Robustness of Memory-Type Charts to Skew Processes

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Abstract: This paper aims to study the robustness of Double Exponentially Weighted Moving Average (DEWMA) in order to detect a change in parameter when process are underlying skew distributions. In general, an Average Run Length (ARL) is used as a common measurement to compare the performance of control chart in term of quick detection. The performance of GWMA chart are compared with Exponentially Weighted Moving Average (EWMA) and Generally Weighted Moving Average Control Chart (GWMA) charts which the former outperforms and give a minimal ARL1 for all magnitudes of shift.

Key words: Performance, monitoring, skewness, control chart.

1. Introduction

In manufacturing process, standards of quality and low cost products are key factors of industrial success. Achieving the quality standards, the manufacture requires either process control and quickly fault detection. In fact, the production still has a variation at any point of time even the process is well controlled. Two types of the variation have been described, which are the nature variation due to chance causes and the variation due to assignable causes, including man, machine, methods, and materials.

The Statistical Quality Control (SQC) play a vital rule in measuring, controlling, monitoring and improving process quality. The SQC is categorized into three parts, which are descriptive statistics, statistical process control charts, and acceptance sampling plans. The most common tool for detecting and monitoring a changing in the process is the control charts. Theirs major functions are the manufacturing standards setting, the production goal's achievement, and the productivity improvement. The control charts have been classified into variables and attributes. The control chart for variables such as $\overline{x}$ -chart, R-chart and S-chart using for detecting the characteristics are measurable and valued continuous. Whereas the control chart for attributes such as p-chart, np-chart, c-chart, and u-chart using for detecting the characteristics that have a discrete value and are countable. Ideally, the parameters changing should be detected as soon as possible, which is the main purpose of a control chart in manufacture. An in-control process setting, the false alarm rate should be sufficient large. Otherwise, the true alarm rate should be minimum when the process is out-of-control.In literature reviews, the control chart known as the Shewhart chart, namely Cumulative Sum [1], Exponentially Weighted Moving Average (EWMA) [2], is commonly used and widely applied in many fields. The Shewhart control chart was discovered by Shewhart in 1931 [3]. This kind of control chart is acceptable to detect the large changes in process, but it is insensitive to minor change detection. To overcome this problem, there are many memory-type effective alternatives to the Shewhart chart, namely

Cumulative Sum (CUSUM) [1], Exponentially Weighted Moving Average (EWMA) [2], Double Exponentially Weighted Moving Average (DEWMA) [4] and Generally Weighted Moving Average (GWMA) [5] charts have been developed to detect small shifts (about 1.5σ or less).

In general, the performance of control chart is measures by Average Run Length (ARL). According to the expectation of the stopping times, ARL is classified into ARL0 and ARL1. The ARL0 (in - control ARL) is the expectation of the stopping time when the process is in-control. On the other hand, ARL1 (out - of - control ARL) is the expectation of delay of true alarm times when the process is out-of-control. From previous studies, there are many literatures studied the ability in order to detect a change in process of control charts which the assumption of process normality [6]. However, in the practice, this assumption always deviated from normal distribution such as skew processes. The performance of the EWMA chart for non-normal distributions has been investigated by [7]. The effect of non-normality and autocorrelation on the performance of EWMA control charts was presented by [8] found that the EWMA chart robust to non-normality assumption for detecting small shifts in a process mean and variance. Then, the robustness of control chart should be investigated to violation of the normality assumption. There are some literatures to compare the performance between DEWMA and EWMA for non-normality process [9], [10]

Consequently, the aims of this paper is to study the robustness of memory-type control charts as EMWA, DEWMA and GWMA charts in order to detect of a change in parameter of skew processes such as gamma and log-normal distributions.

2. Control Charts and Theirs Properties

2.1. Exponentially Weighted Moving Average (EWMA) Charts

The Exponentially Weighted Moving Average (EWMA) statistics is defined as the following form

$$Z_t = \lambda_1 (X_t - \mu_0) + (1-\lambda_1) Z_{t-1}, \quad 0 < \lambda_1 < 1, \tag{1}$$

where λ_1 is a weighting factor for previous observations. The target in-control parameter α_0 is supposed to be steady and the initial value Z_0 is usually chosen to be the process in-control parameter, i.e., $Z_0 = \alpha_0$. If the observations X_i are independent random variables with variance σ^2, then the variance of EWMA statistics Z_t is

$$\sigma_{Z_t}^2 = \sigma_X^2 \left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}], \; t = 1, 2, \ldots$$

Since $0 < 1-\lambda_1 < 1$, we have that $(1-\lambda_1)^{2t} \to 0$ as $t \to \infty$, and therefore the asymptotic value of the variance is

$$\sigma_{Z_t}^2 = \sigma_x^2 \left(\frac{\lambda_1}{2-\lambda_1}\right). \tag{2}$$

Then, an asymptotic standard deviation is used to find the control limits of the EWMA chart is the following:

$$UCL / LCL = \alpha_0 \pm L_1 \sigma_X \sqrt{\frac{\lambda_1}{2-\lambda_1}}$$

where L_1 is the width of EWMA's control limit. The process will be declared to be in an out-of-control state when $Z_t > UCL$ or $Z_t < LCL$.

