Variable Structure Control for Hypersonic Vehicle Based on Model Reference

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Abstract: A hypersonic vehicle has a large flight envelope span, complicated flight environment, glaring external perturbation and unknown factor interference, being difficult to establish its precise model. Furthermore, the advanced airframe-engine integrated design technique makes it sensitive to change in flight conditions, thus challenging the design of its control system. We design the model reference variable structural control (MRVSC) scheme that can resist the internal structural parametric variation and external perturbation of the control system. The scheme has the merits of both adaptive control and variable structural control, can effectively track the reference model and resist the great parametric variation, effectively enhancing the robustness of the control system of the hypersonic vehicle. We design its pitch channel controller and roll channel controller with the model reference variable structural control theory, evaluate the performances of main design points of each channel and perform the 6-DOF simulation to verify the control system. The verification results show preliminarily that the step responses of the MRVSC we designed are basically unchangeable in the two states that the hypersonic vehicle has its cowling or no cowling, while the PID controller cannot ensure that the step responses are unchangeable when the state changes, indicating that the MRVSC scheme is superior to the PID controller when the control has uncertainties. The 6-DOF simulation results further verify that the MRVSC scheme can better resist great parametric variation and has strong robustness.

Key words: Classic PID, hypersonic vehicle, model reference variable structure control, robustness.

1. Introduction

A hypersonic vehicle refers to a type of militarily important flight vehicle whose Mach number exceeds 5. Different from an ordinary low-speed aircraft, the hypersonic vehicle has a large flight envelope span and a complicated flight environment. The advanced airframe-propulsion system integrated design technique causes the mutual interference between the propulsion system and the control rudder surface; and the aerodynamic coefficients are complicated functions between angle of attack and the Mach number. Meanwhile, the hypersonic vehicle flies under highly dynamic pressure with glaring external interference and unknown factor interference, causing the complexity and uncertainty of its aerodynamic characteristics [1]. Therefore, it is difficult to establish its precise model and design its control system.

Due to the above unknown factors and uncertainties of the hypersonic vehicle, variable structural control and adaptive control are widely applied to its effective and robust control. To solve the strong coupling, severe nonlinearity and variable coefficients of the hypersonic vehicle during its re-entry stage, reference [2] proposed an adaptive and robust variable structural control algorithm that can overcome the shortcomings of traditional control algorithms. Reference [3] designed a terminal sliding mode variable structural control scheme by introducing the concept of ultimate attractor in neural networks, enabling the tracking error to converge to the balance point zero within a limited time. Based on the application of the SISO uncertain linear system, Yu et al proposed a rapid terminal sliding mode variable structural control scheme in [4] and [5]. Reference [6] designed a variable structure controller based on Lyapunov stabilization theory and Riccati equation of the generalized correlational large-scale system. It makes the sliding-mode satisfy the reachability condition, and the correlational large-scale system is global structural asymptotic stability. Reference [7] introduced an adaptive parameter adjuster control rule. This method can effectively improve the performance of dynamics and stable characteristics in the pitch channel, and it also has strong adaptability to the changes of aerodynamic parameters and elevator trimming.

Based on the above research, we propose the MRVSC scheme that can resist great parametric variation. This method combines the advantages of model reference adaptive and variable structure control algorithm, and can rapidly track the reference model and resist the great parametric variation, effectively enhancing the robustness of the control system of the hypersonic vehicle.

2. Mathematical Model of a Hypersonic Vehicle

For an axial symmetry aircraft with STT control scheme, coupling of pitch channel, yaw channel and roll channel is small and can be ignored, and kinetic characteristic of yaw channel and roll channel are basically the same and control system of pitch, yaw and roll channel can be designed independently. Mathematical model of longitudinal flight control of a hypersonic vehicle can be seen in [8] and [9].

Dynamic equations of lateral flight control of a hypersonic vehicle are

$$\begin{cases} \dot{\omega}_x = \dfrac{M_x\left(V,\omega_x,\delta_x\right)}{J_x} \\ \dot{\gamma} = \omega_x \end{cases} \tag{1}$$

where V is velocity, γ is roll angle, ω_x is roll rate. δ_x, J_z, and M_x represent aileron deflection, moment of inertia about roll axis and rolling moment respectively.

