Critical Criterion Analysis for Multi-criteria Decision Making

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Abstract: In multi-criteria decision making process, Decision Maker may prefer one criterion more than the others. The preferences can be reflected via the weights of the criteria. Once a decision ranking has been obtained, the Decision Maker may want to change their preferences. The change may or may not affect the current decision ranking. The smallest change in the preferences value that affects the current ranking may determine the critical criterion. To seek for the critical criterion, the sensitivity of ranking to the variety of the criteria weights is analyzed in the paper. Different methods of multi-criteria decision analyses are used for comparison, such as the trade-off ranking, relative distance method and TOPSIS.

Key words: Critical criterion, multi-criteria decision making, relative distance, TOPSIS, trade-off.

1. Introduction

The multi-criteria problem consists of multiple criteria/objectives that need to be optimized simultaneously. The criteria are often conflicting. Hence, the solution is not unique and contains a number of alternatives. However, the Decision Maker needs to choose only one solution. The multi-criteria decision making methods can help the Decision Maker in this way.

The decision making problem is represented by the following decision matrix as in Table 1 where M is the number of alternatives $(1 < i \leq M)$ and N is the number of criteria $(1 < j \leq N)$.

Table 1. Decision Matrix

Alternative	Criterion C_1	C_2	…	C_N
	w_1	w_2	…	w_N
A_1	x_{11}	x_{12}	…	x_{1N}
A_2	x_{21}	x_{22}	…	x_{2N}
⋮	⋮	⋮	⋮	⋮
A_M	x_{M1}	x_{M2}	…	x_{MN}

The weight of each criterion is denoted as w_j $(j = 1, \dots, N)$. The weights represent the importance of each criterion in terms of the Decision Maker preferences and are used in the ranking calculation. Many procedures have been proposed to determine the weights. Von Winterfeldt and Edwards [1] proposed the ratio method and the swing method to determine the average weights. Butler et al [2] suggested three types of weights; random weight, rank order weight and response distribution weight. Olson [3] introduced the equal weights, the weights generated by ordinal rank and the weights generated by a regression technique. Kao [4] calculated the weights by minimizing the sum of squared distances. However, the most popular approach to obtain the weights is that carried out by the Decision Makers themselves [5]-[8]. Once a ranking is obtained, the Decision Makers may be interested in the sensitivity of the ranking to the criteria weights. The analysis of the weight changing versus the current ranking is considered in the paper. The idea of the analysis comes from the work of Triantaphyllou & Sánchez [9]. They carried out a sensitivity analysis for three decision making methods; the weighted sum model, the weighted product model and the analytic hierarchy process.

The rest of the paper is organized as follows. In the next section, three multi-criteria decision making methods used in the paper are described. In Section 3, the methodology used for the analysis purpose is presented. The analysis is given in Section 4 and the conclusion is discussed in Section 5.

2. Multi-criteria Decision Making Methods

Three multi-criteria decision making methods are used in the paper for the analysis and comparison. Each method is introduced briefly in this section.

2.1. Trade-off Ranking

A trade-off ranking method is a new decision making technique in aiding the decision Makers with multiple conflicting criteria problem. One possible application of the method is in group consensus of multi-agent systems [10]. The principle of the method is to have an alternative with the least compromise with the others as the best option. The minimization can be achieved by calculating the differences between each alternative and the others. The distance formula is used to measure the differences.

The general formula for the distance between alternative A_α and alternative A_β is:

$$d(A_\alpha, A_\beta) = \sqrt{\sum_{j=1}^{N} w_j^2 (x_{\alpha j} - x_{\beta j})^2},$$
$$w_j \geq 0,$$
$$\sum_{j=1}^{N} w_j = 1. \tag{1}$$

The sum of differences of one alternative, A_α with all other alternatives, also known as the degree of trade-off, DT is calculated as:

$$DT_\alpha = \sum_{k=1}^{M} [d(A_\alpha, A_k)], \quad \alpha = 1,2,\ldots,M. \tag{2}$$

The trade-off ranking is then determined by the value of DT where the least value holds the highest ranking.

