Determination Optimal Indicators of Expressway Toll Plaza with M/G/1 Queue Model

Meilan Qiu, Dewang Li*, Dazhuo Zhou*
School of Mathematics and Statistics, Huizhou University, Huizhou, Guangdong 516007, China.

* Corresponding author. Email: ldwldw1976@126.com, zhoudazhuo@hzu.edu.cn
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Abstract: Long lines and heavy congestion is always the bottleneck problem of highway toll plaza, the operation efficiency of tollbooths is crucial to the traffic throughout of expressway toll plaza. In this report, we explore M/G/1 model in stochastic service system theory for obtaining the important indicators (eg: delay time, traffic throughput, construction cost and accident prevention) for improving the operation efficiency of tollbooths. Combining with fluid mechanics theory, we get the relationship between some important indicators such as queue length, delay time (including service time and overall waiting time) of the tollbooths system and the single channel capacity, and also the proportion of the number of various windows type. At the same time, we give out the reasonable accident coefficient and safety distance between vehicles. We find that the security distance between vehicles will directly affect the queue length, and thus influence the traffic capacity of toll booths.

Key words: M/G/1 queue model, expressway toll plaza, fluid mechanics theory, optimal indicators.

1. Introduction

With the rapid development of society, the traffic congestion is a bottleneck problem of expressway which is encountered by many countries and should to be solved as soon as possible. Expressway toll plaza is an important component of the highway, and it belongs to the category of traffic construction [1]. During peak hours, the service efficiency of the tollbooths is greatly reduced. When the service level cannot satisfy the demand of traffic capacity, the phenomenon of traffic congestion and long queues will be formed in the toll plaza [2]. How to apply various technical methods to create a smooth and convenient barriers tolls environment is the primary task of freeway toll station design (Fig. 1).

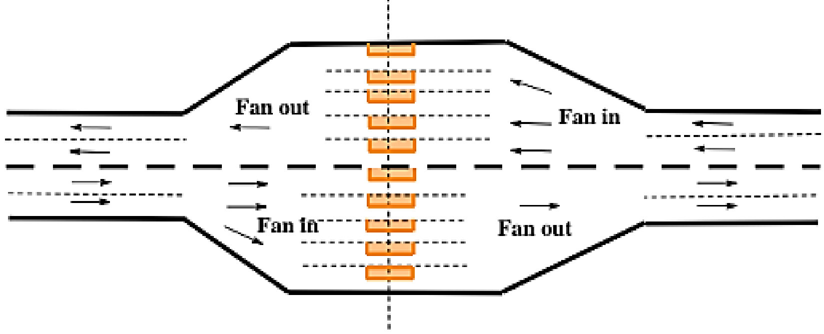

Fig. 1. A traditional graph of the toll plaza.

The previous research works about toll plaza are mainly focused on the traffic capacity, safety and environmental impact. Most researchers have paid attention to the traffic capacity. For example, Zarrillo analyzed the traffic capacity of five freeway tollbooths in Florida by applying SHAKER method [3]. Kazuhito and Takashi discussed the traffic capacity as well as the queues conditions in traffic flow on toll highways with multi-lanes [4]. Moreover, the safety analysis is an indispensable part of expressway toll plaza designation [5]. In literatures [6], [7], Zhang Zhizhao, Ayein and Kiskel et al carried out the application of traffic conflict technology in highway safety evaluation, they simulated the stochastic nature of traffic arrival, toll collection time and driver decision making respectively by using the simulation software VB and NET as well as Hassan, and then established the microscopic traffic simulation model to design, evaluation and operation analysis of expressway toll plaza. As for the delay analysis, Ayein's Mesoscopic approach is more close to the practical delay than the VISSIM method. Such as, in [8], [9], Hassan, Abdelwahab and Chau et al used a simple minded single-lane deterministic discrete traffic model to investigate the impact of the presence of toll stations on average vehicle speed. By using the nagel-schreckenberg model, Dingwei Huang and Weineng Huang simulated the traffic conditions of expressway and analyzed the influence of toll plaza on traffic flow [10]. According to the previous research, about 36 percent of the delay times are caused by the parking fee [11]. Therefore, the most effective method to heighten the service level of expressway is to improve the traffic capacity and to optimize the designation of toll booths on highways. According to different actual situations, Corwen et al studied and simulated the optimal number of tollbooths to minimize the average waiting time [12]. In general, tollbooths need to be designed conforming to encouraged behavior, e.g. Spannet et al suggested that the faster tollbooths should be put on the left of the lane, while Kane proposed that, in order to ensure a relatively smooth flow, tollbooths employ no barrier [13]. Issam Bouganssa et al studied a hardware and software combination to obtain an optimal solution for the edge detection [14].

All the above studies have their own advantages, and the vast majority of them focus on the impact of existing toll plazas. However, these research works did not combine these effects to design a solution to the traffic difficulties of toll plaza. Therefore, we focus on to design the optimal toll plaza. As is known to all, many indicators can affect the operating efficiency of the highway toll booths, for example, the number of lanes, the delay time, the traffic capacity, construct cost and accident rate and so on. In this paper, with the support of technology and fluid mechanics theory, we developed mathematical models to determine the best indicators of designing highway toll plaza, we have done the following three aspect works.

