Some Characteristic Properties of Ruled Surface with Frenet Frame of an Arbitrary Non-cylindrical Ruled Surface in Euclidean 3-Space

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Abstract: In differential geometry of curves and surfaces, a ruled surface represents one of the most fascinating topics in surface theory, it is defined by choosing a curve which called base curve and a line along that curve (ruling). An important number of researchers in many papers have studied one of the moving frames of its base curve.

In this paper, we consider Striction curve of a non-cylindrical ruled surface as base curve of the ruled surface whose rulings are linear combinations of Frenet frame vectors of the first ruled surface. We investigate some important characteristic properties of the new ruled surface such as the Gaussian curvature and the mean curvature. Moreover, we give characterizations relatively to its developability and minimality and achieve by giving an example with illustrations.

Key words: Ruled surface, Frenet frame, striction curve, Gaussian curvature, mean curvature.

1. Introduction

Surfaces and curves theory [1]-[3] has interested and interest until our current days many researchers in the domain of differential geometry. Especially, ruled surface which is generated by a continuously moving of a straight line in the space and which has special characteristics. It has been studied with different frames and characterized in terms of its several properties.

In [4], ruled surface whose characterizations are related to the geodesic curvature, the normal curvature and the geodesic torsion has been investigated introducing Darboux frame, precisely the ruled surface whose ruling are combinations of the first and the third vectors of Darboux frame, they presented several properties and characterizations such as developability when the distribution parameter vanishes. But in [5], the authors have studied a family of ruled surfaces generated by the general and the slant helices [6], [7] with Frenet frame of their base curves, they have investigate properties such as Gaussian curvature, second Gaussian curvature, mean curvature and second mean curvature and gave characterizations in some special cases when the base curve is some special general and slant helices.

Another one of the most important frames which has been introduced for studying ruled surface is Frenet frame of ruled surface, which we can find introduced in [8], it represents a special frame which is defined on striction curve as a special curve that is defined uniquely on a non-cylindrical and regular ruled surface. Frenet frame of ruled surface has been studied in [9] relatively to a special kind of ruled surface which we call slant ruled surface, this last notion which is used in [10] where the authors have introduced

Darboux slant ruled surface when Darboux vector makes an angle constant with a fixed non-zero direction.

Another hand, Bishop frame which is used in the fields of biology and Computer Graphics and which may provide a new way of controlling virtual cameras in Computer animations has been introduced in [11] and related to ruled surface in euclidean 3-space, the authors have studied special ruled surface such as Bishop Darboux ruled surface and its properties. In [12], the parallel ruled surface notion which is used in many industrial process such as manufacturing for toolpath generation in sculptured surface machine and also in rapid prototype has been introduced and its characteristics such as developability and striction have been studied by authors in euclidean 3-space.

Developability is one of the properties of ruled surface, it is defined when the Gaussian curvature vanishes. In [13], the authors have given necessary and sufficient condition for a ruled surface to be developable but this time in Minkowski 3-space by investigating the distribution parameter when the ruled surface is defined with a straight line in Darboux trihedron moving.

In this work, we are interested in study of special ruled surface which is defined with Frenet frame of an arbitrary and given non-cylindrical regular ruled surface in Euclidean 3-space, we investigated its several characteristic properties and give an example.

2. Preliminaries

Let E^3 be a 3-dimensional Euclidean space provided with the metric given by $<,>= dx_1{}^2 + dx_2{}^2 + dx_3{}^2$, where (x_1, x_2, x_3) is a rectangular coordinate system of E^3 and I be an open interval in the real line $\mathbb{R}$. Let $c = c(s)$ and $q = q(s)$ be a regular curve and a unit direction vector of an oriented line in E^3, respectively. Then the parametric representation of a ruled surface is given as follows

$$\phi: (s, v) \in I \times \mathbb{R} \mapsto c(s) + vq(s), \quad \|q(s)\| = 1 \tag{1}$$

The curve $c = c(s)$ and $q(s)$ are called the base curve and rulings of ruled surface, respectively. In particular, if the direction of $q(s)$ is constant, then the ruled surface is said to be cylindrical and non-cylindrical otherwise.