2.2. Relative Distance Method

The relative distance method is proposed by Kao [4]. The method measures the differences of an alternative with the ideal and anti-ideal solutions. The alternative with the least difference with the ideal solution and the most difference with the anti-ideal solution is regarded as the best alternative.

The ideal solution, I_R^+ and the anti-ideal solution, I_R^- are determined by the followings:

$$I_R^+ = (C_1^+, C_2^+, \dots, C_N^+),$$

$$I_R^- = (C_1^-, C_2^-, \dots, C_N^-),$$

where

$$C_j^+ = \min\left(x_{ij}, i = 1,2, \dots, M\right), \;\; j = 1,2, \dots N$$

$$C_j^- = \max\left(x_{ij}, i = 1,2, \dots, M\right), \;\; j = 1,2, \dots N.$$

The difference of each alternative with the ideal and anti-ideal solutions is then calculated respectively by the formulae:

$$dR_i^+ = \sum_{j=1}^{N} w_j \left|C_j^+ - x_{ij}\right|, \quad i = 1,2, \dots, M \tag{3}$$

$$dR_i^- = \sum_{j=1}^{N} w_j \left|C_j^- - x_{ij}\right|, \quad i = 1,2, \dots, M, \tag{4}$$

$$w_j \geq 0,$$

$$\sum_{j=1}^{N} w_j = 1.$$

The constraints of the weight in [4] is different than given in above formulae. In [4], the weights are determined by optimizing the total distance of every alternatives to the ideal solution. However, [4] does not exclude the weights obtained by Decision Maker. Thus, to standardize the weight calculation in relative distance method with the two other methods, this study consider the exclusion, hence the different in the weight constraint.

The alternative with the shortest distance to the ideal solution, I_R^+ and the longest distance to the anti-ideal solution, I_R^- is regarded as the best solution.

2.3. TOPSIS

TOPSIS method also measures the differences of an alternative with the ideal and anti-ideal solutions. The method is proposed by Hwang & Yoon [7]. In comparison with the relative distance approach, the data set in TOPSIS method is standardized as follow:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}}, \;\; i = 1, \dots, M, j = 1, \dots, N.$$

The ideal solution, I_T^+ and the anti-ideal solution, I_T^- for TOPSIS method are determined as:

$$I_T^+ = (v_1^+, v_2^+, \dots, v_N^+),$$

$$I_T^- = (v_1^-, v_2^-, \dots, v_N^-),$$

where

$$v_j^+ = \min\left(v_{ij}, i = 1, \dots, M\right),$$

$$v_j^- = \max(v_{ij}, i = 1, \dots, M), \ (j = 1, \dots, N)$$

$$v_{ij} = w_j r_{ij} \left(\sum_{j=1}^{N} w_j = 1 \text{ and } w_j \geq 0 \right),$$

$$w_j \geq 0,$$

$$\sum_{j=1}^{N} w_j = 1.$$

Another difference between TOPSIS and the relative distance approach is in the distance calculation. The relative distance method uses a comparative distance which deduced to a distance formula in L_1, while TOPSIS is based on the distance formula in L_2. The formulae for the distance of each alternative to the ideal and anti-ideal solutions are:

$$dT_i^+ = \sqrt{\sum_{j=1}^{N} w_j{}^2 \left(C_j^+ - x_{ij}\right)^2}, \quad i = 1,2, \dots, M, \tag{5}$$

$$dT_i^- = \sqrt{\sum_{j=1}^{N} w_j{}^2 \left(C_j^- - x_{ij}\right)^2}, \quad i = 1,2, \dots, M. \tag{6}$$

The full ranking in TOPSIS is then calculated as:

$$D_i = \frac{dT_i^-}{\mathrm{d}T_i^+ + dT_i^-}. \tag{7}$$

An alternative with the largest value of D_i is regarded as the best option.