Determine the number of charging windows: The number of charging windows is determined by the traffic flow, the capacity of each charging window, and the level of service. We choose M / G / 1 queuing model to describe the actual operating state as well as the traffic flow of the toll plaza, so as to divide the service level of toll stations. At a certain level of service, we convert traffic flow into traffic intensity, that is, the matching condition of the number of charging windows.

Optimal cost computation of toll plaza: Vehicles pass through the toll plaza is a process of first slowing down and then accelerating. Therefore, in order to simplify the mathematical model, we assume that our study is carried out in an ideal traffic condition: That is, the safety distance between the front and rear cars as well as the actual braking distance in the process of speed changing. We build an optimal model based on above two conditions and obtain the best indicators of the toll plaza that minimizes the construction cost.

Test models and analyze performance in different traffic environments: Because the service pressure of toll plaza increases with the increase of traffic flow, so we consider the impact of self-driving vehicles on this proposed model. We find that autonomous vehicles can increase the traffic capacity of the toll plaza, and the proportion of the three types of windows plays a key role in the design scheme. Finally, according to the traffic flow characteristics of highway toll plazas，the optimal indicators of traffic performance under different service levels are obtained by exploiting M / G / 1 queuing model, our research is of great economic and social application value.

2. Notation

In this part, we list the symbols which are required for the following text. Let L_q be the average queue length, D is the delay time, Z denotes discharge capacity, λ is the average arrival rate of vehicles on toll plaza, μ is the business service rate, V is the service time, C denotes the total cost, C_1 is the construction cost, $E[V]$ is an expectation of service time, $D[V]$ denotes the variance of service time, t_1 is the vehicle's deceleration time for entering into toll stations, l_1 is the vehicle's deceleration length, W is the vehicle's average stay length on toll points, t_2 is the acceleration time for vehicles leaving out, l_3 is the length for vehicles passing through the toll gates, D denotes the unit price of the land (i.e., D yuan for each square meters), n_M is the number of MTC lanes, n_E is the number of ETC lanes, T_{W_1} is the delay time for MTC charge, T_{W_2} is the delay time for currency charge.

3. M/G/1 Queue Model

3.1. Assumptions and Simplifications

In order to obtain the optimal indicators of the toll booths by using the M/G/1 queue model, we need adopt the following assumptions:

1) The time headway above the square conforms to the distribution characteristics of vehicles arriving at the toll station.
2) Assumption that all the vehicles in the model are standard vehicles, and the actual conversion can be carried out according to the conversion coefficient. As is shown in the following Table 1:

Table 1. Vehicle Conversion Factor [15]

Vehicle type	Conversion factor	Vehicle type	Conversion factor
Car	0.5	Medium truck	1.0
Van	0.5	Big truck	1.0
Bus	1.0	Trailer	1.5
Small truck	1.0	Drag the car	1.0

3.2. M/G/1 Model in Stochastic Service System [16]

3.2.1. Key indicators for the optimal performance

In this part, we derived some indicators for the optimal performance of the toll station system based on the above assumptions.

1) Queue length

The expected queue length of customers waiting for service at any time of the steady-state system is represented by L_q, which is formulated as:

$$L_q = \frac{\lambda^2\{(E[V])^2 + D[V]\}}{2(1-\lambda E[V])} \tag{1}$$

2) The length of the team

The expected average captain of vehicles in the toll station is expressed as L, and the formulation is as following:

$$L = \rho + \frac{\rho^2 + \lambda^2 D[V]}{2(1-\rho)} = \lambda E[V] + \frac{\lambda^2\{(E[V])^2 + D[V]\}}{2(1-\lambda E[V])} = \lambda E[V] + L_q \tag{2}$$

3) Queuing time

The expected average queuing time is denoted by W_q, which is described as:

$$W_q = \frac{\lambda^2\{(E[V])^2 + D[V]\}}{2\lambda(1-\lambda E[V])} = \frac{L_q}{\lambda} \tag{3}$$

4) Residence time

The average residence time of vehicles in the charging station is denoted by W which is equal to W_q plus the service time, the formulation is:

$$W = E[V] + \frac{\lambda^2\{(E[V])^2 + D[V]\}}{2(1-\lambda E[V])} = E[V] + L_q \tag{4}$$

where ρ denotes the toll lane utilization rate, V is the service time, $E[V]$ represents the expectation of service time, $D[V]$ denotes the variance of service time.

In the following text, we will apply the M/G/1 model to calculate the actual problems based on the data in the following Table 2.