Let m be the unit normal vector field on the ruled surface ϕ defined by (1) which is supposed regular, we have

$$m(s, v) = \frac{\phi_s \wedge \phi_v}{\|\phi_s \wedge \phi_v\|} = \frac{\left(c'(s) + vq'(s)\right) \wedge q(s)}{\left\|\left(c'(s) + vq'(s)\right) \wedge q(s)\right\|}. \tag{2}$$

where $\phi_s = \frac{\partial \phi}{\partial s}, \phi_v = \frac{\partial \phi}{\partial v}$.

The first I and the second II fundamental forms or ruled surface ϕ at a regular point $\phi(s, v)$ are defined respectively by

$$I = Eds^2 + 2Fdsdv + Gdv^2, \tag{3}$$

$$II = eds^2 + 2fdsdv + gdv^2, \tag{4}$$

where

$$E = \|\phi_s\|^2, F = \langle \phi_s, \phi_v \rangle\ G = \|\phi_v\|^2, e = \langle \phi_{ss}, m \rangle, f = e = \langle \phi_{sv}, m \rangle, e = \langle \phi_{vv}, m \rangle. \tag{5}$$

The Gaussian curvature K and the mean curvature H at a regular point $\phi = (s, v)$ are defined with the components (5) of the first and the second fundamental forms (3) and (4), respectively by:

$$K = \frac{eg - f^2}{EG - F^2} = -\frac{f^2}{EG - F^2}, \tag{6}$$

$$H = \frac{Eg + Ge - 2Ff}{2(EG - F^2)} = \frac{e - 2Ff}{2(EG - F^2)}. \tag{7}$$

Then, from (6) and (7) we anounce the following definition:

Definition: A regular surface is developable (resp, minimal) if and only if its Gaussian curvature (resp. mean curvature) vanishes identically.

If v decreases infnitely along a ruling $s = s_1$, the unit normal m defined with (2) approaches a limiting direction. This direction is called asymptotic normal (central tangent) direction denoted by $a(s_1)$ and defined with (8) as follows:

$$a(s) = \lim_{v \to -\infty} m(s_1, v) = \frac{q(s_1) \wedge q'(s_1)}{\|q'(s_1)\|}. \tag{8}$$

The point at which the unit normal of the ruled surface ϕ is perpendicular to the asymptotic normal (8) a is called the central point M and the set of central points of all rulings is called striction curve of ruled surface. The parametrization of the striction curve on the ruled surface ϕ is denoted by $\beta = \beta(s)$ and given by

$$\beta(s) = c(s) - \frac{\langle c'(s), q'(s)\rangle}{\|q'(s)\|^2} q(s). \tag{9}$$

The vector h defined by $h(s) = a(s) \wedge q(s)$ is called central normal vector. Thereafter, the orthonormal system $\{\beta(s); q(s), h(s), a(s)\}$ is called Frenet frame of the ruled surface ϕ where $M = \beta(s)$ is the central point of the striction curve (9) and $q(s), h(s)$ and $a(s)$ are unit vector of ruling, central normal and central tangent, respectively. Frenet formulae of the ruled surface ϕ are given by

$$= k_1 h, \ \ h' = -k_1 q + k_2 a, \ \ a' = -k_2 h, \tag{10}$$

where $\boldsymbol{k}_1 = \frac{ds_1}{ds}, \boldsymbol{k}_2 = \frac{ds_3}{ds}$ and $\boldsymbol{s}_1, \boldsymbol{s}_3$ are the arcs of the spherical curves $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ circumscribed by the bound vectors $\boldsymbol{q}$ and $\boldsymbol{a}$.