2.2. Double Exponentially Weighted Moving Average (DEWMA) Chart

This control chart was developed from EWMA chart by taking account to double EWMA statistics which has been proposed by [5] as following

$$Y_t = \lambda_2 Z_t + (1-\lambda_2)Y_{t-1} \tag{3}$$

where $Z_t = \lambda_1(X_t - \mu_0) + (1-\lambda_1)Z_{t-1}$ as Equation (1). If $\lambda_1 = \lambda_2 = \lambda$ from Eq. (1) and (3) then the DEWMA (Y_t) is

$$Y_t = \lambda Z_t + (1-\lambda)Y_{t-1} \tag{4}$$

and $Z_t = \lambda(X_t - \mu_0) + (1-\lambda)Z_{t-1}$. The expectation and variance of DEWMA statistics are as follows:

$$E(Y_t) = E(Z_t) = E(X_t) = \mu_0$$

and

$$V(Y_t) = \sigma_x^2 \lambda^4 \frac{1+(1-\lambda)^2 - (t^2+2t+1)(1-\lambda)^{2t} + (2t^2+2t-1)(1-\lambda)^{2t+2} - t^2(1-\lambda)^{2t+4}}{\left(1-(1-\lambda)^2\right)^3}. \tag{5}$$

If $t \to \infty$ from Eq. (5) then the asymptotical variance of DEWMA statistics is

$$V(Y) = \sigma_x^2 \left(\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3} \right)$$

Therefore, the control limits of DEMWA can be written as

$$UCL/LCL = \mu_0 \pm L_2 \sigma_X \sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}.$$

where L_2 is the width of DEWMA's control limit.

2.3. Generally Weighted Moving Average (GWMA) Chart

The GWMA chart was initial presented by [6] and extensive studied by [11] is weighted moving average of sequential historical observations. Since, each observation is differently weighted that decreases from the present period to past periods then it could be reflected to the important observations on recent process. This chart was extended and developed from EWMA chart by adding an adjustment smoothing constant (α). If the weighted historical observation constant equal to $q = 1-\lambda$ and $\alpha = 1$, then the GWMA chart coincides the EWMA chart.

The GWMA statistic is as following

$$G_t = \sum_{i=1}^{t} (q^{(i-1)^\alpha} - q^{i^\alpha}) X_{t-i+1} + q^{t^\alpha} G_0. \tag{6}$$

Taking geometric series to Eq. (6) then can be rewritten as

$$G_t = \frac{(1-q)(q-1)-(q-1)q(q-1)}{(q-1)(1-q)} X_{t-i+1} + q^\alpha G_0$$

where G_t is the GWMA statistic at time t^{th}, where the initial statistic value $G_0 = \mu_0$.

X_{t-i+1} is the observations of skew process at the $t-i+1^{th}$, $t = 2,3,...$

q is a weighted historical observations constant $(0 < q \le 1)$

α is an adjustment smoothing constant $(\alpha > 0)$.

Mean and variance of GWMA statistic are $E(G_t) = \mu_0$ and $Var(G_t) = \sigma_{G_t}^2 = Q_t \sigma_x^2$, respectively. Therefore, the control limits of GWMA chart are

$$UCL / LCL = \alpha_0 \pm L_3 \sigma_X \sqrt{Q_t}$$

where $Q_t = \sum_{i=1}^{t} (q^{(i-1)^\alpha} - q^{i^\alpha})^2$ and L_3 is the width of GWMA's control limit.

Table 1. Comparison When Processes Are Gamma (1, 2) $\lambda = 0.1$, α=0.8 and $ARL_0 = 370$.

δ	EWMA $L_1 = 3.739$	DEWMA $L_2 = 5.031$	GWMA $L_3 = 2.927$
0.00	370.723 (1.4754)	370.518 (1.422)	370.562 (4.27)
0.01	356.269 (1.4136)	353.274* (1.342)	359.968 (4.213)
0.05	304.372 (1.181)	297.022* (1.09)	320.555 (3.99)
0.1	254.524 (0.9689)	244.337* (0.864)	275.051 (3.704)
0.2	184.943 (0.6582)	175.643* (0.561)	206.229 (3.202)
0.3	142.62 (0.4715)	135.864* (0.389)	155.028 (2.747)
0.5	96.51 (0.2692)	85.117* (0.213)	88.616 (1.973)

Note: the standard error is showed in parentheses.

Table 2. Comparison When Processes Are Gamma (1, 2) $\lambda = 0.1$, α=0.8 and $ARL_0 = 500$.