Table 2. The Average Service Time and Variance

Survey methods	Service time	The variance
Cash payments	22	45
Export tickets	10	15
Entrance card	3	6

3.2.2. The delay time

Define the delay time (denoted by D) as the time when the vehicle leaves the tollbooth, while the subsequent vehicles arrives and stops. This definition satisfies the continuity and linearity of the follow three data, i.e., the deceleration time t_1 when the vehicle enters the toll booth, the acceleration time t_2 when the vehicle leaves the toll booth and the average residence time W, so we obtain:

$$D = W + t_1 + t_2 = (\frac{1}{\lambda} - \frac{3.6m}{v_0})[\lambda E[V] + \frac{\lambda^2\{(E[V])^2 + D[V]\}}{2(1-\lambda E[V])}] \tag{5}$$

where the deceleration time is

$$t_1 = \frac{v_0}{3.6a_1} \tag{6}$$

The displacement a vehicle decelerate as it slows

$$l_1 = \frac{1}{2}a_1 t_1^2 \tag{7}$$

The acceleration time is

$$t_2 = \frac{V_0}{3.6a_2} \tag{8}$$

And the through displacement for a vehicle as it accelerates

$$l_3 = \frac{1}{2} a_2 t_2^2 \tag{9}$$

where V_0 represents the normal velocity, a_1 is acceleration when the vehicle decelerates, a_2 is the acceleration when the vehicle accelerates.

In the following text, we will discuss the service time V

First, we need the probability of customer flow.

The probability distribution of customer flow is the distribution of the time interval between successive customer arrivals. In generally speaking, the probability density function conforms to poison distribution, i.e.:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \tag{10}$$

where, $P_n(t)$ is the probability of n vehicles arriving at the toll plaza within an actual period time, λ is the average arriving rate of vehicle, or the average arriving number of customers in per unit time.

Now we model the probability of service time

The service time of each tollbooth in station obeys a definite probability distribution, the formulation is as following:

$$f(t) = \mu e^{-\mu t} \tag{11}$$

where the distribution function is

$$F(t) = 1 - e^{-\mu t} \tag{12}$$

μ is refer to the average service rate, that is, the average number of customers leaving out the toll plaza after receiving the service in per unit time.

Generally speaking, the smaller the delay time, the higher the service level which is contribution to the high performance of the tollbooths. On the contrary, the larger the delay time, the lower the service level and thus affect the performance of the toll station. Table 3 list out the impact factors of the delay time with different charging methods.

Table 3. Toll Station Service Level and the Delay Time (Denote by D, Hour)

Delay / Charging methods	Service Level			
	First level	Second level	Third level	Fourth level
Cash Payment	D < 35	35 ≤ D < 70	70 ≤ D < 150	D ≥ 150
Export tickets	D < 35	35 ≤ D < 70	70 ≤ D < 150	D ≥ 150
Entrance card	D < 35	35 ≤ D < 70	70 ≤ D < 150	D ≥ 150

3.3. Traffic Capacity Calculation of Toll Station

Expressway toll plaza capacity depends on the choice of charging approach. The traffic capacity can vary greatly depending on the mode of payment. The capacity of the toll plaza is usually measured by the degree of the impact of vehicle conversion. The influence of different vehicles on the capacity of toll plaza is mainly the different departure time.

The basic capacity of the toll lanes refers to the maximum capacity to pass through a toll booth on an ideal condition of road and traffic, it can be expressed as:

$$Z = \frac{3600}{T_j + T_{G_j}} \tag{13}$$

where, Z is the basic traffic capacity of toll lane, T_j denotes the service hours of the standard car j, T_{G_j} represent the standard departure time of car j.

The relationship between toll station types and the capacity is shown in the following Table 4:

Table 4. The Capacity for Various Toll Lanes

Tollbooths type	The capacity (minibus numbers in per unit hour)
The artificial tollbooth	220
Automatic tollbooth	480
Semi-automatic tollbooth	380

In addition, different charging methods also have an impact on traffic capacity, the influence of various charging methods on service level is shown in the following Table 5.

Table 5. The Relationship between Service Level of Station and the Charging Method, Here Z is the Basic Traffic Capacity

Charging method	Service Level			
	First level	Second level	Third level	Fourth level
Labor charges	Z < 60	60 ≤ Z < 130	130 ≤ Z < 148	Z ≥ 148
Semi-automatic charge	Z < 290	29 ≤ Z < 325	325 ≤ Z < 345	Z ≥ 345
Automatic charge	Z < 1100	1100 ≤ Z < 1150	1150 ≤ Z < 1180	Z ≥ 1180

3.4. The Minimum Cost of Toll Plaza Based on the Optimization Model [17]

We must consider the cost factors to ensure that the toll plaza traffic conditions are safe and effective when design the toll plaza, since the highway construction is very expensive. With the principle of economizing on investment, reducing the land use and protecting the natural environment around it, the designation of freeway toll plaza should consider the construction cost, operation cost and delay time cost and so on. We establish the optimization model of the total cost C of toll station as

$$C = C_1 + C_W \tag{14}$$

where $C_1 = C_j/(365 \times 24) + C_y/24$, C_j is the construction cost, C_y denotes the operating cost.