3. Some Characteristic Properties of Ruled Surface with Frenet Frame of an Arbitrary Non-cylindrical Ruled Surface in Euclidean 3-Space

In this section, we define a new special form of ruled surface with striction curve and Frenet frame of a regular arbitrary ruled surface in Euclidean 3-space. It concerned ruled surface which is generated by striction curve of an arbitrary ruled surface and whose rulings are linear combinations with constant components. We presente different outcomes properties of the constructed ruled surface and study I by investigating its Gaussian curvature, mean curvature and striction curve, then we give characterizations relatively to these properties. Moreover, we presente examples with some figures of illustrations in three cases.

Let be $\phi = \phi(s, v)$ a non-cylindrical regular ruled surface of class $C^k (k \geq 3)$ which is defined by the parametric representation:

$$\phi: (s, v) \in I \times \mathbb{R} \mapsto \beta(s) + v q(s), \quad \|q(s)\| = 1, \tag{11}$$

where β is a normal parametrization of its striction curve which is supposed regular.

Considering the ruled surface Ψ with the curve $\beta(s)$ and Frenet frame vectors of the ruled surface ϕ defined with (11):

$$\Psi:(\mathrm{s},\mathrm{v})\in \mathrm{I}\times\mathbb{R}\mapsto \beta(s)+v\big(x_1q(s)+x_2h(s)+x_2a(s)\big), \tag{12}$$

where x_1, x_2 and x_3 are three constants satisfying $x_1^2+x_2^2+x_3^2=1$.

β is the striction curve of ϕ, it verifies $<\beta', q'>=0$, which implies $<\beta', h>=0$, that means β' can be written as follows:

$$\beta^{'}(s)=\alpha_1(s)q(s)+\alpha_2(s)q(s),\ \forall s\epsilon I \tag{13}$$

where $\alpha_1^2+\alpha_2^2=1$.

Differentiating Ψ defined in (12) with respect to s and v respectively and using (10) and (13), we get:

$$\frac{\partial\Psi}{\partial s}=(\alpha_1-vx_2k_1)q+v(x_1k_1-x_3k_2)h+(\alpha_2+vx_2k_2)a, \tag{14}$$

$$\frac{\partial\Psi}{\partial v}=x_1q+x_2h+x_3a. \tag{15}$$

Then, from (14) and (15) we get that the unit normal vector $n(s,v)$ on ruled surface Ψ at a regular point $\Psi(s,v)$ is

$$n=\frac{\Psi_s\wedge\Psi_v}{\|\Psi_s\wedge\Psi_v\|}=\frac{(-x_2\alpha_2+vA)q+(x_1\alpha_2-x_3\alpha_1+vB)h+(x_2\alpha_1-vC)a}{\sqrt{(-x_2\alpha_2+vA)^2+(x_1\alpha_2-x_3\alpha_1+vB)^2+(x_2\alpha_1-vC)^2}}, \tag{16}$$

where

$$A=x_1x_3k_1-(x_2^2+x_3^2)k_2, B=x_2(x_1k_2+x_3k_1), C=(x_1^2+x_2^2)k_1-x_1x_3k_2. \tag{17}$$

The components E, F et G of the first fundamental form of the ruled surface Ψ at a regular point Ψ (s,v) are obtained form (14), (15) and (16) as follows:

$$E=1+2vx_2(\alpha_2k_2-\alpha_1k_1)+v^2[x_2^2(k_1^2+k_2^2)+(x_1k_1-x_3k_2)^2], \tag{18}$$

$$F=x_1\alpha_1+x_3\alpha_2, G=1 \tag{19}$$

Differentiating Ψ_s and Ψ_v with respect to s and v, respectively and using (10) and (13) we get

$$\Psi_{ss}=\frac{\partial^2\Psi}{\partial s^2}=[\alpha_1'-v\{x_2k_1'+k_1(x_1k_1-x_3k_2)\}]q+[k_1\alpha_1-k_2\alpha_2+v\{x_1k_1'-x_3k_2'-x_2(k_1^2+k_2^2)\}]h+ [\alpha_2'+vx_2k_2'+vk_2(x_1k_1-x_3k_2)]a, \tag{20}$$

$$\Psi_{sv}=\frac{\partial^2\Psi}{\partial s\partial v}=-x_2k_1q+(x_1k_1-x_3k_2)h+x_2k_2a,\ \Psi_{vv}=\frac{\partial^2\Psi}{\partial v^2}=0. \tag{21}$$

