Modeling the Effect of Decreasing Dissolved Oxygen on Fish Population Survival in Aquatic Body in the Presence of Nutrients

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Abstract: In this paper a nonlinear mathematical model is proposed to analyze the effect of decreasing level of dissolved oxygen on the survival of aquatic species in the water body in the presence of nutrients. This model has four nonlinear differential equations consisting of four state variables accounting for concentration of nutrients, density of algae, density of fish population, and concentration of dissolved oxygen. The three equilibrium points have been obtained to understand the dynamic behavior of the system. The conditions for existence of these equilibrium points are obtained. The stability analysis is conducted on each equilibrium point by considering variational matrix and using Routh-Hurwitz criteria. The nonlinear stability behavior of an equilibrium point considered to be critical is analyzed by applying Liapunov's direct method.

Key words: Algae, dissolved oxygen, Liapunov's function, mathematical model, nutrients.

1. Introduction

The water runoff from the farmlands and factories contains pollutants in the form of fertilizers and organic matters. These pollutants have nutrients such as nitrogen, phosphorus, carbon etc. The nutrient rich chemicals help the aquatic plants to grow better as they do for agricultural plants. The excessive amount of these nutrients causes excessive growth of algae resulting in the depletion of dissolved oxygen (DO) which in turn causes threat to the survival of aquatic species. The research work in this area started in the year 1925. Streeter and Phelps [1] in 1925 proposed the first dynamical model on depleting DO due to discharge of organic pollutants. Further Dobbins [2] and O' Corner [3] generalized this model. Many researchers [4]-[8] studied depletion of DO by various sources and the survival of aquatic species. Arnold and Voss [9] studied numerical behavior of eutrophied lakes. Marler and Westfield [10] discussed a mathematical model based on ecological problem in which depletion of DO occurs due to growth of bacteria and protozoa. Shukla *et al*. [11], [12] studied the existence and survival of two competing species in a polluted water body and effect of changing habitat on survival of aquatic species.

Phytoplankton also plays an important role in the dynamics of aquatic body. Khare *et al*. [13], [14] discussed the effect of the depleting DO on the survival of interacting planktonic population. However they didnot consider the crowding effect factor in their analysis. Many investigators [15] carried out modeling studies involving zooplankton, phytoplankton and nutrients. However they did not consider the concentration of DO in the model. Keeping in view these aspects, in the present study a mathematical model is proposed to estimate the effect of decreasing DO on fish population survival in aquatic body in the presence of nutrients. It accounts for concentration of nutrients, density of algae, density of fish population, and concentration of DO. Analysis of each equilibrium point is carried out in which the existence criterion of each point is shown. Further, the stability behavior of these points is discussed by using Routh-Hurwitz criteria. Then the nonlinear stability behavior of the critical point is studied by using Liapunov's direct method. To substantiate the feasibility of the model, numerical simulation is carried out.

2. Mathematical Model

Let N be the cumulative concentration of nutrients, A be the density of algae, C be the concentration of DO, and F be the density of fish populations. These are the four state variables. Further let us consider Q be the cumulative rate of discharge of nutrients, q be the rate of growth of DO by various sources. It is further assumed that the growth rate of algae is proportional to the term $[NA/\alpha_1+C_0-C]$. Let ν_2, ν_3, and ν_4 are the natural depletion rate of algae, concentration of DO and fish population respectively. Predation rate of algae by fish is α_2. It is assumed that the growth rate of fish population is proportional to the term $[AF/\ \alpha_4+C_0-C]$. Here α_1 and α_4 are assumed to be half saturation constants, C_0 is the saturation value of DO and C_0-C is oxygen deficit.

Considering the above facts, the system is governed by following nonlinear ordinary differential equations.

$$\frac{dN}{dt} = Q - \nu_1 N - \beta_1 NA, \tag{1}$$

$$\frac{dA}{dt} = \frac{\beta_2 NA}{\alpha_1 + C_0 - C} - \nu_2 A - \alpha_2 AF, \tag{2}$$

$$\frac{dC}{dt} = q - \nu_3 C, \text{ and} \tag{3}$$

$$\frac{dF}{dt} = \frac{\alpha_3 AF}{\alpha_4 + C_0 - C} - \nu_4 F. \tag{4}$$

Here ν_1, ν_2, ν_3, and ν_4 are depletion rate coefficients and β_1, β_2, α_2, and α_3 are positive proportionality constants. It is important to note here that the initial value of each parameter is positive i.e. $N(0)=N_{10}>0$, $A(0)=A_{10}>0$, $C(0)=C_{10}>0$, and $F(0)=F_{10}>0$.