3. Design of MRVSC [10], [11]

3.1. Description of Mathematical Model

3.1.1. Mathematical description of controlled plant

Mathematical model of general uncertainty multivariable system is shown as follow:

$$X_p = (A_p + \Delta A_p)X_p + (B_p + \Delta B_p)u + D_p f_p \tag{2}$$

where $X \in R^n$ is state variable, $u \in R^m$ is control variable, $f_p \in R^l$ is external interference; $A_p \in R^{n\times n}$, and $B_p \in R^{n\times m}$ are nominal matrix and nominal control matrix of controlled plant respectively; $\Delta A_p \in R^{n\times n}$, $\Delta B_p \in R^{n\times m}$, and $D_p \in R^{n\times l}$ are perturbation matrices of A_p, B_p and perturbation distribution matrix.

3.1.2. Mathematical description of reference model

Mathematical model of general reference model is described as follow:

$$X_m = A_m X_m + B_m u_m \tag{3}$$

3.1.3. Mathematical model of error

Mathematical model of error is defined as follow:

$$e(t) = X_m(t) - X_p(t) \tag{4}$$

which can be further described as:

$$e = A_m e(t) + [A_m - A_p]x_p + B_p u(t) - \Delta A_p x_p - \Delta B_p u(t) - D_p f_p(t) \tag{5}$$

In order to simplify control system design, using the elementary row transformation matrix T , $z = Te$,the transformation form of error model is equal to:

$$z = \tilde{A}_m z(t) + A_{mp} x(t) + \tilde{B}_m u_m - \tilde{B} u(t) - \Delta\tilde{A}_p x_p - \Delta\tilde{B}_p u(t) - \tilde{D} f(t) \tag{6}$$

where $\tilde{A}_m = TA_m T^{-1}$, $\Delta\tilde{A}_m = T\Delta A_m T^{-1}$, $A_{mp} = T\left[A_m - A_p\right]$, $B_m = TB_m$, $\Delta B_m = T\Delta B_m$, $D = TD$, $B = TB = \left[0 \quad B_2\right]^T$.

3.2. Choice of Sliding Hyperplane

In allusion to the error model, choosing the following sliding hyperplane:

$$S = Ce \tag{7}$$

where the sliding mode matrix C should be greater than zero, $C = [C_1 \quad C_2] \in R^{m\times n}$ is a full rank matrix, and $C_1 \in R^{m\times(n-m)}$, $C_2 \in R^{m\times m}$.

3.3. Choice of Control Law

The form of MRVSC controller is written as $u = u_{mr} + u_{vs}$ usually, where u_{mr} is matching control law of closed-loop control system of model reference, and u_{vs} is variable structure control law.

3.3.1. Choice of reference model control law

In order to achieve complete tracking, there must be $\lim\limits_{t\to\infty} e = 0$, which means there is a proper u to ensure the following equation having solution.

$$A_{mp} x + B_m u_m - Bu = 0 \tag{8}$$

And the following equation is obtained:

$$u_{mr} = B_2^{-1}\left[0 \quad I_m\right] A_{mp} x_p + B_2^{-1}\left[0 \quad I_m\right] B_m r \tag{9}$$

3.3.2. Choice of variable structure control law

The selected variable structure controller u_{vs} shall ensure to keep the system on the sliding mode stable and reliable , that is to say there is a right u_{vs} to guarantee $V = S^T S < 0$, if $u_{vs} = g(t)\left(CB_p\right)^{-1}\mathrm{sgn}(S)$, V

can be written as:

$$
\begin{aligned}
V &= -g(t)S^T\operatorname{sgn}(S) - g(t)S^T C\Delta B_p (CB_p)^{-1}\operatorname{sgn}(S) - S^T C\Delta B_p u_m + S^T[CA_m z - C\Delta A_p x_p - CD_p f_p] \le \\
&\quad - g(t)\|S^T\|[1-\|\Delta B_p\| \ \|C\| \ \|(CB_p)^{-1}\|] + \|S^T\|\|\Delta B_p\| \ \|C\| \ \|u_m\| + \|S^T\|[\|CA_m\| \ \|z\| \\
&\quad - \|\Delta A_p\| \ \|C\| \ \|x_p\| - \|CD_p\| \ \|f_p\|]
\end{aligned}
\tag{10}
$$

where

$$
g(t) = (1-p_5)^{-1}[p_1\|z\| + p_2\|x_p\| + p_3\|u_m\| + p_4] + \varepsilon \tag{11}
$$

It can make sure that $V < 0$ and keep the system on the sliding mode stable and reliable. Where ε is a small positive number, $p_1 = \|CA_m\|$, $p_2 = \|\Delta A_p\| \ \|C\|$, $p_3 = \|\Delta B_p\| \ \|C\|$, $p_4 = \|f_p\| \ \|CD_p\|$, $p_5 = \|\Delta B_p\| \ \|C\| \ \|(CB_p)^{-1}\|$, and $\|\bullet\|$ means induced norm.