The components e, f and g of the second fundamental form of the ruled surface Ψ at a regular point of its base curve $\beta=\beta(s)$ are abtained form (16), (20) and (21) as follows:

$$e(s,0)=\frac{x_2(\alpha_1\alpha_2'-\alpha_1'\alpha_2)+(k_1\alpha_1-k_2\alpha_2)(x_1\alpha_2-x_3\alpha_1)}{\sqrt{x_2^2+(x_1\alpha_2-x_3\alpha_1)^2}}, \tag{22}$$

$$f(s,0)=\frac{x_2^2(\alpha_2k_1+\alpha_1k_2)+(x_1k_1-x_3k_2)(x_1\alpha_2-x_3\alpha_1)}{\sqrt{x_2^2+(x_1\alpha_2-x_3\alpha_1)^2}}. \tag{23}$$

$$g(s,0)=0. \tag{24}$$

Then, from the components (18) and (19) of the first fundamental forms and those (22), (23) and (24) of the second fundamental form we get the Gaussian and the mean curvatures K and H of the ruled surface Ψ along $\beta=\beta(s)$ using (6) and (7):

$$K(s,0)=-\left(\frac{x_2^2(\alpha_2k_1+\alpha_1k_2)+(x_1k_1-x_3k_2)(x_1\alpha_2-x_3\alpha_1)}{x_2^2+(x_1\alpha_2-x_3\alpha_1)^2}\right)^2, \tag{25}$$

$$H(s,0)=\frac{x_2(\alpha_1\alpha_2'-\alpha_1'\alpha_2)+(k_1\alpha_1-k_2\alpha_2)(x_1\alpha_2-x_3\alpha_1)-2(x_1\alpha_1+x_3\alpha_2)\left[x_2^2(\alpha_2k_1+\alpha_1k_2)+(x_1k_1-x_3k_2)(x_1\alpha_2-x_3\alpha_1)\right]}{2\left(x_2^2+(x_1\alpha_2-x_3\alpha_1)^2\right)^{3/2}}. \tag{26}$$

Another hand, if Ψ is non-cylindrical ruled surface, i.e., $x_2^2(k_1^2+k_2^2)+(x_1k_1-x_3k_2)^2\neq 0$, then, its striction curve denoted by $\mu=\mu(s)$ is given as follows:

$$\mu=\beta-\frac{x_2(-\alpha_1k_1+\alpha_2k_2)}{x_2^2(k_1^2+k_2^2)+(x_1k_1-x_3k_2)^2}(x_1q+x_2h+x_3a). \tag{27}$$

Then from (25), (26) and (27) we get the characterizations of the studied ruled surface in the following corollaries:

Corollary: The ruled surface Ψ is developable along its base curve if and only if k_1 and k_2 satisfy the equation $x_2^2(\alpha_2k_1+\alpha_1k_2)+(x_1k_1-x_3k_2)(x_1\alpha_2-x_3\alpha_1)=0$.

Corollary: The ruled surface Ψ is minimal along its base curve if and only if k_1 and k_2 satisfy the equation

$$\begin{aligned}x_2(\alpha_1\alpha_2'-\alpha_1'\alpha_2)+(k_1\alpha_1-k_2\alpha_2)(x_1\alpha_2-x_3\alpha_1)\\ -2(x_1\alpha_1+x_3\alpha_2)[x_2^2(\alpha_2k_1+\alpha_1k_2)+(x_1k_1-x_3k_2)(x_1\alpha_2-x_3\alpha_1)]=0.\end{aligned}$$

Corollary: If x_2 do not vanishes, then β is the striction curve of ruled surface Ψ if and only if $-\alpha_1k_1+\alpha_2k_2=0$.