3.5. The Component of the Total Cost

3.5.1. The construction cost

The construction cost contains the land acquisition cost and the toll station lane construction cost. We assume that the standard width of MTC channels is 3.2m, and set the standard width of ETC channels as 3.5m, we can get the model of toll station construction cost as following:

$$C_j = n_M C_M + n_E C_E + Sd \tag{15}$$

where

$$S = \frac{(b_2 x_2)^2 - (b_1 x_1)^2}{2\alpha p} + b_2 x_2 l_2 + \frac{(b_2 x_2)^2 - (b_1 x_1)^2}{2\beta p} \tag{16}$$

In the model, C_j is the highway construction cost, d is the price of per unit land, C_M denotes the construction cost of each MTC lane, C_E is the construction cost of each ETC lane, n_M is the number of MTC channels that have been built, and n_E represents the number of ETC lanes that have been built, S is the distance between the rear and front vehicles.

3.5.2. The operation cost

The operation cost mainly includes equipment running cost, depreciation cost, personnel salary and so on. The formula to calculate the operation cost is:

$$C_y = a_1 n_E + a_2 n_M + C_g b n_M / 30 \tag{17}$$

where, C_y is the operating cost of toll station, a_1 is the operation cost of a single ETC lane, a_2 is the operating cost of a single MTC lane, C_g is the monthly salary of the charging staff, b is the average number of personnel assigned per MTC per day.

3.5.3. The cost incurred by time delays

The delay cost mainly refers to a series of costs caused by the long time when the vehicle passes through the toll booth. We calculate the delay cost based on the vehicle's stay time in the toll collection system in the follow formula:

$$C_d = H \times V_{VOT} \times \left(\frac{1}{\lambda} - \frac{3.6m}{v_0}\right)\left[\lambda E[V] + \frac{\lambda^2\{(E[V])^2 + D[V]\}}{2(1 - \lambda E[V])}\right] \tag{18}$$

where, H is the average manned quantity of vehicles and V_{VOT} is the time cost per capita. And we calculate the personnel salary by assuming that the per capita daily working time is 8 hours, i.e., $V_{VOT} = \frac{G}{30 \times 8}$, where G is the per capita monthly income.Some Parameters of the Model are Shown in the Table 6.

Table 6. Some Parameters of the Model

Parameter	The values
a_1/\$	15
a_2/\$	7.5
b	3
d/(\$$\cdot m^{-2}$)	1500
C_E/\$	159506
C_M/\$	107155

3.6. Accident Prevention Based on New Jerseys Freeway toll Plaza [18]

According to the previous work, we realize that we need the following parameters to build an efficient toll plaza: traffic flow, toll lane, proportion of different toll lanes, the length of deceleration and acceleration area and the width of lane. Among these parameters, the most important parameter is the traffic flow. Based on the change of traffic flow at each moment, we obtain the following Fig. 2:

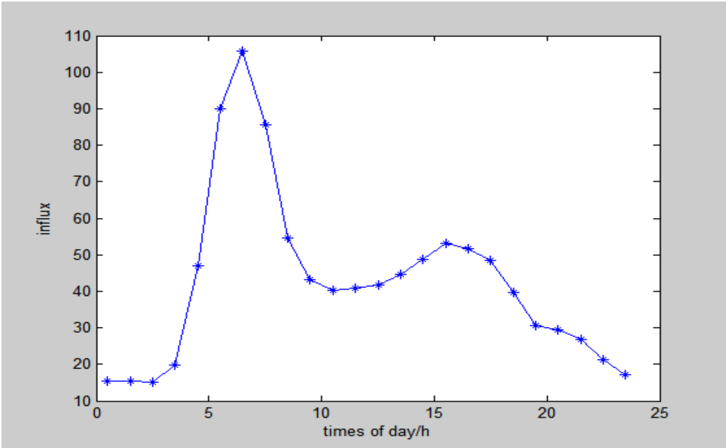

Fig. 2. The change of daily vehicle flow in New Jersey.

When vehicles fan in from B tollbooth egress lanes down to L lanes of traffic, the area following the toll barrier has a significant impact on accident prevention, vehicle flow and so on. Therefore, security analysis is an indispensable part in the design of highway toll plaza. According to the literature [12], the two factors such as cosine value p and the service level ($\frac{V}{C}$) of slope area of toll plaza play very important roles in affecting expressway accidents. Inspired by the above work, we establish the safety assessment model to calculate the evaluation index of accident prevention:

$$N = f(p, \frac{V}{C}) \tag{19}$$

Here, we use linear regression analysis method to verify the function variable f

$$\mathrm{f}\left(\mathrm{p}, \frac{v}{c}\right) = \beta_1 p + \beta_2 \left(\frac{v}{c}\right).$$

