Derivation and Approximation of Coupled Wave Equations for Spontaneous Brillouin Scattering

Christopher Horne
Dept. of Electrical & Computer Engineering, North Carolina A&T State University, Greensboro, NC USA.

Email: ckhorne@ncat.edu
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Abstract: Brillouin Scattering (SBS) is a nonlinear process in which an electromagnetic (pump) wave generates an acoustic wave through the process of electrostriction. The theoretical model for SBS is initiated by thermally excited acoustic waves distributed within a Brillouin-active medium; thermally excited acoustical phonons cause an inelastic scattering of light in a fiber optic tube. This paper provides a derivation of an approximate analytical solution to the system of SBS equations in a lossless medium. The model's solution predicts how the SBS Stokes power depend upon the laser light intensity and upon the physical properties of the medium. A brief study of a numerical solution of the model is presented.

Keywords: Brillouin Scattering (SBS), electrostriction Partial Differential Equations (PDE), nonlinear, optics

1. Introduction

SBS is a process in which an electromagnetic light wave, typically generated by a laser, interacts with a microwave, typically called a Stokes wave, in a material medium. Through the intermediary of an acoustic wave, this leads to the amplification of the Stokes wave and attenuation of the laser wave. A more specific description of scattering is the coupling between the pump, acoustic and Stokes waves. In other words, the scattered Stokes wave will be produced as the acoustic wave generating a modulation in the material. Through electrostriction or radiation pressure [1–3], electromagnetic wave induces very small strains in the medium (material). This in turn stimulates the generation of coherent phonons via photo-elasticity or by physical motion of the waveguide (i.e., fiber optic tube) boundaries. A phonon is a quantity of energy found within a vibration while a photon is a quantum particle within a light wave. A physical example of a phonon includes sound waves. Acoustic phonons are coherent movements of atoms of the lattice out of their equilibrium positions. SBS is sometimes an unwanted phenomena in telecommunications networks. The long lengths and low attenuations which describe modern fiber networks allow for long interaction lengths where SBS can be detrimental even for power levels near 1 mW. SBS creates a situation where small Stokes signals can be amplified, degrading signal quality or damaging upstream optical components. Study and experiment of SBS allows it to be controlled more efficiently.

The geometry of the sBs interaction is shown in Fig. 1. The light wave component is represented by the incident laser light wave EP, the back-scattered Stokes wave, ES, and the acoustic wave, ρ_a. Thus, the pump optical wave, acoustic wave, and scattered Stokes wave will couple to each other and an amplification process will be established when the intensity of pump light is high enough. Based on energy conservation and supported by experimental observations, the optical pump frequency is related to the Stokes and acoustic frequencies via: $\omega_p = \omega_S + \omega_a$. The geometry of the Brillouin scattering medium is typically a fiber optic tube (8–9 um dia.).

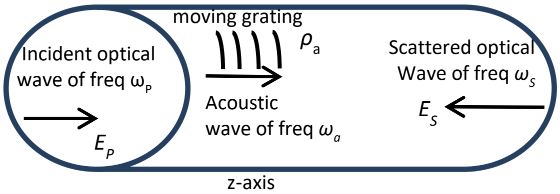

Fig. 1. Geometry of the Brillouin scattering medium.

2. Electrostriction and the Continuity Equation

2.1. Electrostriction

Electrostriction is the tendency of materials to become compressed in the presence of an electric field. An analogy is the parallel plate capacitor where a molecule near the plate placed in the presence of electric field creates energy, U, stored in the polarization of the molecule, which changes linearly as the plates move in one direction. The amplitude function $\boldsymbol{E}$ of the field can be used to express a force, $\boldsymbol{F}$, acting on a molecule near the medium:

$$\boldsymbol{F} = -\nabla U = \frac{\varepsilon_0}{2}\gamma\,\nabla(\boldsymbol{E}\cdot\boldsymbol{E}) \tag{1}$$

So, the electric field is inducing a force which rearranges the molecules and creates a density change. The term ε_0 is the dielectric constant of the fiber medium shown in Fig. 1, while γ is the electrostrictive coefficient. The dielectric constant can be written as a change from average value: $\varepsilon_r = \varepsilon_0 + \Delta\varepsilon_0$ with: $\Delta\varepsilon_0 = (\gamma/\rho_0)\Delta\rho$ where $\Delta\rho$ is a subtraction. Thus, the dielectric constant can be expressed as: $\varepsilon_r = \varepsilon_0 + (\gamma/\rho_0)(\rho - \rho_0)$.

