,  $4/15+7/15=11/15$ ,  $11/15+11/15=22/15$  and  $22/15+1/15=23/15$ . The experiment picks up a lot of two particle states. For  $l > 8$ , the number of populated states becomes small and when  $l >12$ , the population becomes so small that they are not observed. The non-observation of some of the fractions is therefore due to the very small population. The states with  $l = 8$  from eqs. (36) and (37) are  $8/17$  and  $9/17$  and the difference or transition energy is  $1/17$ . The two-particle state of this transition is at  $2/17$ . The two particle states occur at,  $2/17+1/17=3/17$ ,  $2/17+2/17=4/17$ ,  $3/17+2/17 =5/17$  and  $3/17+3/17=6/17$ . For  $l = 9$ , the states are  $9/19$  and  $10/19$ , the transition state of these states is  $1/19$  and its two particle state occurs at  $2/19$ . The two-particle state of  $1/19+2/19$  occurs at  $3/19$ . The two-particle state of  $2/19+2/19$  occurs at  $4/19$ . The two-particle state of  $3/19+2/19$  occurs at  $5/19$  as given in Table IV. Obviously, these states are weak because of large  $l$  and because of combinations involved. Every time we compose a particle, the probability decreases with increase in the number of particles. It is possible to have two particle state with  $5/19+5/19=10/19$ . For  $l =10$ ,  $2l+1=21$  and the predicted energy is  $(1/2)g$  in units of  $\hbar\omega$ . The transition energy between  $10/21$  and  $11/21$  is  $1/21$ . By means of two particle states, the system builds  $2/21$  and  $(2+2+1)/21=5/21$  but these are weak. For  $l = 11$  we get  $11/23$  and  $12/23$  from  $l/(2l+1)$  and  $(l+1)/(2l+1)$ . The difference state occurs at  $1/23$  which gives the two particle state at  $2/23$ . The three particle state of which occurs at  $(2+2+2)/23=6/23$ . Of course, these large  $l$  states are weak. For  $l =12$  we have  $12/25$  and  $13/25$ , the difference of which is  $1/25$ . The two-particle state of which occurs at  $2/25$  given in Table V. Table I. Gives the fractions calculated from 1985 theory of ref.6 by using  $l/(2l+1)$  and  $(l+1)/(2l+1)$ . All of the calculated values are the same as the experimental values. Values marked by \* are predicted but not found in the data. The calculated values use a formula which is quite different from the Lande's formula. Similarly, the theory of Landau levels has to be changed by the spin. This spin occurs in the form of S but not trivially in

the form of components. Usually in the theory of magnetism, the Hamiltonian is written in terms of x, y and z components but in the present theory the spin occurs in the g values. In the case of electron clusters, the spin components become

important as S is modified by the neighbors which requires spin-wave type effect. The quantum Hall effect is thus a property of pure material and a small number of plateaus require clustering.

TABLE I: PRINCIPAL FRACTIONS

<i>l</i>	0	1	2	3	4	5	6	7	8	9	10	11
(1/2) <i>g</i> <sub>-</sub>	0	1/3	2/5	3/7	4/9	5/11	6/13	7/15	8/17	9/19	10/21	11/23 *
(1/2) <i>g</i> <sub>+</sub>	1	2/3	3/5	4/7	5/9	6/11	7/13	8/15	9/17	10/19	11/21 *	12/23 *

TABLE II: TWO-PARTICLE STATES AND TRANSITION STATES GENERATED BY THE SAME EXPRESSION AS USED FOR TABLE I

4/3	5/3	7/3	8/3	4/5	6/5	7/5	8/5	9/5
11/5	12/5	13/5	14/5	16/5	19/5	21/5	24/5	
1/7	2/7	5/7	9/7	10/7	11/7	12/7	16/7	19/7

TABLE III. THE TWO-PARTICLE STATES WITH 9 AND 11 IN THE DENOMINATOR FOR *L* = 4 AND 5, RESPECTIVELY.

1/9	2/9	7/9	11/9	13/9	14/9	25/9	
2/11	3/11	4/11	7/11	8/11	14/11	16/11	17/11

TABLE IV: THE TWO AND THREE-PARTICLE STATES WITH 13, 15, 17, 19 AND 21 IN THE DENOMINATOR.

1/13	2/13	3/13	4/13	5/13	10/13	19/13	20/13
1/15	2/15	4/15	11/15	22/15	23/15		
1/17	2/17	3/17	4/17	5/17	6/17		
1/19	2/19	3/19	4/19	5/19	9/19	10/19	

TABLE V: LARGE *L* OR WEAK FRACTIONS.

