# Sequential Coordinates 

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#### Abstract

This paper proposes a new coordinate system with flexible extensions to multiple dimensions. The main idea is to provide a scalable representations of arbitrary shapes in digitized manner for an effective and efficient data storage. The application areas stem from object recognition to handwriting analysis. A number of demonstrative examples are given, extensions to 3 dimensional and 1 dimensional cases are discussed. The open issues are also highlighted.


Keywords: 3D representation, Cartesian coordinates, non-uniform sampling, sequential coordinates

## 1. Introduction

There are several types of coordinates in the field of mathematics and science since a long time, such as Cartesian coordinates, polar coordinates, cylindrical coordinates, spherical coordinates, etc.) In some cases, one needs abacuses to accomplish a complex calculation. As an example, Smith chart is used to determine the variation of the complex impedance with frequency and to carry out impedance adaptations in the transmission lines. This Abacus allows to perform a complex number calculation for this tedious task. Nowadays with pocket calculators one does not need such an Abacus anymore but for the understanding of the phenomena on high frequency transmission lines it is very useful for engineers in the field. In this coordinate system orthogonal circles define the value of a complex impedance with its real part the resistance and the imaginary part the inductance or capacitance.

The idea of sequential coordinates is based on the sequential movement of a pattern or path that evolves over time as is the case in writing a signature. This sequential movement can be modeled in sequential coordinates for easier recognition later by a computer.

The sequential coordinates contain inclusively the notion of time. This can be used for specific applications as follows:

- The movement of a robot. We follow inclusively the path of a robot that moves and communicates its sequential coordinates. If one loses contact, it is possible to recover and organize the next moves.
- Recognition of signatures. This process is a perfect example for an application of sequential coordinates after discretization of the path by segments that follow each other.
- Navigation. Case of a walker lost in the forest the guide can place him/her on a map with a minimum communication complexity.

In Freeman's early work, a procedure was developed for the description and coding of arbitrary geometric curves, which simplifies the manipulation of these curves by computer [1]. Since then, the methodologies and applications have been diversified. Dziech and Ukasha et al. have proposed a performance measure based on the compression ratio for digitizing arbitrary curves [2]. The application of
curve scanning for contour extraction has been studied by Snarski [3]. Thanikkal and Dubey et al. studies digitized curves for pattern recognition [4]. An overview of recent work on digitizing geometric curves and more generalized drawings has been presented by Moreno-García and Elyan et al. [5]. A specific application of coordinate representation is signature verification by Plamondon and Srihari [6].

Given the wide range of applications and the need for accurate and efficient representations, and we believe that the proposed sequential coordinate system can provide significant advantages both in terms of the representation and applications. The rest of this paper is organized as follows. In the following section, we provide the definition of sequential coordinates. Section 3 introduces the extensions to 3 dimensional and the 1 dimensional cases. The conclusions are given in Section 4.

## 2. Defining Sequential Coordinates

At first, we limit ourselves to two dimensions. An AB starting segment can be traced anywhere on a plane (independence of an origin.) Let us have starting point $A$ and arrival point $B$. Once the point $B$ is obtained, one can turn left ( - ) or right at a certain angle. Then, we continue with another segment length towards point C . This will provide coordinates (length, angle) sequentially $\mathrm{B}\left(r_{0}, 0\right)$; $\mathrm{C}\left(r_{1}, \theta_{1}\right)$; $\mathrm{D}\left(r_{2}, \theta_{2}\right)$; E $\left(r_{3}, \theta_{3}\right)$ etc. For lengths, we can choose a unit or normalize everything in relation to the length of the AB segment (see Fig. 1). We will have $\mathrm{BC} / \mathrm{AB}=d_{1}, \mathrm{CD} / \mathrm{AB}=d_{2}, \mathrm{DE} / \mathrm{AB}=d_{3}$. The series of coordinates will then be: $\left\{\mathrm{B}(1,0) ; \mathrm{C}\left(d_{1}, \theta_{1}\right) ; \mathrm{D}\left(d_{2}, \theta_{2}\right) ; \mathrm{E}\left(d_{3}, \theta_{3}\right)\right\}$.


Fig. 1. Definition of sequential coordinates which contains implicitly time sequence. (Departure from one point to the next one).