2.2. Material Density and Continuity Equation

An incident optical pump wave, $\boldsymbol{E_P}$, with magnitude and direction, is a vector, and induces an acoustic wave with a pressure on the medium. Let P denote the pressure of an elastic acoustic field in the medium, let $\mathbf{u}$ denote the velocity of the elastic motion of the medium along the direction of propagation, namely the z-axis. Assume the direction of propagation is only in the z-direction since longitudinal (compressional) is the dominant mode of SBS and based on based experimental observations [4]. Let ρ_0 denote the average mass density. The density variation $\rho(z,\ t)$ of the fiber from its mean value ρ_0 depend on time t and longitudinal position z, varying from z = 0 at the front face of the fiber to z = L at the rear face. Applying Newton's second law to a unit volume with average density ρ, of the medium, yields the following PDE [5]:

$$\rho_0\frac{\partial\mathbf{u}}{\partial t} + \Gamma\rho_0\mathbf{u} + \nabla P = 0 \tag{2}$$

The first term in Eq. (2) is the force proportional to the mass and acceleration. The second term is the damping force proportional to the mass and velocity of the moving particle. The damping factor Γ has units of sec^{-1} while $\mathbf{u}$ is the velocity vector. P is the pressure imposed on the unit volume of the medium owing to the pressure gradient. The optical pump is connected to the medium left face along the z-axis at z = 0, and assume longitudinal (compressional) is the dominant mode of SBS. Since the intense optical wave exerts a force on the molecules in the fiber medium causing them to move and this change in density would include a forcing term into Eq. (1). Take this force to be proportional to the gradient of the square of the amplitude of the optical wave, $\boldsymbol{E}$. Therefore, using Eqs. (1) and (2) becomes

$$\rho_0 \frac{\partial \mathbf{u}}{\partial t} + \Gamma \rho_0 \mathbf{u} + \nabla P = \frac{\varepsilon_0}{2} \gamma \, \nabla(\boldsymbol{E} \cdot \boldsymbol{E}) \tag{3}$$

The continuity equation for conservation of mass of an element in the medium is

$$\nabla \cdot \mathbf{u} + \frac{1}{\rho_0} \frac{\partial \rho}{\partial t} = 0 \tag{4}$$

where $\rho(z, t)$ is the mass density function in the presence of an optical field.

Clairaut's Theorem states that mixed variable partial derivatives are equal under certain conditions. Here, the velocity, $\mathbf{u}$, density, ρ are functions for which $\partial \mathbf{u}/\partial t$ and $\partial \rho/\partial t$ exist and continuous over their domains. The divergence operator can be written as $\nabla \cdot \mathbf{u} = \partial \mathbf{u}/\partial z \;\; \partial^2 \mathbf{u}/\partial z \partial t = \partial^2 \mathbf{u}/\partial t \partial z$. Taking the divergence of both sides of Eq. (3):

$$\rho_0 \nabla \cdot \left(\frac{\partial \mathbf{u}}{\partial t}\right) + \Gamma \rho_0 \nabla \cdot \mathbf{u} + \nabla^2 \mathrm{P} = \frac{\varepsilon_0}{2} \gamma \nabla^2 (\boldsymbol{E} \cdot \boldsymbol{E}) \tag{5}$$

and using the fact that $\nabla \cdot \mathbf{u} = -\frac{1}{\rho_0} \frac{\partial \rho}{\partial t}$, the new equation is:

$$\nabla^2 P - \Gamma \frac{\partial \rho}{\partial t} - \frac{\partial^2 \rho}{\partial \mathrm{t}^2} = \frac{\varepsilon_0 \gamma}{2} \nabla^2 (\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}}) \tag{6}$$