3. Analysis of equilibrium points of the system

The model discussed in the above section has following three nonnegative equilibrium points.

1) Initially, it is considered that there is no change in the rate of flow of nutrients and concentration of DO, thus $dN/dt = 0$ and $dC/dt = 0$ which give the terms Q/ν_1 and q/ν_3. The 1st equilibrium point E_1 is obtained which always exists and given by $E_1(Q/\nu_1, 0, q/\nu_3, 0)$.
2) The 2nd equilibrium point is given by $E_2(N', A', C', 0)$.

where $N' = \dfrac{\nu_2}{\beta_2 \nu_3}(\alpha_1 \nu_3 + C_0 \nu_3 - q)$, $C' = \dfrac{q}{\nu_3}$, and $A' = \dfrac{Q\beta_2\nu_3 - \nu_1\nu_2\left(\nu_1\nu_3 + C_0\nu_3 - q\right)}{\beta_1\nu_2(\alpha_1\nu_3 + C_0\nu_3 - q)}$.

Thus E_2 exists, provided the following conditions are satisfied:

$$\alpha_1 v_3 + C_0 v_3 - q > 0,\; Q\beta_2 v_3 - v_1 v_2 (\alpha_1 v_3 + C_0 v_3 - q) > 0,\; v_4 (\alpha_4 + C_0 - C') - \alpha_3 A' > 0,$$
$$\alpha_1 + C_0 - C' > 0, \text{ and } Q - v_1 N' > 0.$$

3) The 3rd equilibrium point is given by $E_3(N^*, A^*, C^*, 0)$, where

$$C^* = \frac{q}{v_3},\; A^* = \frac{v_4(\alpha_4 v_3 + C_0 v_3 - q)}{\alpha_3 v_3},$$

$$N^* = \frac{Q\alpha_3 v_3}{[v_1 v_3 \alpha_3 + \beta_1 v_4(\alpha_4 v_3 + C_0 v_3 - q)]},$$

and

$$F^* = \frac{Q\alpha_3 v_3^{\,2}\beta_2}{\alpha_2[\alpha_1 v_3 + C_0 v_3 - q][v_1 v_3 \alpha_3 + \beta_1 v_4(\alpha_4 v_3 + C_0 v_3 - q)]} - \frac{v_2}{\alpha_2}.$$

Thus E_3 exists if these conditions are satisfied:

$$\alpha_4 v_3 + C_0 v_3 - q > 0,\, \beta_2 N^* - v_2(\alpha_1 + C_0 - C^*) > 0,\, (\alpha_1 + C_0 - C^*) > 0, \text{ and } (\alpha_4 + C_0 - C^*) > 0$$

4. Stability Analysis

In this section the stability behavior of each equilibrium point is discussed. The variational matrix $J_i$of The system"(1)–(4)" is given as follows :

$$J_i = \begin{bmatrix} -v_1 - \beta_1 A & -\beta_1 N & 0 & 0 \\ \dfrac{\beta_2 A}{\alpha_1 + C_0 - C} & \dfrac{\beta_2 N}{\alpha_1 + C_0 - C} - v_2 - \alpha_2 F & \dfrac{\beta_2 NA}{(\alpha_1 + C_0 - C)^2} & -\alpha_2 A \\ 0 & 0 & -v_3 & 0 \\ 0 & \dfrac{\alpha_3 F}{\alpha_4 + C_0 - C} & \dfrac{\alpha_3 AF}{(\alpha_4 + C_0 - C)^2} & \dfrac{\alpha_3 A}{\alpha_4 + C_0 - C} - v_4 \end{bmatrix}$$

4.1. Case 1: Stability of E_1 (Q/v_1, 0, q/v_3, 0)

Consider the following jacobian matrix J_1corresponding to E_1

$$J_1 = \begin{bmatrix} -v_1 & -\beta_1 \dfrac{Q}{v_1} & 0 & 0 \\ 0 & \dfrac{\beta_2 Q v_3}{v_1(\alpha_1 v_3 + C_0 v_3 - q)} - v_2 & 0 & 0 \\ 0 & 0 & -v_3 & 0 \\ 0 & 0 & 0 & -v_4 \end{bmatrix}$$

Eigen values are $\lambda_1 = -\nu_1$, $\lambda_2 = -\nu_3$, $\lambda_3 = -\nu_4$, and $\lambda_4 = \frac{\beta_2 Q \nu_3 - \nu_1 \nu_2 (\alpha_1 \nu_3 + C_0 \nu_3 - q_0)}{\nu_1 (\alpha_1 \nu_3 + C_0 \nu_3 - q_0)}$.