$$f(p, \frac{V}{C}) = \beta_1 p + \beta_2 (\frac{V}{C}) \tag{20}$$

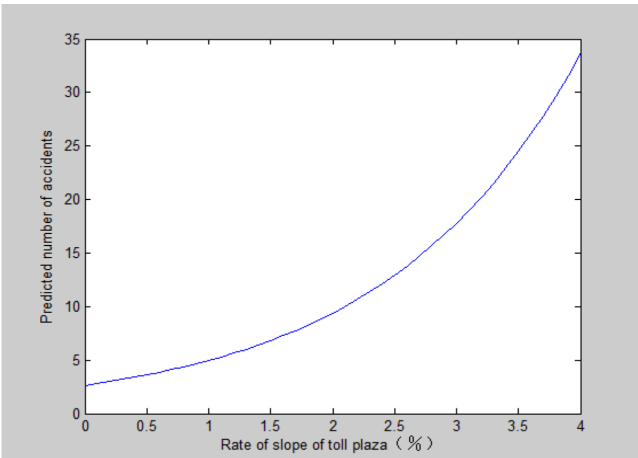

Fig. 3. The relationship between traffic accidents and the slope rate of toll plaza.

We obtain the accident prediction function of toll station caused by slope rate based on the regression analysis results:

$$N' = 2.6254e^{0.638x} \tag{21}$$

The relationship between traffic accidents and the slope rate of toll plaza is shown in Fig. 3, this study uses MATLAB programs to execute the calculations involved in the proposed accident prevention model.

According to the curve in the figure, we observed that the higher the slope rate of the toll plaza, the more traffic accidents occurred and the lower the safety performance.

4. The Model

The previous research studies mainly focus on the impact of existing toll plaza. But these works do not synthesize these effects to design a solution to toll plaza traffic difficulties. Thus we focus on the designation of the toll station.

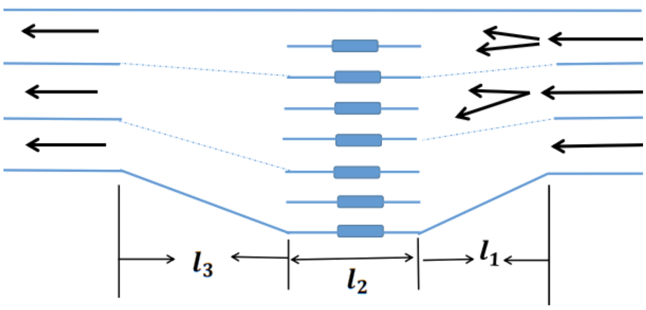

Fig. 4. A plan graph of the toll plaza.

According to the results of extensive literature and Fig. 4, we concluded that the slope of the toll plaza has a great influence on traffic accidents when vehicles enter and exit the toll station. Therefore, we calculate the distance between the rear and front vehicles by the following formulation.

$$S = \frac{(b_2x_2)^2 - (b_1x_1)^2}{2\alpha p} + b_2x_2l_2 + \frac{(b_2x_2)^2 - (b_1x_1)^2}{2\beta p} \tag{22}$$

where x_1 denotes the width of each lane on a highway, x_2 is the distance of every two toll booths, b_1 is the total number of lanes, b_2 is the total number of toll booths, and α, β, p are positive constants.

4.1. The Determination of the Window Type Number [19]

The number of window type should be determined by three main factors: traffic throughput, charging method and service level. The greater the traffic, the more windows needed, and thus need to design window types proportionally. Charging methods and facilities determine the time of charge, the shorter the charging time, the greater the traffic capacity, the less the number of windows required, and accordingly, the window type should be designed in proportion. The service level depends on the road class as well as windows type and the management requirements, which is expressed as the average number of vehicles waiting, the higher the service level, the less toll lanes needed, and thus can assign the window type accordingly. In order to calculate the appropriate number of windows, we need the following assumptions to establish the optimal model:

1. Assume that the traffic system satisfies the conservation of traffic flow.

2. Assume that the average speed is equal to the instantaneous speed when the vehicle leaves the toll plaza.

The toll plaza mainly consists of three areas: fan out, square center and fan in.

4.1.1. Fan-out stage

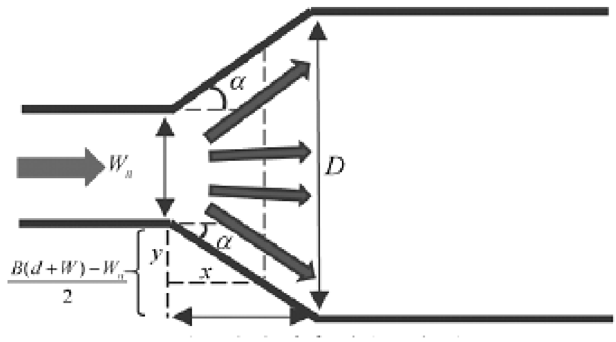

Fig. 5. Schematic diagram of fan-out area.

According to the relationship among traffic flow, traffic density and speed, we obtain the formulation of initial stage as:

$$q_0 = v_0 k_0 \tag{23}$$

And the intermediate stage as:

$$q_0 = v(x)k(x) \tag{24}$$

where q_0 is the initial traffic volume, v_0, k_0 denotes the initial velocity and density respectively, $v(x)$ represents the traffic speed at the point x, $k(x)$ is the traffic density at the point x.