1/21	5/21	10/21	11/21
6/23	11/23	12/23	
1/25	2/25	12/25	13/25

Table I. "Principal fractions" based on single-particle theory valid at low temperatures. *Type 3*. Even denominators. When *s* = 0, 1, 2, ..., which is possible for multielectron states, a factor of 2 drops from the 1/2 in the numerator of eq.(32) to the denominator. For example, *s*=0 which means a two electron state with one spin up and the other down, is formed. The expression (32) for *s* = 0, *l*=0 gives,

$$\frac{1}{2}g = \frac{1}{2} \tag{40}$$

or *g*=1. For *n* = 0 the Landau level  $(n + \frac{1}{2})\frac{1}{2}g\hbar\omega = \frac{1}{4}\hbar\omega$  so

the denominator is even. For *n*=2  $(n+1/2)(1/2)g=5/4$  which is not a paired state. This explains the even denominator state at 5/4. For *n*=3, the predicted fraction is 7/4. Considering two particle states  $5/4+5/4=5/2$  and  $7/4+7/4=7/2$ . If we look at the equation (39) for  $n_1 - n_2 = 2$ , the factor  $(n_1 - n_2)(1/2)g$  gives resonances at *g* and not at (1/2)*g*. This effect generates some degeneracies in the problem. Now using the formula (38) for (1/2)*g*=1 (Table I), the resonance occurs at 5/2 for *n*=2, and at 7/2 for *n*=3. That solves the problem of 5/2 and 7/2. The state of *s*=0 will be non-magnetic and it is similar to a diamagnetic state. The electron pairs of spin = 0 are well known in superconductors which are in the conduction band due to the electron-phonon interaction. However, the present problem is quite different from that of superconductors and the analogy is only to the extent of two-particle state. The two-particle states are not bound whereas in superconductors there is a binding energy. When there are two electrons, we

predict that not only spin zero state occurs but  $s = 1$  state should also occur. The value of  $s = 1$  gives,

$$\frac{1}{2}g = \frac{l + \frac{1}{2} \pm 1}{2l + 1} \quad (41)$$

Which for  $l = 0, s=1$  gives  $(1/2)g_+=3/2$  and  $(1/2)g_-=-1/2$  which have even denominators. For  $l=1$  we get  $(1/2)g_+=5/6$  and  $(1/2)g_-=1/6$ . For  $l=2$ , we obtain,  $(1/2)g_+=7/10$  and  $(1/2)g_-=3/10$ . In fact  $3/10$  is experimentally observed. The even denominator states thus seen above are at  $1/2, 5/2, 7/2, 5/6, 1/6, 3/10$  and  $7/10$ . We can look for more values of  $l$  but they become weak.

Using the formula (1),

For  $s = 0, (1/2)g = 1/2$

For  $l = 0, s = 1, (1/2)g_+ = 3/2$  and  $(1/2)g_- = -1/2$

For  $l = 0, s = 2, (1/2)g_+ = 5/2$  and  $(1/2)g_- = -3/2$ .

The factor of  $(1/2)$  can arise from Bohr magneton. The energy of the electron in a magnetic field is

$g\mu_B H = g \frac{e\hbar}{2mc} H$  which can be written in terms of cyclotron frequency,  $(1/2)g \hbar\omega$  where  $\omega_c = eH/mc$ . The eigen values of the Landau levels become,  $(n + \frac{1}{2}) \frac{1}{2} g \hbar\omega$  in

which case the factor  $(n+1/2)$  can be multiplied by  $(1/2)g$ . For  $n=0$  the factor  $n+1/2$  gives  $1/2$ . When  $g = 2, (1/2)g=1$  in Table I, there is no effect on the harmonic oscillator type eigen values so that the eigen values are obtained from  $(n+1/2)$ . We give in table VI, various energies for a few values of  $n$ . In the serial number 1,  $(1/2)g=1$ . The other values of  $(1/2)g$  are  $1/2, 3/2, 5/2$  and  $(-1/2)$  which are multiplied by  $(n+1/2)$  and given in the Table VI.

TABLE VI: THE ENERGIES FOR  $n=0, 1, 2, 3,$  AND  $4$  FOR VARIOUS VALUES OF  $(1/2)G$ .

