Statistical Design of Double Moving Average Scheme for Zero Inflated Binomial Process

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Abstract: The objective of this paper is to show an explicit formula for Double Moving Average chart of Zero Inflated Binomial process (DMA_{ZIB}). The ARL is a traditional measurement of control chart's performance, the expected number of observations taken from an in-control process until the control chart falsely signals out-of-control is denoted by ARL_0. An ARL_0 will be regarded as acceptable if it is large enough to keep the level of false alarms at an acceptable level. A second common characteristic is the expected number of observations taken from an out-of-control process until the control chart signals that the process is out-of-control is denoted by ARL_1. Especially, the explicit analytical formulas for evaluating ARL_0 and ARL_1 be able to get a set of optimal parameters which depend on width of double moving average (w) and width of control limit (k) for designing DMA_{ZIB} chart with minimum of ARL_1.

Key words: Zero inflated binomial (ZIB) distribution, double moving average control chart (DMA), average run length (ARL).

1. Introduction

Besides traditional Statistical Process Control (SPC) charts, a variety of statistical methods have been developed in many areas of interest including engineering, in industry and manufacturing, epidemiology and health care, sociology, and other fields. Attribute control charts are important technique in SPC to monitor process with the discrete data. When the quality characteristic cannot be measured on a continuous scale, for instance, in counting the number of defective products or the number of nonconformities in a production process, an attribute control chart must be used. Attribute control charts as p, np, c, and u charts are important tools of statistical control to monitor process with discrete data. Additionally, Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) charts for attribute data have also been applied to discrete processes as in [1]. Recently, Khoo [2] first introduced the Moving Average control chart (MA) for monitoring the non-conforming or defective fraction in discrete processes. Later, Khoo and Wong [2] extended Double Moving Average chart (DMA) with moving average of the MA statistic one more time. They proposed this chart with normal observations and also showed the numerical simulations of ARL. According to Khoo and Wong [3], the performance of the DMA chart is superior to the MA, EWMA and CUSUM charts for monitoring small and moderate shifts for process mean.

Due to technological advancement of manufacturing processes, numbers of special statistical methods have been developed such as the number of non-conforming units in a high yield process or health engineering is the occurrence of a large number of zero failures is called rare health events (see in [4], [5]) thus the standard attribute control charts are not very relevant nor effective, due to the occurrence of an excessive number of zeros in the data. According to [4], this excess in zeros results in an over-dispersed distribution and in the under-estimation of process parameters. Therefore, the ordinary p and np charts cannot be efficiently used due to an increased rate of false alarms and, consequently, the development of control charts under a more appropriate probability model is necessary. Therefore some alternative models should be developed. The np chart is widely used to monitor processes with binomial counts. However, as binomial distribution tend to underestimate the mean and variability of the zero-inflated count, the resulting attribute charts have tighter control limits which subsequently lead to a higher false alarm rate in detecting out-of-control signals. This model can be based on Zero Inflation Binomial (ZIB) distribution. The excess number of zeros in binomial count can also be found in the biological control of pests. Furthermore, the explicit formulas for computing the ARL_0 and ARL_1 when the weighted moving average (w) equal to 1 and 2 were proposed by [6]. Consequently, the explicit formulas of ARL_0 and ARL_1 for DMA chart with arbitrary the values of w when observations are binomial distribution also submitted in [7]. In this paper, the explicit analytical formulas for evaluating ARL_0 and ARL_1 DMA chart for Zero Inflation Binomial (ZIB) distribution and a set of optimal parameters which depend on a width of the moving average (w) and width of control limit (k) for designing DMA chart with minimum of ARL_1 are presented.