In order to eliminate the high frequency flutter, substitute the following equation $m_i(s)$ for $\operatorname{sgn}(s_i)$.

$$
m_i(s) = \frac{s_i}{|s_i| + \delta_i} \tag{12}
$$

In summary, the design steps of MRVSC are: obtain the reference model which meets the model matching condition of entirely tracking through model reference planning methods firstly, then design a sliding mode matrix C based on the error system, and finally obtain the variable structure tracking control law to ensure the controlled plant tracks the reference model effectively.

4. MRVSC Design and Simulation of Pitch Channel

4.1. MRVSC Design of Pitch Channel

The detail design procedures are shown in part 3 and no more explanation is presented here. State equation of reference model can be described as:

$$
\begin{bmatrix} \dot{n}_{ym} \\ \dot{\omega}_{zm} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{V\bar{a}_4}{g} \\ -\dfrac{g}{T_m^2 V\bar{a}_4} & -\dfrac{2\xi_m}{T_m} \end{bmatrix} \begin{bmatrix} n_{ym} \\ \omega_{zm} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{g}{T_m^2 V\bar{a}_4} \end{bmatrix} n_{yc} \tag{13}
$$

The reference model control law u_{mr} can be written as follow:

$$
u_{mr} = \left(\frac{g}{a_3 a_4 V T_m^2} - \frac{g a_2}{a_3 a_4 V} \right) n_y - \frac{g}{a_3 a_4 V T_m^2} n_{yc} + \left(\frac{-2\xi_m}{T_m} + a_1 \right) \omega_z \tag{14}
$$

And the variable structure control law u_{vs} is shown in the following equation:

$$
u_{vs} = \frac{g(t)}{-a_3} \operatorname{sgn}(S) \tag{15}
$$

4.2. Performance Check of the Main Design Point of Pitch Channel

State vector of pitch channel is $X = \begin{bmatrix} n_y & \omega_z \end{bmatrix}^T$, and the schematics of control system is shown as Fig. 1.

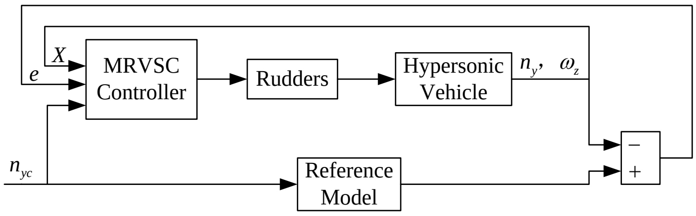

Fig. 1. Control system of pitch channel.

Keep the control parameters unchanged, and compare the control effect of MRVSC and PID under two main design point though digital simulation.

Main design point 1: With cowling, velocity is 5.5Ma, altitude is 24000m and angle of attack is 2º.

Main design point 2: Without cowling, velocity is 6.0Ma, altitude is 26000m and angle of attack is 2º.

Ignore the influence of rate gyro and actuator. Time-domain characteristics of main design point are shown in Fig. 2 and Fig. 3.

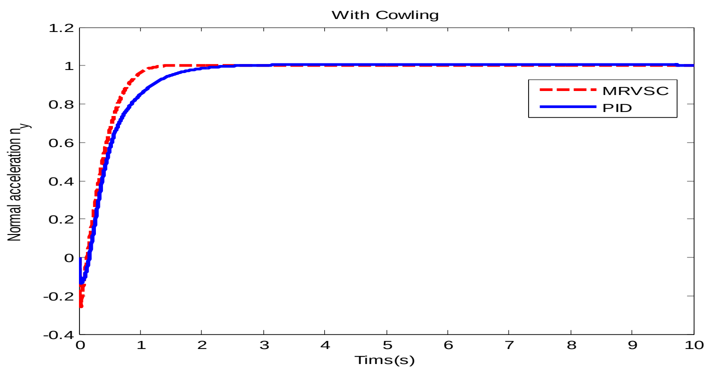

Fig. 2. Normal acceleration step response with cowling.

