## Conflict of Interest

The authors declare that they have no known competing financial interests.

## Autor Contributions

Li Guan put forward the problems we want to study and the research ideas of the problems. Han Dai mainly gives the idea and relevant proof of the algorithm, and is responsible for drafting the paper. Xiaofei Liu is mainly responsible for reviewing and revising the paper, giving a specific example, and is responsible for the final version of the paper.

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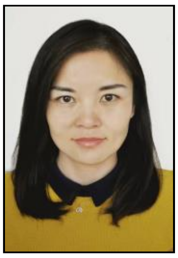

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