Because the traffic volume is just a continuous constant, we get the following equation:

$$k_0 W_n = k(x)W(x) \tag{25}$$

where $W(x)$ is the width of the fan-out area.

And we also obtain the following equation:

$$\frac{y}{x} = \frac{D - W_n}{2S_1} \tag{26}$$

$$W(x) = W_n + 2y \tag{27}$$

$$W(x) = W_n + \frac{D - W_n}{S_1} \tag{28}$$

where, W_n is the width of the highway, the values of the remaining variables are shown in Fig. 5.

Combine with the equations (5-2)-(5-7), we get

$$v(x) = \frac{W(x)}{W_n} v_0 \tag{29}$$

By the definition of velocity, we can calculate the delay time by the follow equation

$$dT = \frac{dx}{v(x)}, T_n = \frac{S_1 W_n}{(D - W_n)v_0} \ln(\frac{D}{W_n}) \tag{30}$$

4.1.2. Stay at toll station stage

In the case of many vehicles, the toll plaza may form a bottleneck and will lead to congestion of highway. Therefore, design a reasonable number of windows type is a very important part for optimizing the toll station. In this paper, we assume that the toll plaza has only three types of window: the ETC gate, the MTC gate and coin gate. According to the theorem of fluid mechanics, we can obtain the optimal indicators of tollbooths, such as the delay time, traffic throughput, construction cost and so on by using the M/G/1 model.

The equation for calculating the delay time of MTC gate is:

$$T_{W1} = \frac{1}{\mu_1 - \lambda_1} = \frac{i_1 B}{\mu_1 i_1 - q_{11}} \tag{31}$$

The formulation for obtaining the delay time of coin gate is:

$$T_{W2} = \frac{1}{\mu_2 - \lambda_2} = \frac{i_2 B}{\mu_2 i_2 - q_{12}} \tag{32}$$

Because the traffic capacity of ETC gate is very efficient, here we ignore the delay time.

where q_{11}, q_{12}, are the traffic volume of the MTC lanes and the coin lanes respectively, i_1 and i_2 denote the probability of the above two gates respectively, and μ_M (M = 1, 2) are the average service rate, λ_M (M = 1, 2) are the average arriving rat

4.1.3. Fan-in stage

Similar to the fan-out stage, the delay time T_n is

$$T_n = \frac{S_1 D}{(D - W_n)v_2} \ln(\frac{D}{W_n}) \tag{33}$$

where v_2 denotes the speed when the vehicle enters into the fan-in zone.

Therefore, the delay time of the three types of windows is respectively:

$$T_1 = T_n + T_{W1} + T_u = \frac{S_1 W_n}{(D - W_n)v_0} \ln(\frac{D}{W_n}) + \frac{i_1 B}{\mu_1 i_1 - q_{11}} + \frac{S_1 D}{(D - W_n)v_2} \ln(\frac{D}{W_n}) \tag{34}$$

$$T_2 = T_n + T_{W2} + T_u = \frac{S_1 W_n}{(D - W_n)v_0} \ln(\frac{D}{W_n}) + \frac{i_2 B}{\mu_2 i_2 - q_{12}} + \frac{S_1 D}{(D - W_n)v_2} \ln(\frac{D}{W_n}) \tag{35}$$

$$T_3 = T_n + T_{W3} + T_u = \frac{S_1 W_n}{(D - W_n)v_0} \ln(\frac{D}{W_n}) + \frac{i_3 B}{\mu_3 i_3 - q_{13}} + \frac{S_1 D}{(D - W_n)v_2} \ln(\frac{D}{W_n}) \tag{36}$$

where i_3 represents the probability of the coin gate.

We use MATLAB programs to execute the calculations in the proposed model to get the optimal number of three types of windows under different traffic volumes.

4.2. The Influence of the Proportion of Three Type Windows on the Model

Because different type windows are different in their work efficiency, and thus the velocity of vehicles passing through different types of windows are different. Therefore, the proportion of different type of windows has great influence on the length of the acceleration area and the total number of windows. Hence, we use the controlling variable method to determine the impact of different type of windows in our program. We set the proportion as automatic windows: semi-automatic windows: artificial windows = 3:1:1. According to the calculation of the number of toll windows and the length of the slope (or acceleration) area, we know that the greater the proportion of automatic windows, the whole toll plaza area will be reduced, so the total number of windows will be reduced, thereby improving the toll plaza capacity.

5. The Impact of the Model on the Autonomous Driving

5.1. Analysis of the Gallop Model

With the rapid development of science and technology, the number of autonomous vehicles in traffic is increasing day by day. We use the letter r to denote the ratio of the autonomous vehicles, then we have the ratio $r = \frac{N_1}{N}$ where N_1 is the number of self-driving vehicles, N is the total number of vehicles, and $0 \le r \le 1$.