2. Methodology

Let observations $X_1, X_2, \ldots, X_m$ be i.i.d. random variables with Binomial distribution, where X_i number of nonconforming is items in sample i of m samples of size n. A simple way to model Zero-Inflated is to include a proportion ω of extra-zeros follow from a Binomial distribution. The Zero-Inflated Binomial density function $X_{ij} ZIB(n,p,\omega)$ can be written as

$$f(x) = \begin{cases} \omega + (1-\omega)(1-p)^n & ; x = 0 \\ (1-\omega)\binom{n}{x} p^x (1-p)^{(n-x)} & ; x = 1, 2, \ldots, n \end{cases}$$

where ω is a probability of extra zeros interpreted as the probability that the site is unoccupied and p is a probability of detection at a single visit given that a site is occupied. For the above distribution, mean and variance of number of non conforming can be calculated by

$$E\left(X_i\right) = (1-\omega)\, np_0,$$

$$Var\left(\overline{X}_i\right) = (1-\omega)\, p_0 \left(1 - p_0 (1 - n\omega)\right).$$

It is assumed that $p = p_0$ while the process is in-control and $p = p_1 > p_0$ when the process goes out-of-control. It is assumed that there is a change-point time $\theta \le \infty$ at which the parameter changes from $p = p_0$ to $p = p_1$. Note that, $\theta = \infty$ means a process always remains in-control state.

Let $E_\theta(.)$ denote the expectation that the change-point from $p = p_0$ to $p = p_1$ for a distribution function $F(x,n,p,\omega)$ occurs at time θ, where $\theta \le \infty$ The quantity $E_\theta(\tau)$ is called the Average Run Length (ARL0) of the chart for the given process.

A typical condition imposed on an ARL_0 is

$$ARL_0 = E_\infty(\tau) = T,$$

where T is given (usually large). For given distribution function, this condition then determines choices for the UCL and LCL.

A typical definition of the ARL_1 is that

$$ARL_1 = E_1(\tau \mid \tau \geq 1),$$

for the change point occurs at $\theta = 1$. One could expect that a sequential control chart has a near optimal performance if the ARL_1 is closed to a minimal value.

A Moving Average control chart for Zero-Inflated Binomial distribution (MAZIB) is defined by the following statistics:

$$MA_i = \begin{cases} \dfrac{\overline{X}_i + \overline{X}_{i-1} + \overline{X}_{i-2} + \ldots}{i} & ; i < w \\ \dfrac{\overline{X}_i + \overline{X}_{i-1} + \ldots + \overline{X}_{i-w+1}}{w} & ; i \geq w \end{cases}$$

where ω is the width of the MAZIB chart. The mean of MAZIB chart is $E(MA_i) = E(\overline{X}_i) = (1-\omega)np_0$, and variance of MAZIB chart is

$$Var(MA_i) = \begin{cases} \dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni} & ; i < w \\ \dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw} & ; i \geq w. \end{cases}$$

The 3σ upper and lower control limits are as the following

$$LCL/UCL = \begin{cases} (1-\omega)np_0 \pm H\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni}} & ;\ i < w \\ (1-\omega)np_0 \pm H\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw}} & ;\ i \geq w \end{cases}$$

where H is width of control limit. The alarm time for the MA_{ZIB} procedure is given by

$$\tau = \inf\{i > 0 : MA_i > UCL \text{ or } MA_i < LCL\}.$$

A Double Moving Average (DMA) control chart defined by the following statistics:

$$DMA_i = \begin{cases} \dfrac{MA_i + MA_{i-1} + MA_{i-2} + \ldots}{i} & ;\ i \leq w \\ \dfrac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & ;\ w < i < 2w-1. \\ \dfrac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & ;\ w \geq 2w-1 \end{cases}$$

The DMA chart for Zero-Inflated Binomial distribution is so-called DMA_{ZIB} chart which the mean of DMA_{ZIB} for all period is $E(DMA_i) = (1-\omega)np_0$ and its variance is

$$Var(DMA_i)=\begin{cases}\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{i^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j} & ; i\le w\\ \dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{w^2}\left[\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(i-w+1)\left(\frac{1}{w}\right)\right] & ; w<i<2w-1.\\ \dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{w^2} & ; w\ge 2w-1\end{cases}$$

where ω is the width of the DMA$_{\text{ZIB}}$ chart. The 3σ upper and lower control limits are as the following

$$LCL/UCL=\begin{cases}(1-\omega)np_0\pm k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}} & ; i\le w\\ (1-\omega)np_0\pm k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}\left[\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(i-w+1)\left(\frac{1}{w}\right)\right]} & ; w<i<2w-1\\ (1-\omega)np_0\pm k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}} & ; i\ge 2w-1\end{cases}$$