The first term in Eq. (6) can be simplified by recognizing the Laplacian of pressure is proportional to the modulus of elasticity, β divided by average density variation, ρ_0. Thus, Eq. (6) becomes after multiplying through by ρ_0/β .

$$\nabla^2 \rho - \frac{\rho_0}{\beta} \Gamma \frac{\partial \rho}{\partial t} - \frac{\rho_0}{\beta} \frac{\partial^2 \rho}{\partial \mathrm{t}^2} = \frac{\rho_0}{\beta} \frac{\varepsilon_0 \gamma}{2} \nabla^2 (\boldsymbol{E} \cdot \boldsymbol{E}) \tag{7}$$

The velocity of acoustic wave in a medium depends on the square root of the restoring force, that is the ratio of elasticity to density. Velocity squared is the ratio of elasticity, divided by average density variation, $v_a^2 = \beta/\rho_0$. The acoustic wave attenuation, α, in the fiber medium, is related to the damping factor, Γ (units of sec^{-1}) and velocity as [6]: $\Gamma = v_a \alpha_a$. Eq. (7) can be revised with real mass density, so the fiber medium modulus of elasticity, $\beta = \rho_0 v_a^2$. Therefore, the material density disturbance is described by the wave equation:

$$\nabla^2 \rho - \frac{\alpha_a}{v_a} \frac{\partial \rho}{\partial t} - \frac{1}{v_a^2} \frac{\partial^2 \rho}{\partial t^2} = \frac{\varepsilon_0}{v_a^2} \frac{\gamma}{2} \nabla^2 (\boldsymbol{E} \cdot \boldsymbol{E}) \tag{8}$$

where

- ρ [kg/m3] is the material density fluctuation with average value of ρ_0 over the fiber medium of length zero to L.
- α_a is the attenuation factor of the acoustic wave
- v_a^2 is the squared longitudinal sound velocity.
- The "forcing term" for the induced pressure which can be written as $-\varepsilon \gamma \nabla^2 (\boldsymbol{E} \cdot \boldsymbol{E})$where ε_0 is the permittivity and γ is the electrostrictive coupling constant (induced pressure constant) [7]

3. Consequence of Maxwell's Equations

3.1. Optical Wave Equation

The wave equation of the optical field $\boldsymbol{E}$ (pump or Stokes) can be written in an isotropic and transparent medium as $\nabla^2\boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\varepsilon_r\boldsymbol{E}) = 0$, where $\varepsilon_r = \frac{\gamma}{\rho_0}(\rho - \rho_0) + \overline{\varepsilon_r}$ is the relative permittivity. Substituting ε_r into the wave equation and taking two derivatives w.r.t time yields:

$$\nabla^2\boldsymbol{E} - \frac{\overline{\varepsilon_r}+\gamma}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E} = \frac{1}{c^2}\frac{\gamma}{\rho_0}\frac{\partial^2}{\partial t^2}(\rho\boldsymbol{E}) \tag{9}$$

Eq. (9) is the wave equation of an optical field $\boldsymbol{E}$ in a medium involving an induced 'photoelastic effect', That is, the refractive index in the fiber optic medium is modified due to elastic strain in the material. Elastic strain changes when material density changes.

The optical (pump) and Stokes waves propagate in the longitudinal z direction. This is because extensive experimental observations show the optical wave to propagate 'down' the tube and assume one spatial dependence, z, due to extensive experimental observations of Stokes wave behavior [6]. Summarizing, the material density disturbance is described by the density Eq. (8) along with the optical wave Eq. (9). The optical wave will take the form:

$$\boldsymbol{E}(z,t) = Re\left[\boldsymbol{E_p}(z,t)\, e^{i(\omega_p t - k_p z)} + \boldsymbol{E_S}(z,t)\, e^{i(\omega_S t + k_S z)}\right] \tag{10}$$

where $\boldsymbol{E_p}(z,t)$ and $\boldsymbol{E_S}(z,t)$ are the pump and Stokes amplitudes. k_P and k_S are the wavenumbers while ω_p and ω_S are the pump and Stokes frequencies, respectively. Since the optical wave and acoustic wave interact with each other (through the SBS process) in which the waves decay exponentially with fiber length (verified by experiments by the author and others), the solution is assumed exponential. Accordingly, a solution to the optical field and the induced acoustic wave in Eqs. (8) and (9) is:

$$\boldsymbol{E_P} = E_P(z)e^{i\omega_p t}; \tag{11-a}$$

$$\boldsymbol{E_S} = E_S(z)e^{-i\omega_s t}; \tag{11-b}$$

$$\rho_{\boldsymbol{a}} = \rho_a(z)e^{-i\omega_a t} \tag{11-c}$$

Considering Fig. 1, the initial condition, $E_P(0)$ is the optical wave intensity at the entrance (left front face) to the fiber medium. $E_P(L)$ is the optical wave intensity at the end of the fiber medium. Take z = 0 to be the laser source location. That is, the laser is fixed on the far left side of the fiber medium at z = 0. Since back-scattered Stokes wave grows from right to left, the boundary conditions are: $\boldsymbol{E_P}(z = 0, \mathrm{t}) = E_P(0)e^{i\omega_p \boldsymbol{t}}$ and $\boldsymbol{E_S}(z = L, t) = 0$.

3.2. Acoustic Solution: One Wave

Substituting the induced acoustic field Eq. (11-c) into the acoustic wave Eq. (11-c) into Eq. (8) and differentiating the left-hand-side yields:

$$[\nabla^2 + k_a^2 + i\alpha_a k_a]\rho_a(z) = \frac{\varepsilon_0}{v_A^2}\frac{\gamma_e}{2}\nabla^2\left[E_P(z)e^{i\omega_p t}E_S(z)e^{i\omega_s t}\right] \tag{12}$$

where $k_a = \omega_a/v_a$ is the magnitude of acoustic wave.

3.3. Optical Solution: Two Waves

Substituting the acoustic solution form Eq. (11-c) into the optical wave Eq. (9) and assuming a slowly varying acoustic amplitude such that $v_a \ll c/n$ and $\omega_a \ll \omega_P$ yields a separate pump and Stokes components. Then, this result can be separated into the optical laser (pump) and Stokes waves:

$$\left[\nabla^2 + \frac{\varepsilon_r \omega_p^2}{c^2}\right]\left(E_P(z)e^{-i\omega_p t}\right) = -\frac{\omega_p^2}{c^2}\frac{\gamma}{\rho_0}\left[\rho_a(z)E_s(z)\right]$$
$$\left[\nabla^2 + \frac{\varepsilon_r \omega_s^2}{c^2}\right]\left(E_s(z)e^{-i\omega_s t}\right) = -\frac{\omega_s^2}{c^2}\frac{\gamma}{\rho_0}\left[\rho_a(z)E_p(z)\right] \tag{13}$$

The forward (incident) optical pump, backward Stokes wave and the induced acoustic wave are propagating along the z-axis. Recall, the amplitude functions of these waves are:

$$\boldsymbol{E} = \boldsymbol{E}_P(z)e^{-i\omega_p t} + \boldsymbol{E}_S(z)e^{-i\omega_s t} \text{ and } \rho = \rho_a(z)e^{-i\omega_a t} \tag{14}$$

Substituting Eq. (14) into the acoustic density Eq. (8) yields:

$$\left[\frac{\partial^2}{\partial z^2}\rho_a\, e^{-i\omega_a t} + i\alpha_a k_a \rho_a(z)\right] = -k_a^2 \rho_a\, e^{-i\omega_a t} + \frac{\varepsilon_0}{v_A^2}\frac{\gamma_e}{2}\frac{\partial^2}{\partial t^2}\left[\boldsymbol{E_P}(z)\cdot\, \boldsymbol{E_S}(z)\right] \tag{15}$$