3. Methodology

The methodology used for the analysis is given in this section. Let δ_1 denotes the change in the current weight w_1 associated with criterion C_1. Thus, a new weight for criterion C_1 is $w_1^* = w_1 + \delta_1$ where $\delta_1 \geq -w_1$ since $w_j^* \geq 0$ $(j = 1, \dots, N)$. Note that the weights w_j are normalized such that $\sum_{j=1}^{N} w_j = 1$. Hence, the new normalized weights, $w_j^{'}$ $(j = 1, .., N)$ for the case of the weight change in criterion C_1 are then given by the formulae:

$$w_1^{'} = \frac{w_1^*}{w_1^* + w_2 + \dots + w_N} \tag{8}$$

$$w_j^{'} = \frac{w_j}{w_1^* + w_2 + \dots + w_N} \quad \text{for } j \neq 1. \tag{9}$$

The new ranking is calculated by substituting the new normalized weights, formulae (8) and (9) into formulae (1)-(7). The weight change analysis is done for each criterion C_j $(j = 1, \dots, N)$ with any possible value δ_j $(j = 1, \dots, N)$. Formula (8) and (9) are replaced by a different value of *j* according to the new weight in each analysis case.

The critical criterion is determined after the whole analysis has been completed. The critical criterion is defined by referring to Triantaphyllou & Sánchez [9]. Triantaphyllou & Sánchez introduced four definitions of critical criterion based on absolute term, relative term, top ranking and any ranking. In this study, the critical criterion is defined as a criterion with the smallest changes in the current weights which affect the current ranking. The absolute value is used to determine the smallest changes.

4. Critical Criterion Analysis

Consider the data as in Table 2 taken from Triantaphyllou & Sánchez [9]. The data consist of four alternatives and four criteria with associated weights. Criterion C_1 has the largest weight, while C_4 has the smallest.

Table 2. Data of the Problem

	C_1 $w_1 = 0.3277$	C_2 $w_2 = 0.3058$	C_3 $w_3 = 0.2876$	C_4 $w_4 = 0.0790$
A_1	0.3088	0.2897	0.3867	0.1922
A_2	0.2163	0.3458	0.1755	0.6288
A_3	0.4509	0.2473	0.1194	0.0575
A_4	0.0240	0.1172	0.3184	0.1215

Table 3. Current Ranking for Each Method

Alternative	Trade-off ranking	TOPSIS	Relative distance method
A_1	1	4	4
A_2	2	2	3
A_3	3	3	2
A_4	4	1	1

Table 3 shows the current ranking for each decision making method calculated from the formulae (1)-(7). The current ranking for the trade-off method is different in comparison to the other two methods, TOPSIS and the relative distance ranking. The best alternative in the trade-off method is ranked the lowest in the other two methods. In turn, their best solution is ranked the worst in the trade-off approach (see Note in the Appendix).

The analysis of the changes in each of the weights (w_1, w_2, w_3, w_4) separately may give insight into determining the critical criterion. The results of the analysis are given in Tables 4-7.

Table 4. New Ranking for Each Method with Weight Change in w_1.

δ_1	-0.3			-0.2			-0.1			0.2			0.4			0.6		
$\begin{pmatrix} w'_1 \\ w'_2 \\ w'_3 \\ w'_4 \end{pmatrix}$	$\begin{pmatrix} 0.04 \\ 0.44 \\ 0.41 \\ 0.11 \end{pmatrix}$			$\begin{pmatrix} 0.16 \\ 0.38 \\ 0.36 \\ 0.10 \end{pmatrix}$			$\begin{pmatrix} 0.25 \\ 0.34 \\ 0.32 \\ 0.09 \end{pmatrix}$			$\begin{pmatrix} 0.44 \\ 0.25 \\ 0.24 \\ 0.07 \end{pmatrix}$			$\begin{pmatrix} 0.52 \\ 0.22 \\ 0.21 \\ 0.06 \end{pmatrix}$			$\begin{pmatrix} 0.58 \\ 0.19 \\ 0.18 \\ 0.05 \end{pmatrix}$		
Method	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
A_1	2	4	4	1	4	4	1	4	4	1	3	4	1	3	3	1	3	3
A_2	4	3	3	2	3	3	2	3	3	2	2	2	2	2	2	2	2	2
A_3	1	1	1	3	2	2	3	2	2	3	4	3	3	4	4	3	4	4
A_4	3	2	2	4	1	1	4	1	1	4	1	1	4	1	1	4	1	1

