Hall Effect on Oscillatory Hydromagnetic Free Convective Flow of a Non-Newtonian Fluid past an Infinite Vertical Porous Flat Plate with Heat Sources / Sink and Constant Suction

P. K. Mishra, S. Biswal, G. S. Ray, and N. Dash

***Abstract*—This paper deals with the hall effect on oscillatory hydromagnetic free convective flow of non-Newtonian fluid past an infinite vertical porous flat plate with heat sources/sinks. In the formulation part, equations of continuity, motion and energy have been developed. These equations have been made dimensionless with the introduction of non-dimensional parameters concerned and solved applying complex variable technique under the boundary conditions employed. Profiles of primary velocity, secondary velocity and temperature have been plotted after computerization with the numerical values of the non-dimensional parameters. The values of skin frictions and the rates of heat transfer have been entered in the tables. It is observed that both the primary and secondary velocity increase with the increase of the non-Newtonian parameters (R_c) as well as the reciprocal of the porosity parameters ($1/K^*$). The temperature of the fluid falls with the Prandtl number (P_r).**

***Index Terms*—Hall effect, visco-elastic fluid, porous plate, heat transfer**

I. Introduction

The effects of Hall current on the free convection flow of a viscous fluid have been studied by a number of researchers. The literature is replete with copious instances of such investigations. A few of them are the works of Pop and Angew[1], Hossain and Mohammad[2], Datta and Jana[3], Mishra and Mishra[4], Mishra and Mohapatra[5], Datta and Mazumdar[6], Dwivedi and Dube[7] and Mohapatra and Tripathy[8]. The effect of Hall current is also significant in case of MHD flow of elastic fluid which is revealed from the results of the following investigations. Biswal and Pattnaik[9] have analysed the Hall effect on oscillatory hydromagnetic free convective flow of visco-elastic fluid past an infinite vertical porous flat plate. Biswal and Sahoo[10] have investigated the problem of Hall effect on oscillatory hydromagnetic free convective flow of a visco-elastic fluid past an infinite vertical porous flat plate with mass transfer. However not much studies have been reported on the impact of heat sources / sink and constant suction upon free convection flow of a non-Newtonian fluid past an infinite vertical porous flat plate. Keeping in view the application of such problems in the field of metallurgy in general and polymer technology in particular, we have briefly reported here our study on Hall effect on oscillatory hydromagnetic free convection flow of non-Newtonian fluid past an infinite vertical porous plate with heat sources / sinks along with uniform suction at the plate.

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II. Mathematical Formulation

Under the physical situations taken up, the X-axis is chosen along the vertical plate, Y-axis normal to the plate and Z-axis lying on the plate, perpendicular to both X – and Y – axes. The equation of continuity takes the form

$$\frac{\partial V}{\partial y} = 0\,, \tag{1}$$

$$\Rightarrow \text{V = Constant = - } V_0 \text{ (say)}, V_0 > 0;$$

where, ($-V_0$) is the suction velocity of the fluid and that remains constant throughout the motion of the fluid.

The equations of motion and energy governing the flow under the usual Boussinesq approximation are

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = \eta_0 \frac{\partial^2 u}{\partial y^2} - K_0 \frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma B_0^2}{\rho}\frac{(u+mw)}{(1+m^2)} + g\beta(T-T_\infty) - \frac{\nu}{K'}u\,, \tag{2}$$

$$\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial y} = \eta_0 \frac{\partial^2 w}{\partial y^2} - K_0 \frac{\partial^2 w}{\partial y^2 \partial t} + \frac{\sigma B_0^2}{\rho} - \frac{\nu}{K'}w, \tag{3}$$

and

$$\frac{\partial (T-T_\infty)}{\partial t} + V\frac{\partial (T-T_\infty)}{\partial y} = \frac{K}{\rho C_P}\frac{\partial (T-T_\infty)}{\partial y^2} + S'(T-T_\infty), \tag{4}$$

Subjected to the boundary conditions

$$t \le 0\,,\ u(y,t) = W(y,t) = 0\,,\ \theta = 0 \text{ for all } y$$

$$t>0\,, \begin{cases} u(0,t) = W(0,t) = 0\ ,\ \theta = a e^{i\omega t}\ , \text{ as } y = 0 \\ u(\infty,t) = W(\infty,t) = 0\ ,\ \theta(\infty,t) = 0\ , \text{ as } y \to \infty \end{cases} \tag{5}$$