3.4. Amplitude Functions of Three Waves

To solve Eqs. (13) and (15), assume plane waves satisfy the electromagnetic wave equations in a homogeneous media (optical fiber). Specifically, the backward Stokes waves travels in the $-z$ direction while the optical pump and acoustic wave travels in the $+z$ direction. Based on a solution to the optical field and the induced acoustic wave as shown in Eqs. (11-a), (11-b) and (11-c), substituting these three equations into Eq. (15) and keeping only the leading terms according to these relationships immediately above yields a simplified expression for the density wave:

$$\left[i2k_a \frac{\partial \rho_a(z)}{\partial z} + i\alpha_a k_a \rho_a(z)\right] = -\frac{\varepsilon_0}{v_A^2}\frac{\gamma}{2}(+k_s)^2\left[E_P(z)E_S^*(z)\right]e^{i(k_p z + k_s z - k_a z)} \tag{16}$$

where E_S^* denotes the complex conjugate. Ideally, $k_P = k_S + k_a$ however, the backward Stokes wave may not always be exactly 180 degrees oriented relative to the pump wave. The wave-vector difference $\Delta k = k_p + k_s - k_a$ is called a phase mismatch. Moreover, by experimental observations (by author), as the fiber length approaches infinity (i.e., from 1 m to 1000 m), $\omega_a \ll \omega_p \approx \omega_S$. Then, the exponent in Eq. (16) can be rewritten. After dividing each side of Eq. (16) by $i2k_a$ and integrating both sides w.r.t. z and using integrating factor $e^{\int \alpha_a z/2}$, yields:

$$\rho_a(z) = \frac{i\varepsilon_0\gamma_e}{4}\frac{k_a}{v_A^2}\int_0^z \left[E_P(z')E_S(z')\right]e^{-i(\Delta k z')}e^{-i(\alpha_a(z-z')/2}dz' + \rho_a(0)e^{-\alpha_a z/2} \tag{17}$$

A critical assumption on phase (exponential terms) is the acoustic loss, α_a [units of m^{-1}] is only significant in a portion of fiber medium length, L [3]. Due to the factor $e^{(\alpha_a(z-z')/2}$ in the integrand, the major contribution to the integral in Eq. (17) must come from the range $(z - z') \le 2/\alpha_a$. Furthermore, for most experiments observed by the author, the rate of acoustic loss is much larger than rates of change for $E_P(z)$and $E_S(z)$ [7]. Under these circumstances, the product of these two amplitude functions $E_P(z)$ and $E_S(z)$ can be moved outside the integral. Evaluating the integral with $z \ge 2/\alpha_a$ yields:

$$\rho_a(z) = \frac{i\varepsilon_0\gamma_e}{4}\frac{k_a}{v_A^2}E_P(z)E_S^*(z)\frac{1}{i\Delta k+(\alpha_a/2)}\left[e^{-i(\Delta kz)}\right] \tag{18}$$

4. Final Exact Solutions: Acoustic and Optical

The density Eq. (8) when substituted into the optical Eq. (9) and neglecting the 2nd order spatial derivatives along with the case for maximum photoelastic coupling (when $\Delta k = 0$) yields:

$$\frac{\partial E_P(z)}{\partial z} \approx -\frac{\omega_p^2\varepsilon_0\gamma^2 k_a}{8c^2\beta k_p}\left[\frac{1}{i\Delta k+(\alpha_a/2)}\right]E_P(z)|E_s(z)|^2 \tag{19}$$

Similar to deriving Eq. (19), the spatial variation in the back-scattered Stokes wave is:

$$\frac{\partial E_S(z)}{\partial z} \approx \frac{\omega_p^2\varepsilon_0\gamma^2 k_a}{8c^2\beta k_s}\left[\frac{1}{-i\Delta k+(\alpha_a/2)}\right]E_s(z)\left|E_p(z)\right|^2 \tag{20}$$

In order to arrive at a useful PDE for the pump and Stokes wave, which can be used in MATLAB simulation, the following relations for wave intensity, I are useful:

$$I \approx \frac{1}{2}\varepsilon_0 c n_0|E|^2 = \frac{1}{2}\varepsilon_0 c n_0(EE^*);\ \frac{\partial}{\partial z}|E|^2 = \frac{\partial}{\partial z}(EE^*) = E\frac{\partial}{\partial z}(E^*) + E^*\frac{\partial}{\partial z}(E) \tag{21}$$

4.1. Summary of Coupled Solutions to SBS

Eqs. (19) and (20) can be rewritten with the density Eq. (18) and in the terms of Eq. (21):

$$\frac{\partial I_P(z)}{\partial z} \approx -g_0 I_P(z)I_s(z) \tag{22a}$$

$$\frac{\partial I_S(z)}{\partial z} \approx -g_S I_s(z)I_p(z) \tag{22b}$$

where I_P and I_s are the optical pump and back-scattered Stokes wave Intensities. $I_P(0)$ is the power (intensity) at incident plane ($z = 0$).