where $p=p_0$ is denoted that process is in-control state and k is width of control limit. The alarm time for the DMA$_{\text{ZIB}}$ procedure is given by

$$\tau=\inf\{i>0: DMA_i>UCL \text{ or } DMA_i<LCL\}.$$

The ARL values of DMA$_{\text{ZIB}}$ control chart can be derived as follows: Let $ARL=N$, then

$$\begin{aligned}\frac{1}{ARL}&=\frac{1}{N}P(out-of-control\text{ signal at time } i\le w)+\frac{1}{N}P(out-of-control\text{ signal at time } w<i<2w-1)\\&\quad+\left(\frac{N-(2w-2)}{N}\right)P(out-of-control\text{ signal at time } i\ge 2w-1)\\&=\frac{1}{N}\left(\sum_{i=1}^{w}\left(P\left(\frac{\sum_{j=1}^{i}p_j}{i}>UCL_i\right)+P\left(\frac{\sum_{j=1}^{i}p_j}{i}<LCL_1\right)\right)\right)+\frac{1}{n}\left(\sum_{j=w+1}^{2w-2}\left(P\left(\sum_{j=i-w+1}^{i}\frac{p_j}{w}>UCL_2\right)+P\left(\sum_{j=i-w+1}^{i}\frac{p_j}{w}<LCL_2\right)\right)\right)\\&\quad+\left(\frac{N-(w-1)}{N}\right)\left(P\left(\sum_{j=i-w+1}^{i}\frac{p_j}{w}>UCL_3\right)+P\left(\sum_{j=i-w+1}^{i}\frac{p_j}{w}<LCL_3\right)\right).\end{aligned}$$

The solution can be obtained by central limit theorem, and then the explicit formula of ARL_0 for DMA$_{\text{ZIB}}$ chart is

$$ARL_0=\left[1-1-\sum_{i=1}^{w}\left(P\left(Z>\frac{(1-\omega)np_0+k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}-(1-\omega)np_0}{\sqrt{\dfrac{(1-\omega)np_0(1-p(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.\right.$$

$$
\begin{aligned}
&+P\left(Z<\frac{(1-\omega)np_0-k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}-(1-\omega)np_0}{\sqrt{\dfrac{(1-\omega)np_0(1-p(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}}\right)\Bigg)\Bigg)\\
&-\left(\sum_{j=w+1}^{2w-2}\left(P\left(Z>(1-\omega)np_0+\frac{k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-(1-\omega)np_0}{\sqrt{\dfrac{(1-\omega)np_0(1-p(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right)\right.\\
&+P\left(Z<(1-\omega)np_0-\frac{k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-(1-\omega)np_0}{\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\Bigg)\Bigg)\\
&\times\left(P\left(Z>(1-\omega)np_0+\frac{k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}}-(1-\omega)np_0}{\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}}}\right)\right.\\
&\left.+P\left(Z<(1-\omega)np_0-\frac{k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}}-(1-\omega)np_0}{\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}}}\right)\right)^{-1}+(2w-2).
\end{aligned}
\tag{1}
$$

and the explicit formula of ARL_1 of DMA$_{ZIB}$ for a width of control limit H, can therefore be written as follows:

$$
\begin{aligned}
ARL_1=&\left[1-1-\sum_{i=1}^{w}\left(P\left(Z>\frac{(1-\omega)np_0+k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}-(1-\omega)np}{\sqrt{\dfrac{(1-\omega)np(1-p(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.\right.\\
&+P\left(Z<\frac{(1-\omega)np_0-k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}-(1-\omega)np}{\sqrt{\dfrac{(1-\omega)np(1-p(1-n\omega))}{ni^2}\displaystyle\sum_{j=1}^{i}\frac{1}{j}}}\right)\Bigg)\Bigg)\\
&-\left(\sum_{j=w+1}^{2w-2}\left(P\left(Z>(1-\omega)np_0+\frac{k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-(1-\omega)np}{\sqrt{\dfrac{(1-\omega)np(1-p(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right)\right.\\
&+P\left(Z<(1-\omega)np_0-\frac{k\sqrt{\dfrac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-(1-\omega)np}{\sqrt{\dfrac{(1-\omega)np(1-p(1-n\omega))}{nw^2}\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\Bigg)\Bigg)
\end{aligned}
$$

$$\times\left(P\left(Z>(1-\omega)np_0+\frac{k\sqrt{\frac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}}-(1-\omega)np}{\sqrt{\frac{(1-\omega)np(1-p(1-n\omega))}{nw^2}}}\right)\right.$$

$$\left.+P\left(Z<(1-\omega)np_0-\frac{k\sqrt{\frac{(1-\omega)np_0(1-p_0(1-n\omega))}{nw^2}}-(1-\omega)np}{\sqrt{\frac{(1-\omega)np(1-p(1-n\omega))}{nw^2}}}\right)\right)+(2w-2). \quad (2)$$