Eqns. (2) – (4) have been transformed to their corresponding non-dimensional forms as

$$\frac{\partial u}{\partial t}-4\frac{\partial u}{\partial \eta}=4\frac{\partial^2 u}{\partial \eta^2}-Rc\frac{\partial^3 u}{\partial \eta^2\,\partial t}-\frac{M(mw+u)}{1+m^2}+G\theta-\frac{u}{K^*}, \quad (6)$$

$$\frac{\partial w}{\partial t}-4\frac{\partial w}{\partial \eta}=4\frac{\partial^2 w}{\partial \eta^2}-Rc\frac{\partial^3 w}{\partial \eta^2\,\partial t}+\frac{M(mu-w)}{1+m^2}-\frac{w}{K^*} \quad (7)$$

and

$$\frac{\partial \theta}{\partial t}-4\frac{\partial \theta}{\partial \eta}=\frac{4}{Pr}\frac{\partial^2 \theta}{\partial \eta^2}+S\theta\,, \quad (8)$$

The modified boundary conditions become

$$t\le 0 : u(\eta,t)=w(\eta,t)=0\ ,\ \theta=0\ ,\ \text{for all}\ \eta$$

$$t>0:\begin{cases} u(0,t)=w(0,t)=0\ ,\ \theta(0,t)=e^{i\omega t}\ ,\ \text{at}\ \eta=0 \\ u(\infty,t)=w(\infty,t)=0, \theta(\infty,t)=0 \quad ,\ \text{as}\ \eta\to\infty \end{cases} \quad (9)$$

III. Solutions of the Equations

The equations (6) and (7) are combined using the complex variable

$$\psi = u+iw\,, \quad (10)$$

giving

$$\frac{\partial^2\psi}{\partial\eta^2}-\frac{Rc}{4}\frac{\partial\psi}{\partial\eta^2\partial t}-\frac{1}{4}\frac{\partial\psi}{\partial\eta}+\frac{\partial\psi}{\partial\eta}-\frac{M}{4}\frac{(1-im)}{(1+m^2)}-\frac{1}{K^*}\psi=-\frac{1}{4}G\theta \quad (11)$$

Solving equation (8) and (11), we obtain

$$\psi(\eta)=P_8\,e^{i\Omega t}(e^{-P_2\eta}-e^{-P_7\eta})$$
$$=u(\eta)+iW(\eta)\,, \quad (12)$$

where $u(\eta)=P_8\ \cos\ \Omega t\left(e^{-P_2\eta}-e^{-P_7\eta}\right)$

(13)

$$w(\eta)=P_8\ \sin\ \Omega t\left(e^{-P_2\eta}-e^{-P_7\eta}\right)$$

(14)

and $\theta(\eta,t)=e^{\left[i\Omega t-\frac{\eta}{2}\left(P_r+\sqrt{P_r^2+(S-i\Omega)P_r}\right)\right]}$.

(15)

The constants involved here are omitted in order to save space.

IV. Results and Discussion

The effects of various fluid parameters like non-Newtonian parameters (R_c), Hartmann number (M), Hall parameters (m), Prandtel number (P_r), Grashof number (G), Source / sink parameters (S) and permeability parameter (K^*) on the oscillatory hydromagnetic free convective non-Newtonian flow past an infinite vertical porous flat plate with heat sources / sinks and constant suction have been studied through graphs and tables. Velocity and temperature profiles have been shown by graphs.

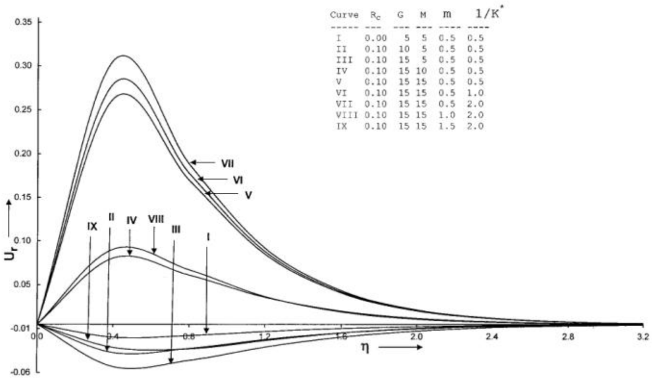

Fig. 1. Profiles of mean primary velocity U_r for P_r = 2.0, S=0.05, V_0 = 0.1, Ω=1.0, t = 15

Fig. 1 shows the profiles of mean primary velocity (u_r) for the different values of R_c, G, M, m and $(K^*)^{-1}$. It is observed that the mean primary velocity decreases with the elastic parameter R_c and increases with the values of $(K^*)^{-1}$.