The same notation can also be used in the case of disconnected contours, representing the corresponding distance by $\bar{d}$. For example, the set of coordinates in Fig. 2 becomes: $\left\{\mathrm{B}(1,0) ; \mathrm{C}\left(d_{1}, \theta_{1}\right) ; \mathrm{D}\left(\bar{d}_{2}, \theta_{2}\right)\right.$; E $\left.\left(d_{3}, \theta_{3}\right)\right\}$.


Fig. 2. Example of generalization with disconnected contours.

### 2.1. Recognition of Shapes

In Cartesian coordinates, a figure formed by segments can move, rotate in one direction, and change scale
in a plane; everything will be expressed according to the figures given in Fig. 3. It is very difficult to predict that these different figures are similar just by looking at the Cartesian coordinates:

$$
\begin{gathered}
\mathrm{A}\left(x_{1}, y_{1}\right) ; \mathrm{B}\left(x_{2}, y_{2}\right) ; \mathrm{C}\left(x_{3}, y_{3}\right) ; \mathrm{D}\left(x_{4}, y_{4}\right) ; \mathrm{E}\left(x_{5}, y_{5}\right) \\
\mathrm{A}^{\prime}\left(x^{\prime}{ }_{1}, y_{1}^{\prime}\right) ; \mathrm{B}^{\prime}\left(x_{2}^{\prime}, y_{2}^{\prime}\right) ; \mathrm{C}^{\prime}\left(x_{3}^{\prime}, y_{3}^{\prime}\right) ; \mathrm{D}^{\prime}\left(x_{4}^{\prime}, y_{4}^{\prime}\right) ; \mathrm{E}^{\prime}\left(x_{5}^{\prime}, y_{5}^{\prime}\right) \\
\mathrm{A}^{\prime \prime}\left(x^{\prime \prime}{ }_{1}, y^{\prime \prime}{ }_{1}\right) ; \mathrm{B}^{\prime \prime}\left(x^{\prime \prime}{ }_{2}, y^{\prime \prime}{ }_{2}\right) ; \mathrm{C}^{\prime \prime}\left(x^{\prime \prime}{ }_{3}, y^{\prime \prime}{ }_{3}\right) ; \mathrm{D}^{\prime \prime}\left(x^{\prime \prime}{ }_{4}, y^{\prime \prime}{ }_{4}\right) ; \mathrm{E}^{\prime}\left(x^{\prime \prime}{ }_{5}, y^{\prime \prime}{ }_{5}\right)
\end{gathered}
$$

If we use sequential coordinates, we will notice that the lengths change only in the same proportion and the angles remain the same. Since we have no origin, we can place these figures anywhere on the plane (see Fig. 3).


Fig. 3. Illustration of various positions and orientations of similar figures without origin problem in sequential coordinates (a) Original shape (b) Rotated shape (c) Shape at another scale and rotated.

### 2.2. Exemplary Cases

In Fig. 4 an equilateral triangle (Fig. 4(a)) and a square (Fig. 4(b)) are shown. The coordinates that define these figures will be:

- In the case of the equilateral triangle (by changing the place of length $r$ and angle $\theta$ $\left\{\left(0^{\circ}, r_{0}\right) ;\left(120^{\circ}, r_{1}\right) ;\left(120^{\circ}, r_{2}\right)\right\}$ with $r_{0}=r_{1}=r_{2}=r$.
- For the square we simply have $\left(0^{\circ}, r\right) ;\left(90^{\circ}, r\right) ;\left(90^{\circ}, r\right) ;\left(90^{\circ}, r\right)$.
- For a segmented spiral, we can use the limit $n \rightarrow+\infty$

$$
\lim _{n \rightarrow \infty} \bigcup_{i=0}^{n}\left(\frac{360^{o}}{M_{i}}, i \times r\right)
$$

for $r$ constant. The values of $M_{i}$ can be selected at each step to obtain the desired shape.


Fig. 4. Simple geometric figures, such as equilateral triangles, squares in sequential coordinates, easy to recognize (a) An equilateral triangle (b) A square.