Upon implementing Routh-Hurwitz criteria, all the eigen values of matrix J_1 must be negative. It is observed that $\lambda_1, \lambda_2,$ and λ_3 are negative. Therefore, E_1 will be stable only if $\boldsymbol{\lambda_4}$ is also negative and thus the following condition is obtained $(\nu_1\nu_2\nu_3\alpha_1 + C_0\nu_1\nu_2\nu_3) > (\beta_2 Q\nu_3 + \nu_1\nu_2 q_0)$.

4.2. Case 2: Stability of E_2 (N', A', C', 0)

Corresponding to the point E_2, jacobian matrixJ_2 is obtained.

$$J_2 = \begin{bmatrix} -a_{11} & -\beta_1 N' & 0 & 0 \\ a_{21} & a_{22} & a_{23} & -\alpha_2 A' \\ 0 & 0 & -\nu_3 & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

where $a_{11} = \alpha_1 + \beta_1 A'$. and $a_{22} = \frac{\beta_2 N'}{\alpha_1 + C_0 - C'} - \nu_2$.

Upon substituting the values of N' and C' we get

$$a_{22} = 0,\ a_{21} = \frac{\beta_2 A'}{\alpha_1 + C_0 - C'},\ a_{23} = \frac{\beta_2 N' A'}{(\alpha_1 + C_0 - C')^2},\ \text{and}\ a_{44} = \frac{\alpha_3 A'}{\alpha_4 + C_0 - C'} - \nu_4.$$

Characteristic equation corresponding to the above jacobian J_2 is

$$(-\nu_3 - \lambda)\ (\frac{\alpha_3 A'}{\alpha_4 + C_0 - C'} - \nu_4 - \lambda)\ \left(\lambda^2 + a_{11}\lambda + a_{21} N' \beta_1\right) = 0. \qquad (5)$$

The eigen values of "(5)"are given by the following equations

$$\lambda_1 = -\nu_3,\ \lambda_2 = \frac{\alpha_3 A'}{\alpha_4 + C_0 - C'} - \nu_4 = -\left[\frac{\nu_4(\alpha_4 + C_0 - C) - \alpha_3 A'}{(\alpha_4 + C_0 - C)}\right].$$

Consider $\left(\lambda^2 + a_{11}\lambda + a_{21} N' \beta_1\right)$ = 0, which gives the following values

$$\lambda_3 = \frac{-a_{11} + \sqrt{a_{11}^2 - 4a_{21} N' \beta_1}}{2} = \frac{-(a_{11} - \sqrt{a_{11}^2 - 4a_{21} N' \beta_1})}{2}, \text{ and}$$

$$\lambda_4 = \frac{-a_{11} - \sqrt{a_{11}^2 - 4a_{21} N' \beta_1}}{2}.$$

Upon applying Routh–Hurwitz criteria, it is found that all the eigenvalues with respect to E_2 are negative. Therefore, the equilibrium point $E_2$is locally asymptotically stable.

4.3. Case 3: Stability of $E_3\left(N^*, A^*, C^*, F^*\right)$

Consider the Jacobian matrix J_3 corresponding to the point E_3

$$J_3 = \begin{bmatrix} -a_{11} & -\beta_1 N* & 0 & 0 \\ a_{21} & 0 & a_{23} & -\alpha_2 A* \\ 0 & 0 & -\nu_3 & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

The characteristic equation is $(-\nu_3-\lambda)[-\lambda^3 - a_{11}\lambda^2 - (\alpha_2\nu_4 F^* + a_{21}N^*\beta_1)\lambda - a_{11}\alpha_2\nu_4 F^*] = 0.$