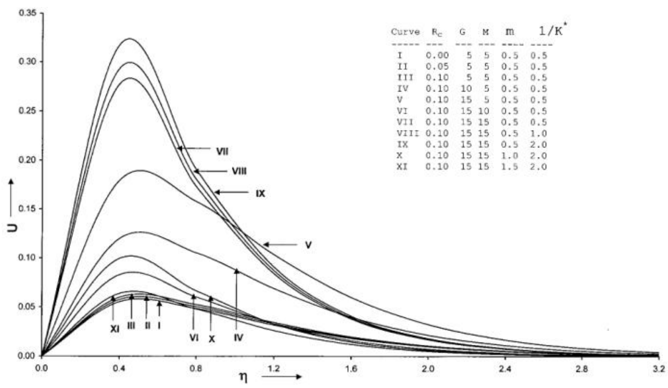

Fig. 2. Profiles of mean primary velocity U for Pr = 2.0, S=0.05, $V0$ = 0.1, Ω=1.0, t = 15

Profiles of primary velocity u are sown in Figure 2 for various values of fluid parameters. It is seen that the primary velocity u rises with R_c and falls with the Hall parameters.

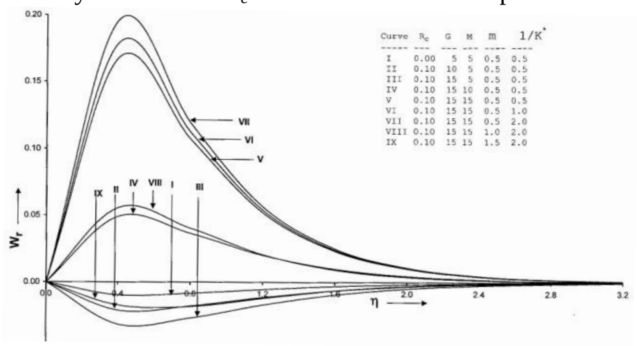

Fig. 3. Profiles of mean secondary velocity WR for Pr = 2.0, S=0.05, $V0$ = 0.1, Ω=1.0, t = 15

Fig. 3 illustrates the nature of the mean secondary velocity w_r for different values of the fluid parameters mentioned earlier. It is noticed that w_r decreases with Hall parameter (m)

and assumes negative values for m=1.5.

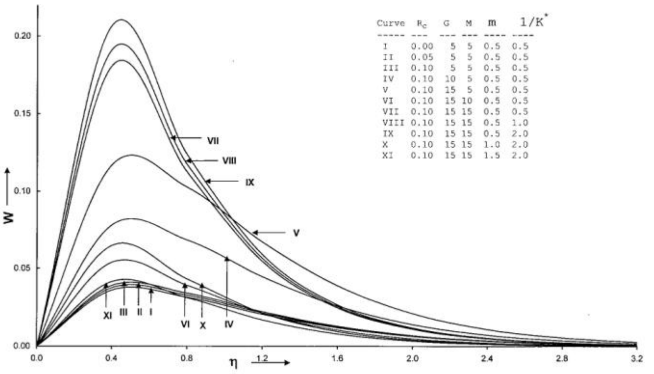

Fig. 4. Profiles of secondary velocity W for Pr = 2.0, S=0.05, $V0$ = 0.1, Ω=1.0, t = 15

Profiles of secondary velocity w are drawn in Figure 4 for different values of the fluid parameters. It is gleaned from these profiles that w increases with the non-Newtonian parameter (R_c) and similar effect is marked in case of $(K^*)^{-1}$. However, increase in the Grashof number produces rise in the secondary velocity W.

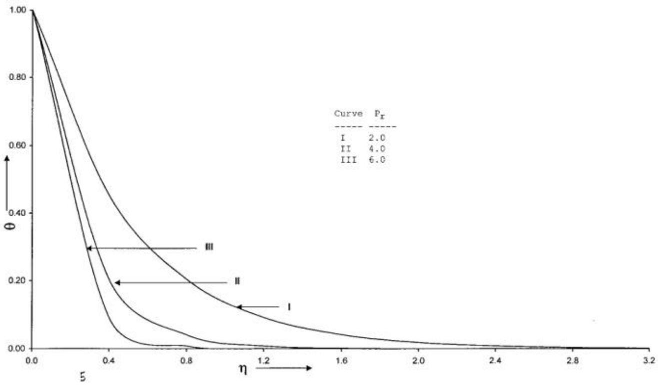

Fig. 5. Temperature profiles for S=0.05, $V0$ = 0.1, Ω=1.0, t = 15

Fig. 5 exhibits the behaviour of the temperature as noticed from the profiles drawn. It is observed that the temperature falls with the rise of Prandtl number P_r.

V. Conclusions

Findings of the present theoretical study of Hall effct on oscillatory hydromagnetic free convective flow of a non-Newtonian fluid past an infinite vertical porous flat plate with heat sources/sinks and constant suction are the following.

1) The reciprocal of the porosity parameter increases the mean primary velocity.
2) The primary velocity (u) rises with the elastic parameter (R_c).
3) Mean secondary velocity decreases with Hall parameter (m).
4) Increase in Grashof number produces rise in the secondary velocity (w).
5) Fluid temperature falls with the rise of Prandtl number (P_r).

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