3.1. Examples

Considering the regular and non-cylindrical ruled surface defined in the real Euclidean 3-space R^3 by

$$\phi(\mathrm{s},\mathrm{v})=(\cos\mathrm{s},\sin\mathrm{s},0)+\frac{\mathrm{v}}{\sqrt{2}}(\sin\mathrm{s},-\cos\mathrm{s},1), \tag{28}$$

where the unit circle $\beta(s)=(\cos\mathrm{s},\sin\mathrm{s},0)$ is its striction curve.

The three Frenet frame vector of ϕ defined with (28) and the associated curvatures and constant functions are given, reespectively as follows:

$$q(s)=\frac{1}{\sqrt{2}}(\sin\mathrm{s},-\cos\mathrm{s},1),h(s)=(\cos\mathrm{s},\sin\mathrm{s},0),a(s)=\frac{1}{\sqrt{2}}(-\sin\mathrm{s},-\cos\mathrm{s},1), \tag{29}$$

where

$$k_1=k_2=\frac{1}{\sqrt{2}},\alpha_1=-\frac{1}{\sqrt{2}},\alpha_2=\frac{1}{\sqrt{2}}. \tag{30}$$

In this case, the new ruled surface studied which is defined with (29) and (30) and the base curve of ϕ

takes the following form

$$\Psi(s,v) = (\cos s, \sin s, 0) + v\left(\frac{x_1 - x_3}{\sqrt{2}}\sin s + x_2 \cos s, -\frac{x_1 - x_3}{\sqrt{2}}\cos s + x_2 \sin s, \frac{x_1 - x_3}{\sqrt{2}}\right), \tag{31}$$

where $x_1^2 + x_2^2 + x_3^2 = 1$.

Its Gaussian curvature K and the mean curvature H of the ruled surface Ψ are given respectively as follows:

$$K(s,0) = -\left(\frac{(x_1 - x_3)(x_1 + x_3)}{2x_2^2 + (x_1 + x_3)^2}\right)^2, \tag{32}$$

$$H(s,0) = \frac{(x_1 + x_3)((x_1 - x_3)^2 - 1)}{(2x_2^2 + (x_1 + x_3)^2)^{\frac{3}{2}}}. \tag{33}$$

Another hand, if Ψ is non-cylindrical, then its striction curve μ is

$$\mu = (\cos s, \sin s, 0) + \frac{2x_2}{2x_2^2 + (x_1 + x_3)^2}\left(\frac{x_1 - x_3}{\sqrt{2}}\sin s + x_2 \cos s, -\frac{x_1 - x_3}{\sqrt{2}}\cos s + x_2 \sin s, \frac{x_1 - x_3}{\sqrt{2}}\right). \tag{34}$$

Hence, from (32), (33) and (34) we get the following characterizations of ruled surface Ψ defined with (31):

Corollary: Ψ is developable along its base curve $\beta(s)$ if and only if $(x_1 - x_3)(x_1 + x_3) = 0$.

Corollary: Ψ is minimal along its base curve $\beta(s)$ if and only if $(x_1 + x_3)((x_1 - x_3)^2 - 1) = 0$.

Corollary: The unit circle is the striction curve of the ruled surface

$$(s,v) \longmapsto (\cos s, \sin s, 0) + v\left(\frac{x_1 - x_3}{\sqrt{2}}\sin s, -\frac{x_1 - x_3}{\sqrt{2}}\cos s, \frac{x_1 - x_3}{\sqrt{2}}\right).$$

In the following, some illustrations of special cases of the last ruled surface Ψ which is defined by the unit circle as base curve and whose ruling are linear combinaison of Frenet frame vectors of ϕ.