Therefore, one value of λ is $\lambda_1 = -\nu_3$ which is negative. Now consider the following cubic equation

$$\lambda^3 + k_1\lambda^2 + k_2\lambda + k_3 = 0.$$

where $k_1 = a_{11} > 0,\ k_2 = \alpha_2\nu_4 F* + a_{21}N*\beta_1 > 0,\ k_3 = a_{11}\alpha_2\nu_4 F* > 0,$ and $k_1k_2 - k_3 = a_{11}a_{21}N*\beta_1 > 0.$

Upon implementing Routh-Hurwitz criteria it is shown that the coefficients of λ^2, λ, and the constant term in the above cubic equation are positive and the product of coefficients of λ^2 and λ is greater than the constant term. This shows that all the eigen values of jacobian J_3 are negative and thus the equilibrium point E_3 is asymptotically stable. The dynamic behavior of point $E_3$is analyzed by using Liapunov's direct method in the following theorem.

5. Nonlinear Stability Analysis of E_3

Theorem: The equilibrium point E_3 is nonlinearly stable if the following conditions are satisfied:

$$\left[\frac{n_1\beta_2\nu_3}{(\alpha_1\nu_3 + C_0\nu_3 - q\ )} - \beta_1 N*\right]^2 < \frac{2n_1\nu_1\alpha_2}{3},\quad \left[\frac{n_1\beta_2 N*}{(\alpha_1\nu_3 + C_0\nu_3 - q\ )(\alpha_1 + C_0 - C*)}\right]^2 \nu_3 < \frac{n_1n_2\alpha_2}{3},$$

$$\left[\frac{n_3\alpha_2\nu_3}{(\alpha_4\nu_3 + C_0\nu_3 - q\ )}\right]^2 < \frac{n_1{}^2\alpha_2{}^2}{6},\ \text{and}\ \left[\frac{n_3\alpha_3 A*}{(\alpha_4\nu_3 + C_0\nu_3 - q\ )(\alpha_4 + C_0 - C*)}\right]^2 \nu_3 < \frac{n_1n_2\alpha_2}{2}.$$

Proof: Let us consider the positive definite function

$$V = \frac{1}{2}(N - N*)^2 + n_1\left(A - A* - A*\ln\frac{A}{A*}\right) + \frac{1}{2}n_2(C - C*)^2 + n_3\left(F - F* - F*\ln\frac{F}{F*}\right) \quad (6)$$

where n_1, n_2 and n_3 are positive constants, to be chosen appropriately. Let us consider the derivative

$$\frac{dV}{dt} = (N - N*)\frac{dN}{dt} + n_1\frac{(A - A*)}{A}\frac{dA}{dt} + n_2(C - C*)\frac{dC}{dt} + n_3\frac{(F - F*)}{F}\frac{dF}{dt} \quad (7)$$

$$\frac{dV}{dt} = Y_1\frac{dN}{dt} + n_1\frac{Y_2}{A}\frac{dA}{dt} + n_2Y_3\frac{dC}{dt} + n_3\frac{Y_4}{F}\frac{dF}{dt} \quad (8)$$

where $Y_1 = (N - N^*),\ Y_2 = (A - A^*),\ Y_3 = (C - C^*),\ Y_4 = (F - F^*)$

Upon substituting the values of *dN/dt, dA/dt, dC/dt* and *dF/dt* from the model "(1)–(4)" and then applying some algebraic manipulations by using the inequality $x^2+y^2 \geq 2xy$, *dV/dt* is reduced into following inequality

$$\frac{dV}{dt} \leq -\beta_1 A Y_1^2 - \frac{1}{2} 2\nu_1 Y_1^2 + Y_1 Y_2 \left[\frac{n_1 \beta_2}{(\alpha_1 + C_0 - C)} - \beta_1 N^* \right] - \frac{1}{2}\frac{n_1\alpha_2}{3} Y_2^2 - \frac{1}{2}\frac{n_1\alpha_2}{3} Y_2^2$$
$$+ \frac{n_1 \beta_2 N^*}{(\alpha_1 + C_0 - C)\ (\alpha_1 + C_0 - C^*)} Y_2 Y_3 - \frac{1}{2} n_2 \nu_3 Y_3^2 - \frac{1}{2}\frac{n_1\alpha_2}{3} Y_2^2 + \frac{n_3\alpha_3}{(\alpha_4 + C_0 - C)} Y_2 Y_4 - \frac{1}{2}\frac{n_1\alpha_2}{2} Y_4^2 - \frac{1}{2} n_2 \nu_3 Y_3^2$$
$$+ \frac{n_3\alpha_3 A^*}{(\alpha_4 + C_0 - C)\ (\alpha_4 + C_0 - C^*)} Y_3 Y_4 - \frac{1}{2}\frac{n_1\alpha_2}{2} Y_4^2.$$