The relationship between the human-driven mixed traffic flow and the density on a single lane with different mixing ratios was simulated by numerical methods. Six groups of mixing ratios were selected to simulate, the results are shown in the following Fig. 6:

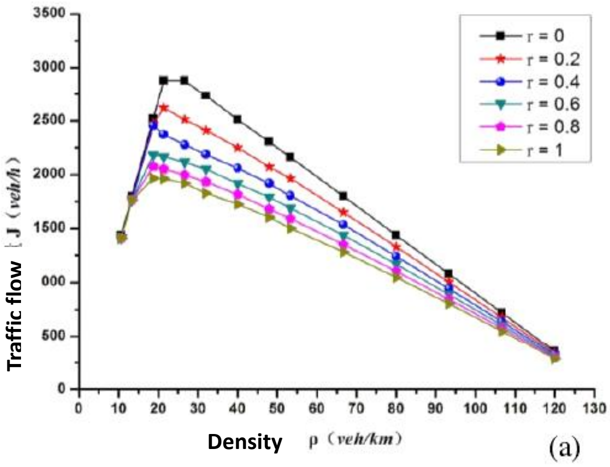

Fig. 6. The diagram of traffic flow changing with density.

It can be seen that, from the curve in the Fig. 6, with the increase of density p, the traffic flow strengthens at the beginning, then starts to decline after reaching the peak value, and reaches the peak value at the critical density.

5.2. Assumptions

In the same lane, we assume that the same direction of the vehicle traveling at the same speed, and there is safe minimum distance between vehicles. This situation constitutes a stable traffic flow. If the space of the following vehicle is too small, it is prone to crash, if the space is too large, it will affect the capacity of the highway.

5.3. Minimum Safe Distance

The minimum safe distance refers to the shortest distance between vehicles that must be maintained to avoid rear-end collision with the current vehicle. We design the shortest safety distance between any two vehicles as in the below Fig. 7.

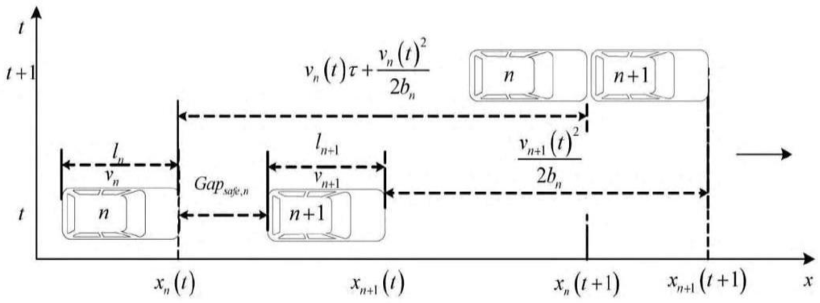

Fig. 7. The shortest safety distance between any two vehicles.

By the principle of calculating safety distance, we establish the safety distance model as follow:

$$Gap_{safe,n} = x_{n+1}(t) - x_n(t) - l_{n+1} = v_n(t)\tau_n + \frac{v_n(t)^2}{2b_n} - \frac{v_{n+1}(t)^2}{2b_n} \tag{37}$$

where, the $Gap_{safe,n}$ is the safety distance required by the Nth vehicle, $x_n(t)$ denotes the position of the Nth car at time t, $x_{n+1}(t)$ represents the position of the vehicle in front of the Nth vehicle at time t, l_{n+1} is the length of the vehicle, b_n is the maximum deceleration speed of the Nth car, τ_n is the reaction time of the driver in the Nth vehicle, $v_n(t)$ is the instantaneous velocity of the Nth vehicle at time t, $v_{n+1}(t)$ is the instantaneous velocity of the vehicle in front of the Nth vehicle at time t.

The safety distance required for safe driving is closely related to the velocity of the vehicle and the reaction time of the driver. The relationship surface among the safety distance, the velocity and the reaction time are shown in Fig. 8, here we assume that the current vehicle is traveling at the same speed as the vehicle in front of it, i.e: $v(t) = v_n(t) = v_{n+1}(t)$.

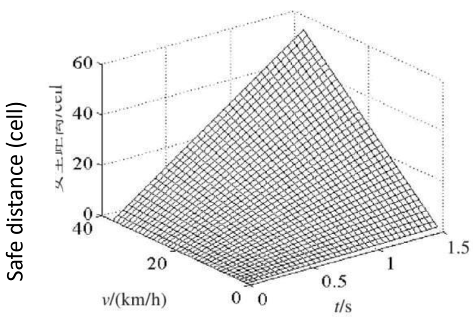

Fig. 8. The relationship among the safety distance, the velocity and the reaction time.

It can be seen from the surface graph that the greater the speed and the longer the reaction time, the greater the safety distance required, and then affect the capacity of highway toll plaza. We obtain the results as in Fig. 9 by using the simulation analysis.