5. MATLAB Simulation of Sbs

Fiber optic systems like the ones used to carry internet data, operate under some threshold in order to prevent significant power losses. Above an input power threshold, the Stokes backscattering limits the transmitted power the laser power then saturates. In fact, higher input power is redirected toward the Brillouin scattering direction. Therefore, the Brillouin threshold becomes an important metric.

5.1. MATLAB Solver and Free Boundary Condition

One issue with the usage of the ODE45 solver in MATLAB is related to the definition of the boundary conditions for I_P and I_s in Eq. (22) which describe signals propagating in forward and backward directions. Thus, the boundary conditions are defined on the opposite sides of the fiber section. For Eq. (22) only the pump power $I_P(0)$ is known, as this is the power supplied by the source (pump) laser. The Stokes power $I_S(0)$ is a solution of this problem and results from SBS. It is unknown until the coupled equations are solved.

Free Boundary Condition: Analogously, for Eq. (22), the known boundary condition is $I_S(L) = 0$, as the scattered signal propagates in the backward direction. The value $I_P(L)$ is unknown which makes this a free boundary problem (missing boundary information). Nevertheless, an effort is made to characterize the total optical power in MATLAB. Specifically, the following parameters are specified: Stokes power, Psp; the pump power, Pin; the length of the fiber, L; the Brillouin gain coefficient, gB, and the fiber loss, α. The approximation is the total optical power versus fiber length.

MATLAB was chosen to simulate the Stokes power because the MATLAB subroutine BVP (boundary value problem) is a sophisticated solver. The *bvp4c* algorithm relies on an iteration structure for solving nonlinear systems of equations. In particular, *bvp4c* is a finite-difference code that implements the three-stage Lobatto III-a formula. This is a collocation formula and the collocation polynomial provides a 'C1 –continuous' solution that is fourth-order accurate uniformly in $z \in$ [L1, L2]. Mesh selection and error control are based on the residual of the continuous solution. Since it is an iteration scheme, its effectiveness will ultimately rely on the ability to provide the algorithm with an initial guess for the solution [6]. In the code, *sol = bvp4c(@bvp4ode, @bvp4bc,solinit)* integrates a system of differential equations of the form $y' = f(x,y)$ specified by *bvp4ode*, subject to the boundary conditions described by *bvp4bc* and the initial solution guess, solinit. That is, an initial guess for the Stokes and pump powers is input to the bvpinit command. For each component of the solution, bvpinit replicates the corresponding element of the vector as a constant guess across all mesh points. Following the code is the simulation result for the total optical power plotted against fiber length. A goal here is determine the Brillouin threshold power depending on fiber length. To run the code, the function file 'SBS' with five input parameters are required. Those parameters are: Psp, the Stokes power in Watts; Pin, the pump power in Watts; L, the length of the fiber in meters; g_B, the Brillouin gain coefficient; alpha, the fiber loss in $(mW)^{-1}$.

5.2. MATLAB Code for SBS Simulation

The MATLAB code includes fiber attenuation, α, so the exact PDEs are used:

$$\frac{\partial I_P(z)}{\partial z} \approx -g_0 I_P(z) I_s(z) - \alpha_P\, I_s(z) \tag{23a}$$

$$\frac{\partial I_S(z)}{\partial z} \approx -g_S I_s(z) I_p(z) - \alpha_S\, I_P(z) \tag{23b}$$