$$\frac{dV}{dt} \leq -\beta_1 A Y_1^2 - \frac{1}{2} P_{11} Y_1^2 + P_{12} Y_1 Y_2 - \frac{1}{2} P_{22} Y_2^2 - \frac{1}{2} P_{22} Y_2^2 + P_{23} Y_2 Y_3 - \frac{1}{2} P_{33} Y_3^2 - \frac{1}{2} P_{22} Y_2^2 + P_{24} Y_2 Y_4$$
$$- \frac{1}{2} P_{44} Y_4^2 - \frac{1}{2} P_{33} Y_3^2 + P_{34} Y_3 Y_4 - \frac{1}{2} P_{44} Y_4^2.$$

where

$$P_{11} = 2\nu_1,\ P_{22} = \frac{n_1\alpha_2}{3},\ P_{33} = n_2\nu_3,\ P_{44} = \frac{n_1\alpha_2}{2},\ P_{23} = \frac{N^* n_1 \beta_2}{(\alpha_1 + C_0 - C)(\alpha_1 + C_0 - C^*)},$$

$$P_{24} = \frac{n_3\alpha_3}{(\alpha_4 + C_0 - C)},\ P_{34} = \frac{n_3\alpha_3 A^*}{(\alpha_4 + C_0 - C)\ (\alpha_4 + C_0 - C^*)},\ \text{and}\ P_{12} = \frac{n_1\beta_2}{(\alpha_1 + C_0 - C)} - \beta_1 N^*.$$

Hence, the sufficient conditions for *dV/dt* to be negative definite are given by the following inequalities

$$P_{12}^2 < P_{11}.P_{22}\ ,\ P_{23}^2 < P_{33}.P_{22}\ ,\ P_{24}^2 < P_{44}.P_{22}\ ,\ \text{and}\ P_{34}^2 < P_{33}.P_{44}.$$

Hence, *V is* a Liapunov's function with respect to E_3.

6. Numerical Simulation

The validity of the proposed model for verifying the results obtained by stability analysis is carried out. The following values for various parameters of the model are taken from the literature [16].

$$Q = 3,\ \beta_1 = 0.5,\ \beta_2 = 0.35,\ \nu_1 = 0.1,\ \nu_2 = 0.009,\ \nu_3 = 3,\ \nu_4 = 0.01$$

$$\alpha_1 = 0.51,\ \alpha_2 = 0.41,\ \alpha_3 = 0.33,\ \alpha_4 = 0.3,\ q = 24,\ C_0 = 30$$

The values obtained for E_3 (N^*, A^*, C^*, F^*) are N^*= 6.8191, A^*= 0.6799, C^* = 7.9994, and F^* = 0.2367.

These points are further checked for the conditions of existence of equilibrium point E_3. This gives

$\alpha_4\nu_3 + C_0\nu_3 - q = 66.9 > 0$, $\left(\alpha_4 + C_0 - C^*\right) = 22.3 > 0$, $\beta_2 N^* - \nu_2\left(\alpha_1 + C_0 - C^*\right) = 2.19491 > 0$, and

$$\left(\alpha_1 + C_0 - C^*\right) = 22.51 > 0$$

It shows that the non negativity conditions are satisfied and thus E_3 is nonlinear stable.

7. Conclusion

In this paper, a mathematical model is proposed and analyzed to understand the effect of the growth of algae population in aquatic body in the presence of nutrients and its impact on the survival of fish population. Three equilibrium points are identified and the criteria of their existence are obtained. By applying stability analysis, it is shown that all the feasible equilibrium points are locally asymptotically stable if they satisfy these conditions. The nonlinear stability behavior of the point $E_3$is studied by applying Liapunov's direct method. By numerical solution of the model, it is shown that E_3 is a nonlinear stable for the considered values of parameters. This shows that the proposed model is valid and can effectively estimate the effect of depleting dissolved oxygen on the survival of fish population in aquatic body in the presence of nutrients.

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