S.No.	n	0	1	2	3	4
1	$n+(1/2)$	1/2	3/2	5/2	7/2	9/2
2	$[n+(1/2)](1/2)$	1/4	3/4	5/4	7/4	9/4
3	$[n+(1/2)](3/2)$	3/4	9/4	15/4	21/4	27/4
4	$[n+(1/2)](5/2)$	5/4	15/4	25/4	35/4	45/4
5	$[n+1/2](-1/2)$	-1/4	-3/4	-5/4	-7/4	-9/4

Let us make two particle states,  $\omega = \omega_1 + \omega_2$ . From  $n=4, (1/2)g=1, g=2$  from serial number 1 and the first member of the third row,  $9/2+1/4=19/4$ . The values of  $3/4$  and  $5/4$  are already in the Table. Therefore, we can obtain the values  $3/4, 5/4$  and  $19/4$ . In the expression for the flux quantization, there is an area in which flux is quantized in units of  $hc/e$ . It is perfectly valid to say that flux is quantized in units of integer multiple of  $hc/e$ . When we take  $n'=2$  in  $n'hc/e$ , the flux quanta also correspond to  $(1/2)e$ . Hence, the flux is quantized with half the charge of the electron. In this way a factor of  $1/2$  appears in the charge. In the previous Table, the charges of  $3/4, 5/4$  and  $19/4$  were obtained. Considering  $n'=2$ , these values become  $3/8, 5/8$  and  $19/8$ . The quantized Hall effect formula becomes,  $h/[(1/2)e]e$  so that weak plateaus are predicted at  $3/8, 5/8$  and  $19/8$ .

The helicity is defined as the sign of the product of the linear momentum and the spin, p.s. In quantum Hall effect the total angular momentum  $j=l \pm s$  shows the importance of the sign of the spin. The data are symmetric with respect to the sign of the spin and hence display helicity. The sign of the velocity of the electron is the same as the sign in front of  $s$ . Hence particles differing in the sign of  $s$  travel in different directions, such as  $+$  sign for downstream particles implies that the particle with  $-$  sign in  $s$  travels upstream. It may be noted that the sign of the  $z$  component of the spin which has

two values for spin  $1/2$  is not used to define the helicity. The interpretation of the quantum Hall effect in GaAs as well as in graphene is in agreement with the  $g$  values which have particle-hole symmetry.

## VI. CONCLUSIONS

The quantum Hall effect was discovered by von Klitzing et al experimentally but it had no theory in those days. The value of  $h/e^2$  can be calculated by a pencil from the known values of  $h$  and  $e$ . The experimental value of the resistivity of  $h/e^2$  multiplies by a lot of unknown integers or fractions. It is important to find out as to what these fractions or integers are. We have found the theory which predicts the fractions which occur in the resistivity at the plateaus in the Hall effect. Basically, the Lande's formula requires to be changed by a more accurate formula which has certain symmetries such as those of helicity. The Landau levels as given in the books are single valued. The constant in the frequency must be double valued. Hence we have changed the Landau level theory suitably. The effect of the spin which is not trivially contained in the Landau levels has been found by us and applied to the quantum Hall effect. The spin thus not only occurs in the Hamiltonian as components but also as a constant in the Zeeman effect which at high fields is subject

to helicity type doubling of states.

REFERENCES

[1] K. Von Klitzing, G. Dorda, and M. Pepper, "New method for high-accuracy determination of the fine structure constant," *Phys. Rev. Lett.*, vol. 45, pp. 494-497, 11 August 1980.

[2] D. C. Tsui, H. L. Stormer, and A. C. Gossard, "Two-dimensional magnetotransport in the extreme quantum limit," *Phys. Rev. Lett.* vol. 48, pp.1559-1562, 1982.

[3] R. B. Laughlin, "Anomalous quantum Hall effect: An incompressible quantum fluid with fractionally charged excitations," *Phys. Rev. Lett.* vol. 50, pp. 1395-1398, 1983.

[4] P. W. Anderson, "Remark on Laughlin theory of the fractionally quantized Hall effect," *Phys. Rev. B*, vol. 28, pp. 2264-2265, 1983.

[5] D. Arovav, J. R. Schrieffer, and F. Wilczek, *Phys. Rev. Lett.* vol.53, 722-723, 1984.

[6] K. N. Shrivastava, "Rational numbers of the fractionally quantized Hall effect," *Phys. Lett. A*, vol.113, pp. 435-436, 1986.

[7] K. N. Shrivastava, "Negative-spin quasiparticles in quantum Hall effect," *Phys. Lett. A*, vol. 326, pp. 469-472, 2004.

[8] K. N. Shrivastava, "Particle-hole symmetry in quantum Hall effect," *Mod. Phys. Lett. B* vol. 13, pp.1087-1090, 1999.

[9] K. N. Shrivastava, "Theory of quantum Hall effect with high Landau levels," *Mod. Phys. Lett. B*, vol. 14, pp. 1009-1013, 2000.

[10] K. N. Shrivastava, "Frontiers of fundamental physics 4," 1<sup>st</sup> ed. B. G. Sidharth, ISBN 0-306-46641-4, Academic/Plenum Pub. N.Y. 2001, pp. 235-249.