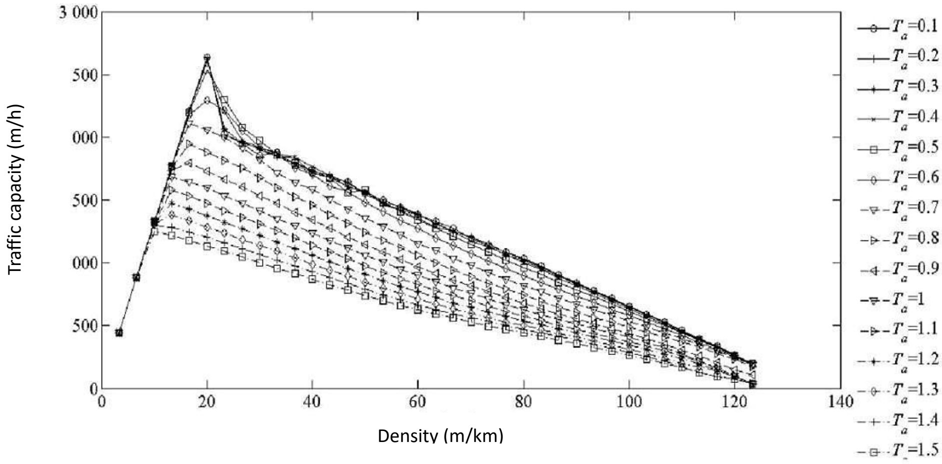

Fig. 9. The relationship between traffic capacity and density.

As can be seen from the Fig. 9, when the response time is in the range of 0~0.5, the traffic capacity does not change significantly. When the reaction time is greater than 0.5, with the increasing of density p, the traffic flow strengthens at the beginning, then starts to decline after reaching the peak value, and reaches the peak value at the critical density, which is consistent with the results of the galloping model. This shows that our proposed model is effective.

6. Conclusion

In this paper, we designed the optimal model for calculating the proportion of the different type toll windows, we find that the number of tollbooths can be determined by the throughput, traffic capacity and service level of each toll lane. So it is an important task to accurately calculate the number of various windows for designing the toll plaza. Through the sensitivity analysis, the traffic volume is positively related to the number of tollbooths, in turn, the traffic volume has a lot of influence on the number of tollbooths, and thus it also affects the proportion settings of various types of windows. Moreover, we apply the M/G/1 model in stochastic service system theory to analyze the model and get some important indicators such as traffic throughput, construction cost and delay time to improve the operation efficiency of tollbooths.

By the analysis of the traffic volume and accident rate of existing toll station, we build the optimal model based on the maximum traffic flow on designing the toll plaza and calculate the evaluation index of accident prevention. When analyzing the impact of traffic flow on accident rate, we find that the accident rate increases with the increase of traffic flow at a very small rate, while the braking distance and the safety distance of vehicles on the toll plaza determines the size of the toll station. In view of the above analysis results, we establish the optimal model to obtain the minimum total cost.

Finally, we verify our model and find that throughput is increasing as traffic flow increased, the entry of autonomous vehicles will optimize the capacity of the toll plaza, and the proportion of the three types of windows will play a perfect role in the design scheme.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Meilan Qiu and Dewang Li conducted the research; Meilan Qiu and Dazhuo Zhou analyzed the data; Dewang Li and Dazhuo Zhou wrote the paper; all authors had approved the final version.

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Dewang Li was born in Anyuan, Jiangxi, China in January 1976. He studied at Yunnan University. He obtained a probability theory Ph.D. in mathematics and mathematical statistics in 2015. He is mainly engaged in the fields of mathematical statistics and Bayesian statistics. He is a lecturer. He worked in the Department of Mathematics, Hechi College, Guangxi, China from 2008 to 2012. From 2015 to now, he works in the Department of Statistics, School of Mathematics and Statistics, Huizhou University, Guangdong, China. He has published SCI articles in communications in statistics - theory and methods, stat papers, and advances in mathematical physics.

Meilan Qiu was born in 1981, Ganzhou, Jiangxi. She is hold with a Ph.D. and is a lecturer of Huizhou University. She mainly studies the theory of partial differential equations and complex flow coupling model as well as its numerical solution, profound understand the basic theory and programming calculation of the finite difference method, the finite element method (FEM) and LDG (local discontinuous finite element) method and so on. She can write the corresponding Matlab code to enhance the implementation capability and have a deep understanding of the design of numerical calculation method for complex coupled flow equations and its numerical theoretical analysis. At present, she is mainly engaged in modeling, theoretical analysis and numerical simulation of complex porous media flow and fluid coupling in large cracks or pipelines. She published 9 SCI papers, presided over one Natural Science Foundation project of Guangdong Province (2018A030310038) and participated in four projects of the National Natural Science Foundation of China.

Dazhuo Zhou was born in 1971, Langfang, Hebei. She is a Ph.D. and professor of Huizhou University. She mainly studies the theory and algorithm of data mining. She can write corresponding Python programs to enhance the implementation capability, and has an in-depth practice and understanding of the actual data analysis. At present, she is mainly engaged in data analysis preprocessing and clustering and classification, as well as association rule algorithm research. She has published three EI papers, participated in one Natural Science Foundation project of Guangdong Province. This paper is supported by Huizhou University Project (151090153).