The Fig. 1 corresponds to the case of the ruled surface Ψ where $[x_1 = 1, x_2 = x_3 = 0]$, which is defined by

$$(s,v) \in]-2\pi, 2\pi[\times [-1,1] \mapsto (\cos s, \sin s, 0) + \frac{v}{\sqrt{2}}(\sin s, -\cos s, 1),$$

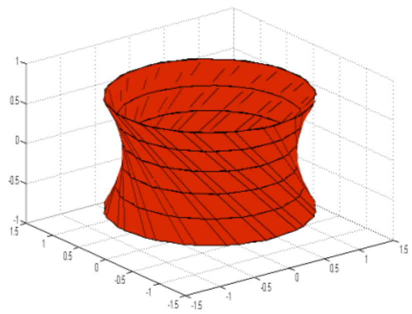

Fig. 1. Ruled surface Ψ with $[x_1 = 1, x_2 = x_3 = 0]$。

The Fig. 2 corresponds to the ruled surface Ψ in the case where $[x_2 = 1, x_1 = x_3 = 0]$, which is defined by

$$(\mathrm{s},\mathrm{v}) \in \,]-2\pi, 2\pi[\, \times [-4,4] \mapsto (\cos \mathrm{s}, \sin \mathrm{s}, 0) + v(\cos \mathrm{s}, \sin \mathrm{s}, 0)$$

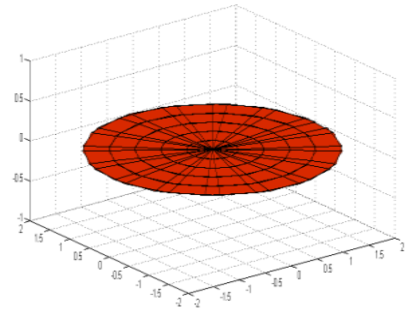

Fig. 2. Ruled surface Ψ with $[x_2 = 1, x_1 = x_3 = 0]$.

The third case in this example is shown in Fig. 3 for ruled surface Ψ where $\left[x_1 = x_2 = x_3 = \frac{1}{\sqrt{3}}\right]$, and which is defined by

$$(\mathrm{s},\mathrm{v}) \in \,]-2\pi, 2\pi[\, \times [-4,4] \mapsto (\cos \mathrm{s}, \sin \mathrm{s}, 0) + \frac{1}{\sqrt{3}}(\cos \mathrm{s}, \sin \mathrm{s}, 0)$$

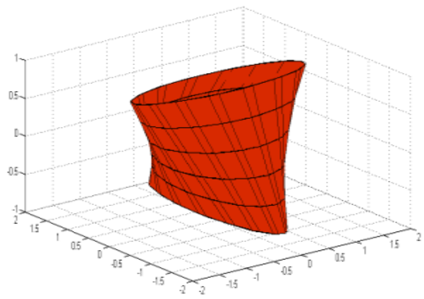

Fig. 3. Ruled surface Ψ with $\left[x_1 = x_2 = x_3 = \frac{1}{\sqrt{3}}\right]$.

4. Conclusion

We presented a study of a special form of ruled surface which we constructed with a given non-cylindrical and regular ruled surface. The essential point of interest was Frenet frame of ruled surface as a special frame relatively to surface, which is defined precisely along striction curve and whose vectors are investigated with a special manner. The special king of the fabricated ruled surface with the first given surface allowed us to investigate its properties and characterize it depending on the first surface. Thus, we were able to show illustrations of our studied surface for an example.

Conflict of Interest

The authors declare that there are any conflicts of interest.

Author Contributions

The two authors of the present paper have contributed to this work.

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Soukaina could until now publish an original paper: OUARAB, S., OUAZZANI, A., IZID, M. (2018). Ruled Surfaces with Alternative Moving Frame in Euclidean 3-Space, International J. of Math. Sci. & Engg. Appls. (IJMSEA). Another one is submitted from Januray 2017 and two others papers are in preparation for a publication project. Its current and previous interests focus on study of ruled surfaces as a special kind of surfaces which have many important characteristics.

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