[11] K. N. Shrivastava, *Introduction to quantum Hall effect*, ISBN 1-59033-419-1, Nova Sci. Pub. New York 2002.

[12] K. N. Shrivastava, *Quantum Hall effect: Expressions*, ISBN 1-59454-399-2, Nova Science Pub. New York 2005.

[13] K. N. Shrivastava, "Additional Dirac matrix in quantum Hall effect," in *Proce. AIP Conference*, vol. 1017, pp 47-56, 2008

[14] K. N. Shrivastava, "The theory of the quantum Hall effect," in *Proc. AIP Conference*, vol. 1017, pp. 422-428, 2008.

[15] K. N. Shrivastava, "Infrared of thin film grapheme in a magnetic field and the Hall effect," in *Proc. SPIE*, vol. 7155, pp. 71552F, 2008.

[16] K. N. Shrivastava, *Proc. Conf. Honor C. N. Yang's 85<sup>th</sup> Birthday, Statistical Physics, High Energy, Condensed matter and Mathematical Physics*, ISBN 9812794174, World Scientific Pub. Co. Singapore, pp. 526, 2008.

[17] K. N. Shrivastava, *Proc. Conf. Honor Murray Gell-Mann's 80<sup>th</sup> Birthday, quantum mechanics, elementary particles, quantum cosmology and complexity*, ISBN 13-978-981-4335-60-7 and ISBN 10-981-4335-60-6, World Sci. Pub. Co. Singapore, pp. 511-517, 2011.

[18] A. N. Rosli and K. N. Shrivastava, "von Klitzing's constant as a special case of generalized constants," in *Proc.AIP conference*, vol. 1136, pp469-473, 2009.

[19] K. N. Shrivastava, "Interpretation of fractions in quantum Hall effect:Wei Pan's data," in *Proc. AIP conference*, vol. 1150, pp. 59-67, 2009.

[20] K. N. Shrivastava, "The fractional quantum Hall effect: The cases of 5/2 and 12/5," in *Proc. AIP conference*, vol. 1169, pp. 48-54, 2009.

[21] K. N. Shrivastava, "Laughlin's wave function and the angular momentum," *International J. Mod. Phys. B*, vol. 25, pp. 1301-1357, 2011.

[22] A. Kumar, G. S. Csathy, M. J. Manfra, L. N. Pfeiffer, and K. W. West, "Nonconventional odd-denominator fractional quantum Hall states in the second Landau level," *Phys. Rev. Lett.*, vol. 105, pp. 246808, 2010.

[23] W. Pan, J. S. Xia, H. L. Stormer, D. C. Tsui, C. Vicente, E. D. Adams, N. S. Sullivan, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, "Experimental studies of the fractional quantum Hall effect in the first excited Landau level," *Phys. Rev. B.*, vol.77, pp. 075307, 2008.



**Keshav N. Shrivastava** obtained B.Sc. degree from the Agra University in 1961, M.Sc. degree from the University of Allahabad in 1963, Ph.D. from the Indian Institute of Technology Kanpur in 1966 and D.Sc. from Calcutta University in 1980.

He worked at the Clark University during 1966-68, at the Harvard University during the summer of 1968, at the Montana State University during 1968-69, and at the University of Nottingham during 1969-70. He was Associate Professor at the Himachal Pradesh University Simla, India during 1970-74. He worked at the University of California Santa Barbara during 1974-75 and at the State University Utrecht during 1975-76. He was Professor at the University of Hyderabad where he worked from 1978 till 2005. He worked in Tohoku University during 1999-2000. He visited the University of Zurich for short times during 1984, 1985, 1988 and 1999. He worked in the University of Houston in the summer of 1990. He visited the Royal Institute of Technology Stockholm and the University of Uppsala during 1994. He visited the Princeton University and the University of Cincinnati during 2003. He was Professor in the University of Malaya Kuala Lumpur during 2005-2011. He is affiliated with the University of Hyderabad, India. He has worked in electron spin resonance, spin-phonon interaction, relaxation, Moessbauer effect, exchange interaction, superconductivity and the quantum Hall effect. He discovered the flux quantized energy levels in superconductors, the spin dependent flux and the correct theory of the quantum Hall effect. He has published more than 227 papers in journals and he is the author of the following books. (i) Superconductivity: Elementary Topics, World Scientific Singapore 2000. (ii) Introduction to Quantum Hall effect, Nova Sci. Pub N.Y. 2002 and (iii) The quantum Hall effect: Expressions, Nova Sci. N.Y. 2005.

Prof. Shrivastava is a Fellow of the Institute of Physics London, Fellow of the National Academy of Sciences India and member of the American Physical Society.