I = trade-off ranking; II = TOPSIS; III = Relative distance

Table 4 shows the result of the rankings for the weight changes in w_1 for criterion C_1 analysis. As can be seen, the current ranking for the trade-off ranking method starts to change with $\delta_1 = -0.3$. The Decision

Maker may analyze the change in the range of $\delta_1 = [-0.3, -0.2]$ to determine the smallest changes in w_1 that affects the current trade-off ranking.

The current ranking by TOPSIS does not retain with any of the changes in w_1. However, the best solution for TOPSIS (alternative A_4) remains the first ranking until $\delta_1 = -0.3$ where it starts downgrading into the second choice in contrast to alternative A_3. A drastic change in the TOPSIS ranking for alternative A_3 starts to occur in the range of $\delta_1 = [-0.1, 0.2]$ as it changes from the worst option to the second one. More analysis in the specific range is needed if the Decision Maker wants to determine the exact value of δ_1 at which the change happens.

The current ranking for the relative distance approach retains with $\delta_1 = [-0.2, -0.1]$. However, at $\delta_1 = -0.3$, the current ranking changes with the switch of the first and second rankings. The first current option (alternative A_4) swaps with the second current option (alternative A_3). With $\delta_1 = 0.2$, the same shift occurs with the second and third rankings, while the best and worst alternatives retain. For $\delta_1 = 0.4$ and $\delta_1 = 0.6$, all the second, third and fourth current rankings change while the first one remains. The Decision Maker may want to analyze the changes in the whole ranking or may be interested in looking into the changes of the best ranking only.

Table 5. New Ranking for Each Method with Weight Change in w_2.

δ_2	-0.3			-0.2			-0.1			0.2			0.4			0.6		
$\begin{pmatrix} w'_1 \\ w'_2 \\ w'_3 \\ w'_4 \end{pmatrix}$	$\begin{pmatrix} 0.47 \\ 0.01 \\ 0.41 \\ 0.11 \end{pmatrix}$			$\begin{pmatrix} 0.41 \\ 0.13 \\ 0.36 \\ 0.10 \end{pmatrix}$			$\begin{pmatrix} 0.36 \\ 0.23 \\ 0.32 \\ 0.09 \end{pmatrix}$			$\begin{pmatrix} 0.27 \\ 0.42 \\ 0.24 \\ 0.07 \end{pmatrix}$			$\begin{pmatrix} 0.23 \\ 0.50 \\ 0.21 \\ 0.06 \end{pmatrix}$			$\begin{pmatrix} 0.20 \\ 0.57 \\ 0.18 \\ 0.05 \end{pmatrix}$		
Methods	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
A_1	2	4	4	2	4	4	2	4	4	1	4	4	1	4	4	1	4	4
A_2	1	2	2	1	2	2	1	2	3	2	2	3	2	3	3	3	3	3
A_3	4	3	3	3	3	3	3	3	2	3	3	2	3	2	2	2	2	2
A_4	3	1	1	4	1	1	4	1	1	4	1	1	4	1	1	4	1	1

I = trade-off ranking; II = TOPSIS; III = Relative distance

Table 5 shows the new rankings of the weight changes in w_2 for criterion C_2 analysis. Current ranking for the trade-off ranking method preserves in the range of $\delta_2 = [0.2, 0.4]$. If $\delta_2 = 0.6$, the second and third choices of the current trade-off rankings swap their places. However, the alternative A_1 remains as the first choice. Similarly, the current ranking starts changing with $\delta_2 = -0.1$, where alternative A_1 is not the best option anymore, and replaced by alternative A_2. In the analysis done, the change of $\delta_2 = -0.1$ is regarded as the smallest change in w_2 that affects the ranking for the trade-off ranking method.