### 2.3.Characteristics of the System

The characteristics of such a system are as follows;

1. Independence of origin, 0
2. Independence of the rotation of the figure one way or the other.
3. Easy recognition by similar lengths ( $r$ lengths).
4. In addition, if one normalizes (e.g., $\frac{r_{1}}{r_{0}}=d_{1}, \frac{r_{2}}{r_{0}}=d_{2}, \frac{r_{2}}{r_{0}}=d_{3}$ ), the same number will appear in the lengths of the sides. This leads to the recognition of these kinds of figures in an immediate way.
The expressions concerning polygons with rotational symmetry will be of type $\left\{(r, 0) ;\left(r, \theta_{1}\right) ;\left(r, \theta_{1}\right) ;\left(r, \theta_{1}\right) ;\left(r, \theta_{1}\right) ;\left(r, \theta_{1}\right)\right\}$. Definition of a symbol for this kind of sequence thus becomes:

$$
\bigcup_{i=0}^{n}(r, \theta)_{i}=\left\{\left(r_{0}, \theta_{0}\right),\left(r_{1}, \theta_{1}\right),\left(r_{2}, \theta_{2}\right),\left(r_{3}, \theta_{3}\right), \ldots\right\},
$$

where $\theta_{0}=0$. Any arbitrary form can be represented by the formulation given above.

## 3. Extensions to Other Dimensions

In this section, the 3-dimensional and 1-dimensional cases are defined.

### 3.1.Three Dimensional Case

Move to a third dimension while in a given plane, one can proceed as follows:
When we arrive from a point A to a point B in Fig. 1, we can draw the normal to the segment AB in this plane and taking this normal as a hinge we can rotate around this normal in space ( + ) upwards by an angle $\varphi$ and continue with the sequential coordinates in this new plane of space. $\varphi$ will be positive ( + ) if we have turned upwards or negative ( - ) if we have turned downwards (See Fig. 4).

$$
\lim _{n \rightarrow \infty} \bigcup_{i=0}^{n}(r, \theta, \varphi)_{i}=\left\{\left(r_{0}, \theta_{0}, \varphi_{0}\right),\left(r_{1}, \theta_{1}, \varphi_{1}\right),\left(r_{2}, \theta_{2}, \varphi_{2}\right),\left(r_{3}, \theta_{3}, \varphi_{3}\right), \ldots\right\}
$$

with $\theta_{0}=0$ and $\varphi_{0}=0$. If we don't change the plan of course $\phi_{0}=\phi_{1}=\phi_{2}=\ldots=0$.

### 3.2.One Dimensional Case

For the one-dimensional sequential coordinates, it is sufficient to discretize the curved line and express with the + or - sign the direction of displacement and indicate only the lengths of the segments with a sign. As there is no origin this sequence will keep its values whatever the starting point (See Fig. 5).

(a)

(b)

Fig. 5. Sequential coordinates in 3 dimensions. Passage from two dimensions to three dimensions in space.

## 4. Conclusion

In an era of machine learning, it is essential to have efficient representations of arbitrary shapes. This manuscript introduces the concept of sequential coordinates for the first time in the literature. This flexible form allows multidimensional shapes to be represented in an evolutionary manner. Open issues include coordinate sampling and transformation between coordinate systems. The authors believe that nonuniform sampling theory contains the corresponding keys and plan to work on this topic in the future.

## Conflict of Interest

The authors declare no conflict of interest.

## Author Contributions

Cevdet Akyel introduced the main idea and developed the 3-D representation. Gunes Karabulut Kurt contributed to the introduction of the notation.

## References

[1] Freeman, H. (1961). On the encoding of arbitrary geometric configurations. IRE Transactions on Electronic Computers, 2, 260-268.
[2] Dziech, A., Ukasha, A., \& Wassermann, J. (2006). A new method for contour extraction and image compression in spectral domain. IEEE Proceedings ELMAR (pp. 41-44).
[3] Snarski, A. C. (2020). Recursive path following in log polar space for autonomous leaf contour extraction. Master of Science Dissertation, University of New Hampshire.
[4] Thanikkal, J. G., Dubey, A. K., \& Thomas, M. T. (2020). Unique Shape Descriptor Algorithm for Medicinal Plant Identification (SDAMPI) with abridged image database. IEEE Sensors Journal, 20(21), 1310313109.
[5] Moreno-García, C. F., Elyan, E., \& Jayne, C. (2019). New trends on digitization of complex engineering drawings. Neural Computing and Applications, 31(6), 1695-1712.
[6] Plamondon, R., \& Srihari, S. N. (2000). Online and off-line handwriting recognition: A comprehensive survey. IEEE Transactions on Pattern Analysis and Machine Intelligence, 22(1), 63-84.

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