We first describe a procedure for obtaining optimal designs for DMA_{ZIB} chart. The criterion used for choosing optimal values for is the width of the double moving average chart (w) and width of control limit (k) is minimization of ARL_1 for a given in-control parameter value p_0 = 0.01, 0.02 and $ARL_0 = T$ and a given out-of-control parameter value ($p = p_1$). We compute optimal (w,k) values for T = 370.4 and 500 and magnitudes of change. Tables of the optimal parameters values are shown in Tables 2 - 5.

The numerical procedure for obtaining optimal parameters for DMA_{ZIB} designs

1) Select an acceptable in-control value of ARL_0 and decide on the change parameter value (p_0) for an out-of-control state.
2) For given p_0 and T, find optimal values of w and k to minimize the ARL_1 values given by Equation 2 subject to the constraint that $ARL_0 = T$ in Equation 1, i.e. w and k are solutions of the optimality problem.

3. Numerical Results

In this section, the numerical results for ARL_0 and ARL_1 for a DMA_{ZIB} chart were calculated from Equation (1) and Equation (2) as shown in Table 1. The parameter values for DMA chart was chosen by given desired ARL_0 = 370, in-control parameter p_0 = 0.05 and out-of-control parameter values p_1 from 0.06 to 0.2. The performance of DMA_{ZIB} chart show that for the shift increasing DMA_{ZIB} performs better as the value of w decreases. For example, when $\delta = 0.05$ DMA chart with $w = 5$ shows the best performance because of given minimum ARL_1. Note that, calculations with explicit formula from Equation (1) and (2) is simple and very fast to calculate which the computational times take less than 1 second. The numerical results of optimal parameters for DMA_{ZIB} chart was calculated from Equations 1-2 as shown in Tables 2-5. The parameter values for DMA_{ZIB} chart was chosen by given desired ARL_0 = 370 and 500, in-control parameter p_0 = 0.01, 0.2 and $\omega = 0.3, 0.5$. In tables 2-5, the results in terms of the width of the double moving average chart (w) and width of control limit (k) and minimum ARL_1 for ARL=370 and 500 are shown. For example, if we want to detect a parameter change from $p_0 = 0.01$ to $p_1 = 0.05$, $\omega = 0.3$ and the ARL value is 370 then the optimality procedure given above will give optimal parameter values k =3 and w = 6 and ARL_1^* value = 11.824. As shown in Tables 2 - 5 the use of the suggested explicit formulas for ARL_0 and ARL_1 are useful to practitioners especially finding optimal parameters of DMA_{ZIB} chart.

4. Conclusions

Using the explicit formulas, we have been able to provide tables for the width of the double moving average chart (w) and width of control limit (k) and minimum ARL_1 for Double Moving Average chart of Zero Inflated Binomial process (DMA_{ZIB}).

Table 1. ARL_0 and ARL_1 for DMAZIB Chart with Magnitudes w given ARL_0 = 370

δ	w =1	w =3	w =5	w =10	w =15	w =20	w =25
0.00	370.4	370.4	370.4	370.4	370.4	370.4	370.4
0.01	319.74	208.3	117.6	40.29	**32.13**	37.56	45.42
0.02	248.86	82.10	31.90	**18.26**	24.94	31.56	35.85
0.03	182.14	35.34	**13.44**	15.50	20.38	21.57	20.74
0.04	129.99	17.84	**9.24**	13.30	14.29	13.18	12.24
0.05	92.46	10.60	**7.56**	10.56	9.82	9.16	8.97
0.06	66.28	7.29	**6.69**	8.11	7.42	7.27	7.26
0.07	48.13	**5.61**	6.03	6.41	6.11	6.10	6.10
0.08	35.46	**4.68**	5.41	5.33	5.24	5.24	5.24
0.09	26.53	**4.10**	4.81	4.61	4.59	4.59	4.59
0.10	20.13	**3.71**	4.25	4.08	4.08	4.08	4.08
0.15	6.12	**2.59**	2.62	2.62	2.62	2.62	2.62