For the analysis with TOPSIS method, the current ranking starts changing from $\delta_2 = 0.4$, where the changes only occur in the second and third options. While, the best and worst rankings retain as the same alternatives. The same ranking changes occur in the analysis with the relative distance approach. The first and last choices retain throughout the changes. However, the second and third options start to change their ranking places at $\delta_2 = -0.2$.

Table 6 shows the result of the rankings for the weight changes in w_3 for criterion C_3. Current ranking for the trade-off method changes at $\delta_3 = 0.2$ onwards where alternative A_2 becomes the first choice instead of alternative A_1. Further analysis in the range of $\delta_3 = [-0.1, 0.2]$ can be done to find the exact value where the change starts to occur. In this analysis the value of $\delta_3 = 0.2$ is the smallest change in w_3 that affects the current ranking.

The current ranking for the TOPSIS retains with $\delta_3 = 0.2$. As $\delta_3 = -0.1$, the third and the fourth rankings shift towards each other, while the highest ranking retains. In the relative distance method, the current ranking changes at $\delta_3 = -0.2$. Again, it is only the third and fourth options swap. The highest ranking in the relative distance approach (alternative A_3) downgrades into the second choice with $\delta_3 = 0.6$.

Table 6. New Ranking for Each Method with Weight Change in w_3.

δ_3	-0.2			-0.1			0.2			0.4			0.6			0.8		
$\begin{pmatrix} w'_1 \\ w'_2 \\ w'_3 \\ w'_4 \end{pmatrix}$	$\begin{pmatrix} 0.41 \\ 0.38 \\ 0.11 \\ 0.10 \end{pmatrix}$			$\begin{pmatrix} 0.36 \\ 0.34 \\ 0.21 \\ 0.09 \end{pmatrix}$			$\begin{pmatrix} 0.27 \\ 0.25 \\ 0.41 \\ 0.07 \end{pmatrix}$			$\begin{pmatrix} 0.23 \\ 0.22 \\ 0.49 \\ 0.06 \end{pmatrix}$			$\begin{pmatrix} 0.21 \\ 0.19 \\ 0.55 \\ 0.05 \end{pmatrix}$			$\begin{pmatrix} 0.18 \\ 0.17 \\ 0.61 \\ 0.04 \end{pmatrix}$		
Methods	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
A_1	1	3	3	1	3	4	2	4	4	3	4	4	3	4	4	3	4	4
A_2	2	2	4	2	2	3	1	2	3	1	1	3	1	1	3	1	2	3
A_3	3	4	2	3	4	2	4	3	2	4	2	2	4	2	1	4	1	1
A_4	4	1	1	4	1	1	3	1	1	2	3	1	2	3	2	2	3	2

I = trade-off ranking; II = TOPSIS; III = Relative distance

Table 7. New Ranking for Each Method with Weight Change in w_4.

δ_4	0.1			0.2			0.4			0.6			0.8			1.0		
$\begin{pmatrix} w'_1 \\ w'_2 \\ w'_3 \\ w'_4 \end{pmatrix}$	$\begin{pmatrix} 0.30 \\ 0.28 \\ 0.26 \\ 0.16 \end{pmatrix}$			$\begin{pmatrix} 0.27 \\ 0.25 \\ 0.24 \\ 0.23 \end{pmatrix}$			$\begin{pmatrix} 0.23 \\ 0.22 \\ 0.21 \\ 0.34 \end{pmatrix}$			$\begin{pmatrix} 0.20 \\ 0.19 \\ 0.18 \\ 0.42 \end{pmatrix}$			$\begin{pmatrix} 0.18 \\ 0.17 \\ 0.16 \\ 0.49 \end{pmatrix}$			$\begin{pmatrix} 0.16 \\ 0.15 \\ 0.14 \\ 0.54 \end{pmatrix}$		
Methods	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
A_1	1	4	3	1	3	3	1	3	3	1	3	3	1	3	3	1	3	3
A_2	2	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
A_3	3	2	2	2	2	2	3	2	2	3	2	2	3	2	2	3	2	2
A_4	4	1	1	3	1	1	2	1	1	2	1	1	2	1	1	2	1	1