δ	EWMA $L_1 = 3.761$	DEWMA $L_2 = 5.066$	GWMA $L_3 = 3.00$
0.00	500.858 (2.04)	500.523 (1.97)	500.884 (4.875)
0.01	480.741 (1.957)	475.99* (1.867)	486.752 (4.816)
0.05	406.439 (1.636)	393.45* (1.514)	435.169 (4.586)
0.1	334.012 (1.312)	361.091* (1.169)	379.678 (4.312)
0.2	235.835 (0.879)	217.873* (0.742)	285.485 (3.765)
0.3	175.992 (0.613)	162.396* (0.50)	217.583 (3.284)
0.5	113.354 (0.339)	107.513* (0.262)	127.412 (2.448)

Note: the standard error is showed in parentheses.

3. Average Run Length (ARL)

Generally, the performance of control chart are compared by considering the Average Run Length (ARL). The ARL is the expected number of samples obtained before a change in process is detected. It has two values under two states- ARL before an out-of-control state is detected when the process is in control defined as ARL0 and ARL1 before an out-of-control state is detected after process mean changed defined as ARL1. In this research, Monte Carlo simulation is used to evaluate the ARL which is classical method and very useful while the closed-form formula and the explicit expression of ARL are not exist. In addition, the results obtained from MC use for checking an accuracy the results from other approaches.

Table 3. Comparison When Processes Are Log-Normal (0, 1) $\lambda = 0.1$, α=0.8 and $ARL_0 = 370$.

δ	EWMA $L_1 = 4.211$	DEWMA $L_2 = 2.638$	GWMA $L_3 = 14.66$
0.00	370.815	370.87	370.015
	(1.654)	(1.632)	(4.05)
0.01	350.20	348.36*	353.833
	(1.559)	(1.529)	(4.124)
0.05	281.107	266.197*	292.833
	(1.261)	(1.171)	(4.03)
0.1	212.893	187.309*	211.713
	(0.950)	(0.83)	(4.087)
0.2	126.54	94.19*	120.352
	(0.576)	(0.426)	(3.855)
0.3	75.805	46.361*	68.214
	(0.35)	(0.232)	(3.747)
0.5	27.716	1.00*	22.885
	(0.14)	(0.001)	(1.141)

Note: the standard error is showed in parentheses.

Table 4. Comparison When Processes Are Log-Normal (0, 1) $\lambda = 0.1$, α=0.8 and $ARL_0 = 500$.

δ	EWMA $L_1 = 4.678$	DEWMA $L_2 = 2.907$	GWMA $L_3 = 14.7$
0.00	500.666	500.293	493.032
	(2.208)	(2.194)	(4.966)
0.01	474.086	464.936*	471.364
	(2.098)	(2.048)	(4.051)
0.05	378.745	343.288*	362.258
	(1.690)	(1.52)	(4.005)
0.1	286.307	235.821*	241.455
	(1.282)	(1.044)	(3.992)
0.2	167.791	116.417*	125.564
	(0.754)	(0.522)	(3.552)
0.3	99.181	58.363*	62.484
	(0.449)	(0.279)	(3.234)
0.5	36.432	1.00*	11.478
	(0.177)	(0.001)	(1.1)

Note: the standard error is showed in parentheses.

The approximation ARL by MC is given by

$$ARL = \frac{\sum_{t=1}^{N} RL_t}{N}.$$

The standard deviations of ARL (SDRL) as

$$SDRL = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(RL_t - ARL)^2}.$$

where RL_t is the number of observations used to monitoring before out-of-control in simulation t^{th} round and N =50,000 runs is the number simulation each situations.

4. Numerical Results

In this section, the performance of EWMA, DEMWA and GWMA charts in order to detect a change in parameter are compare by considering an out-of-control average run length (ARL_1) when observations are underlying gamma (2, 1) and log-normal (0, 1). The weighted parameter (λ) of EWMA, DEWMA and GWMA chart is given to equal 0.1, adjustment smoothing constant (α) is 0.8 and the magnitudes of shift are given to be δ = 0.01, 0.05, 0.1, 0.2, 0.3 and 0.5. The comparison of ARL_1 for gamma distribution when ARL_0 = 370 and 500 are shown on Table 1 and 2, respectively. When processes are log-normal distributed, the numerical comparison of ARL_1 are presented on Table 3 and 4.

5. Conclusions

According to the numerical results, the performance of memory-type control charts as EWMA, DEWMA and GWMA are investigated to study the robustness to skew process such gamma and log-normal distributions. The historical weighted of those control charts are given equally to 0.1 that means the past information are concerned similarly of all three control charts. Since, the value of ARL_1 obtained from DEWMA is minimum it can explain that the DEWMA outperforms to detect a change in parameter when observations are both gamma and log-normal distributions. Therefore, DEWMA's performance robust to the skew processes.

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