Table 2. Optimal Design Parameters and Minimum ARL_1 for DMAZIB Chart Given p_0 = 0.01, ω = 0.3

δ	ARL_0 =370			ARL_0 =500		
	k	w	ARL_1^*	k	w	ARL_1^*
0.01	3	21	47.397	3.0905	21	50.284
0.02	3	12	26.811	3.0905	13	28.259
0.03	3	9	18.824	3.0905	10	19.877
0.04	3	7	14.582	3.0905	8	15.260
0.05	3	6	11.824	3.0905	7	12.455
0.06	3	6	10.028	3.0905	6	10.403
0.07	3	5	8.525	3.0905	5	8.908
0.08	3	5	7.624	3.0905	5	7.884
0.09	3	4	6.668	3.0905	4	6.993
0.10	3	4	6.003	3.0905	4	6.237
0.15	3	3	3.985	3.0905	3	4.132
0.20	3	2	1.617	3.0905	3	3.183

Table 3. Optimal Design Parameters and Minimum ARL_1 for DMAZIB Chart Given p_0 = 0.01, ω = 0.5

δ	ARL_0 =370			ARL_0 =500		
	k	w	ARL_1^*	k	w	ARL_1^*
0.01	3	25	57.059	3.0905	25	60.687
0.02	3	15	32.707	3.0905	15	35.551
0.03	3	11	23.144	3.0905	11	24.406
0.04	3	9	17.96	3.0905	9	18.858
0.05	3	7	14.742	3.0905	8	15.39
0.06	3	7	12.467	3.0905	7	12.972
0.07	3	6	10.685	3.0905	6	11.167
0.08	3	5	9.443	3.0905	6	9.962
0.09	3	5	8.375	3.0905	5	8.737
0.10	3	5	7.673	3.0905	5	7.941
0.15	3	4	5.208	3.0905	3	5.519
0.20	3	2	2.353	3.0905	3	3.913

Table 4. Optimal Design Parameters and Minimum ARL_1 for DMAZIB Chart Given p_0 = 0.02, ω = 0.3

δ	ARL_0 =370			ARL_0 =500		
	k	w	ARL_1^*	k	w	ARL_1^*
0.01	3	12	25.114	3.0905	12	26.404
0.02	3	7	13.511	3.0905	7	14.196
0.03	3	5	9.236	3.0905	5	9.734
0.04	3	4	6.999	3.0905	4	7.374
0.05	3	4	5.626	3.0905	4	5.816
0.06	3	3	4.677	3.0905	3	4.906
0.07	3	3	3.956	3.0905	3	4.099
0.08	3	3	3.503	3.0905	3	3.611
0.09	3	3	3.165	3.0905	3	3.257
0.10	3	3	2.883	3.0905	3	2.967
0.15	3	2	1.901	3.0905	2	1.955
0.20	3	2	1.455	3.0905	2	1.491

Table 5. Optimal Design Parameters and Minimum ARL_1 for DMAZIB Chart Given p_0 = 0.02, ω =0.5

δ	ARL_0 =370			ARL_0 =500		
	k	w	ARL_1^*	k	w	ARL_1^*
0.01	3	15	32.864	3.0905	15	34.738
0.02	3	9	18.089	3.0905	9	19.007
0.03	3	7	12.523	3.0905	7	13.078
0.04	3	6	9.712	3.0905	6	10.042
0.05	3	5	7.741	3.0905	5	8.013
0.06	3	4	6.363	3.0905	4	6.643
0.07	3	4	5.543	3.0905	4	5.726
0.08	3	3	4.860	3.0905	3	5.113
0.09	3	3	4.237	3.0905	3	4.411
0.10	3	3	3.817	3.0905	3	3.948
0.15	3	2	2.642	3.0905	2	2.748
0.20	3	2	1.969	3.0905	2	2.025

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