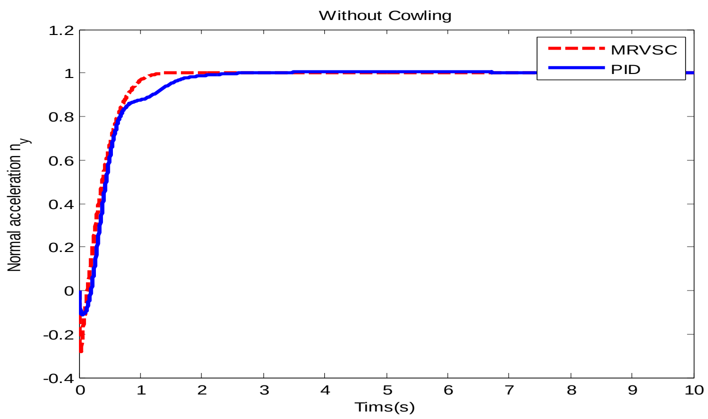

Fig. 3. Normal acceleration step response without cowling.

5. MRVSC Design and Simulation of Roll Channel

5.1. MRVSC Design of Roll Channel

The design procedures of roll channel are the same as pitch channel and no more explanation is presented here. The state variable of reference model is $X_m = [\gamma_m \quad \omega_{xm}]^T$, and the state equation of reference model can be described as:

$$\begin{bmatrix} \dot{\gamma}_m \\ \dot{\omega}_{xm} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{T_m^2} & -\frac{2\xi_m}{T_m} \end{bmatrix} \begin{bmatrix} \gamma_m \\ \omega_{xm} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{T_m^2} \end{bmatrix} \gamma_c \tag{16}$$

The reference model control law can be written as follow:

$$u_{mr} = \frac{1}{c_3 T_m^2}(\gamma - \gamma_c) + \frac{2\xi_m - T_m c_1}{c_3 T_m}\omega_x \tag{17}$$

And the variable structure control law u_{vs} is shown in the following equation:

$$u_{vs} = \frac{g(t)}{-c_3}\operatorname{sgn}(S) \tag{18}$$

5.2. Performance Check of the Main Design Point of Roll Channel

Variable structure control scheme of hypersonic vehicle roll channel is roll angle + roll rate control, and the schematics of control system is shown as Fig. 4.

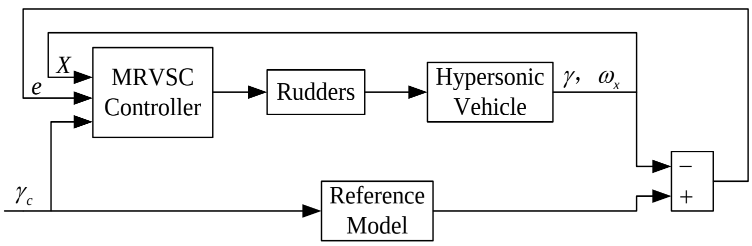

Fig. 4. Control system of roll channel.

Simulation conditions are the same as mentioned above. Time-domain characteristics of main design point are shown in Fig. 5 and Fig. 6.

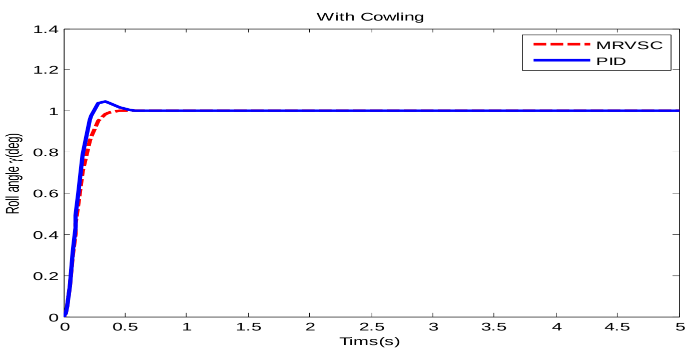

Fig. 5. Roll angle step response with cowling.

6. Six Degree of Freedom Simulation and Analysis

To validate the performance of MRVSC, examine system dynamic quality and steady performance of the three-channel control systems, 6-DOF simulation results of a STT hypersonic vehicle are shown below.

Simulation conditions: initial position is $X_0 = 0m$, $Y_0 = 26000m$, $Z_0 = 0m$; initial velocity is $V_{x0} = 6Ma$, $V_{y0} = 0 m/s$, $V_{z0} = 0 m/s$. Time constant of the actuator is 0.008s, and damping coefficient is 0.6. Time constant of the rate gyro 0.002s, and damping coefficient is 0.7. Lateral maneuver will be performed at the time of 50s and 60s for 3s, and the magnitude is 0.5. The simulation results are shown in Fig. 7 and Fig. 8.