I = trade-off ranking; II = TOPSIS; III = Relative distance

Table 7 shows new rankings of the weight w_4 changes for criterion C_4. The current ranking for the trade-off method changes at $\delta_4 = 0.2$. However, the best alternative (alternative A_1) retains throughout the changes. The affected rankings are the second, third and fourth ranks only. The top ranking for TOPSIS (alternative A_4) also retains throughout the changes. However, the full current ranking is not preserved in any of these changes. Even with the smallest change of $\delta_4 = 0.1$, the second and third ranks are reversed. The same ranking situation occurs in the relative distance approach. There are no changes in the weight which preserve the current ranking. Nevertheless, from the change of $\delta_4 = 0.1$ onwards, the full rankings of the relative distance method remain the same with alternative A_4 as the best option and alternative A_2 as the worst one.

From the results of the analysis in Tables 4-7, the critical criterion in the trade-off ranking method is the criterion C_2 where the smallest change of $\delta_2 = 0.1$ affects the current full ranking. For TOPSIS, the changes of the weights in criteria C_1, C_3 and C_4 have the same critical value at $\delta_1 = \delta_3 = \delta_4 = 0.1$. After further analysis, the critical criterion in TOPSIS is criterion C_4 with the smallest change of $\delta_4 = 0.07$. Criterion C_2 is the most non-critical criterion in TOPSIS method. In the relative distance approach, criterion C_4 is the critical criterion with the smallest change of $\delta_4 = 0.1$ affecting the current full ranking. It is interesting to note that criterion C_1 has the largest weight in the current data. However, it is not the critical criterion in either the trade-off ranking, TOPSIS or the relative distance method.

5. Conclusion

The paper has presented a sensitivity analysis of the weight changes in three multi-criteria decision making methods: the trade-off ranking, TOPSIS and the relative distance approach. The analysis is done to determine the critical criterion with the smallest changes in the weights that affect the current ranking. The study showed that, in the analysis with three decision making methods, the critical criterion in each method is not a criterion with the most weight. Thus, it is not an important criterion. Due to that, with the knowledge of the critical criterion, the Decision Maker may discard that criterion in order to obtain a more robust ranking due to preference perturbation.

Appendix

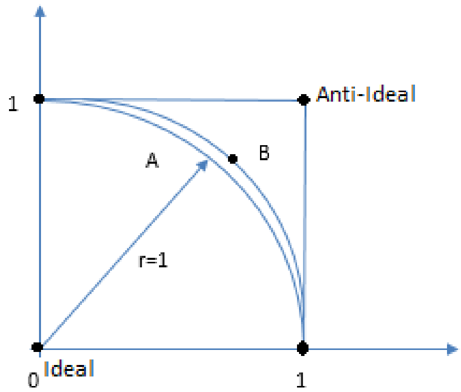

Fig. 1. Graph of a quarter circle A and an arc B.

Next, consider an example of how the current ranking in Table 3 occurs. Figure 1 shows two curves A and B where A is a quarter circle and B is an arc. The middle point of B has the distance greater than 1 to the ideal solution. With the trade-off method, the middle point holds the first rank in the B arc. In turn, with TOPSIS and the relative distance approach, the point holds the worst rank as it is the closest to the anti-ideal and the farthest from the ideal.

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