The function SBS computes the evolution of optical power in the fiber that includes a single 'signal' beam and backward scattered light due to SBS, by solving the Brillouin power differential equations as a boundary value problem (BVP). The boundary conditions are given by the input optical power and the spontaneous Brillouin power that 'seeds' the SBS process. The function inputs are as follows: Psp the spontaneous Brillouin power in Watts (aka Stokes power); Pin is the input signal power in Watts (W), the pump signal ; L is the fiber length in m. g_B is the Brillouin gain parameter in $(m*W)^{-1}$ while alpha is a length 2 vector specifying the fiber attenuation for the signal and Brillouin wavelengths in dB/km. P_{out} is a 3 column array with the first column being values of position z in the fiber, the second column being signal power as a function of z, and the third column being the SBS power as a function of z. The code converts attenuation to units of m^{-1}. The code is shown here:

function Pout = SBS(Psp,Pin,L,gB,alpha)
*alphap=alpha/(4.3429*1000); % signal (pump)*
*alphas=alpha/(4.3429*1000); % Stokes loss factor in units %of m^{-1}*
% Set up the initial solution [Ps(L),P_P(0)]
solinit = bvpinit(linspace(0,L,20),[1,1]); % bvpinit is the %initial guess for BVP4C
% Solve the boundary value problem (calls functions below)
sol = bvp4c(@bvp4ode,@bvp4bc,solinit);

```
% Evaluate the solution - zint is a vector corresponding to %fiber position
% and Szint is an array with columns 1-2 the corresponding values for the pump power and Stokes power
zint = linspace(0,L,1000); % L pts between 0 and 1000 %meters
Szint = deval(sol,zint);% evals the soln of DE at all entries %of vector zint
Pout = Szint; %DE solution is passed to the variable Pout
% ODE definition routine where P(1) and P(2) are the %pump power and Stokes power,
% respectively
function dPdz=bvp4ode(z,P) %solve boundary value %problem for dPdz ODE
dPdz = [-(gB)*P(1)*P(2)-(alphap*P(1)) ... % next line for %the PDE
 -(gB)*P(1)*(P(2)+Psp)+(alphas*P(2))];
End %boundary condition definition routine - P0/L is the power at z=0/L and indices 1 & 2 are the incident
(pump) and Stokes optical intensities
function res=bvp4bc(P0,PL)
res = [P0(1)-Pin PL(2)-0];
end
end
plot(zint,Szint(1,:));
xlabel('fiber length(m)'); ylabel('Optical Power(W)')
title('Optical Power vs. Fiber Length')
end
```

5.3. MATLAB Simulation Result

The simulation results are the total optical power due to the input optical power and the spontaneous Brillouin power that 'seeds' the SBS process. The input parameters used to run the code are SBS(0.1, 1, 100, 0.1, 0.1). As shown in Fig. 2, the 'half-power' point occurs at 3.7 meters from the fiber face where the optical power is 0.708 Watts (708 mW). See Fig. 2 below.

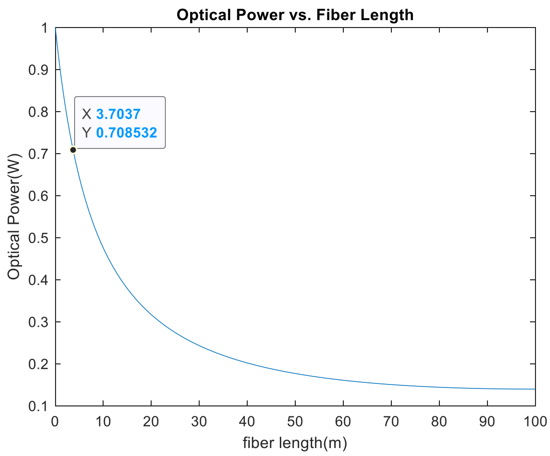

Fig. 2. SBS optical power versus fiber length (MATLAB simulation).

6. Conclusion

The current literature survey reveals various experimental schemes for fiber optic experiments that exploit SBS. Only a few of the papers and one or two textbooks go into detail of the derivations or the coupled SBS PDEs.

This study started with Maxwell's equations and derived a theoretical model of Brillouin scattering that shows how spontaneous Brillouin scattering is initiated by acoustic waves within a fiber optic medium. Provided is a derivation of an approximate analytical solution to the system of SBS equations in a lossless medium. This model predicts how the SBS Stokes intensity depend upon the laser pump intensity and upon the physical properties of the SBS medium. A brief study of a numerical approximation to the Brillouin wave equations was presented as MATLAB code and simulation result.

Conflict of Interest

The author declares no conflict of interest.

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