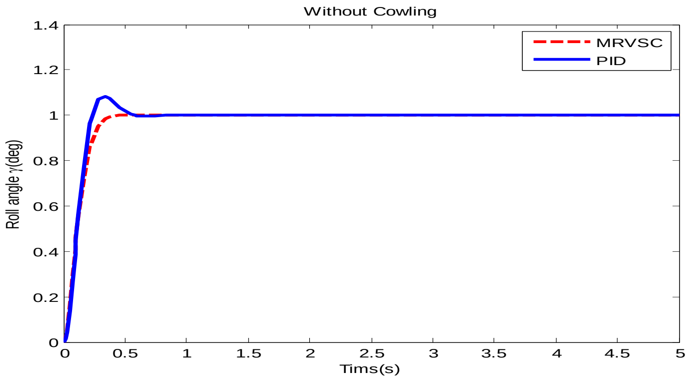

Fig. 6. Roll angle step response without cowling.

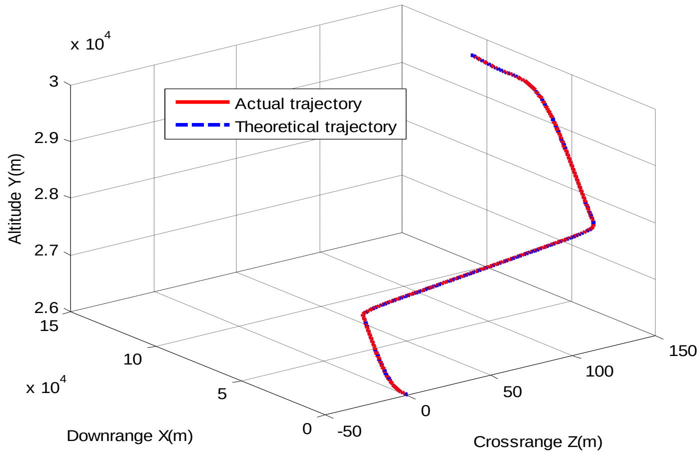

Fig. 7. Curve of trajectory tracking.

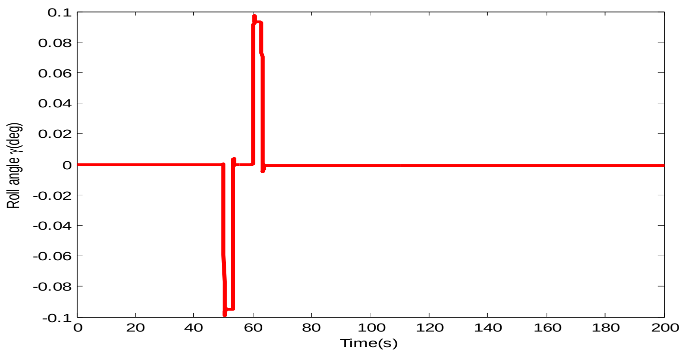

Fig. 8. Response curve of roll angle.

Though Fig. 7 and Fig. 8, we can find that, MRVSC can effectively control the STT hypersonic vehicle track the ideal trajectory, and almost keep roll angle unchanged. It means that MRVSC enables to control the system enter the sliding mode in a short time. And in case of dramatic changes in altitude, it can overcome parameter perturbation easily, and show good dynamic property and strong robustness.

7. Conclusion

We use the MRVSC scheme to design the model reference variable structural controller for the pitch and roll channel of a hypersonic vehicle. The scheme utilizes the adaptiveness of model reference and the merits of variable structural control. We compare the performances of main design points of each channel. The comparison results show that MRVSC can keep the control effects unchangeable under the circumstance of changeable parameters of the control system. But under the circumstance of parametric variation, PID controller has poorer control effects; the control system needs more control time or has larger overshooting. This shows that under the circumstance of parametric variation, MRVSC can ensure the robustness of a control system and its design by overcoming the shortcomings of the conventional PID controller such as uncertainties and poor adaptiveness when interference exceeds a certain limit. Finally, we give the overall-ballistic 6-DOF simulation results, which further verify that MRVSC has strong robustness to uncertainties.

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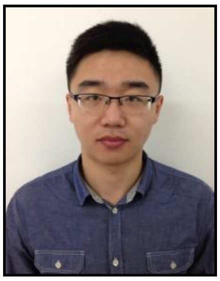

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