Specially Structured Three Stage Flowshop Scheduling to Minimize the Rental Cost

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Abstract—**This paper pertain to a specially structured n-jobs, 3- machines flow shop scheduling in which the processing times are associated with probabilities to minimize the rental cost of the machines. Further the expected processing times are not random but bear a well defined relationship to one another. Many heuristics approaches have already been discussed in literature to minimize the makespan. But, in many cases minimization of makespan does not lead to minimum rental cost of the machines under a specified rental policy. The present work is an attempt to develop a heuristic algorithm to minimize the rental cost of the machines under a specified rental policy and will provide an important tool for the decision makers.**

Index Terms—**Specially structured flow shop scheduling; processing time; rental cost; idle time;makesspan; utiliztaion time.**

I. Introduction

Scheduling problems have been extensively investigated by many researchers with an aim to determine the sequence for processing jobs on a given set of machines. Its importance and relevance to industry has promoted researchers to study it from different perspectives. Flowshop is the classical and most studied manufacturing environment in scheduling literature. Makespan and total flow time are commonly used performance measure in flowshop scheduling literature. Only some efforts had been made to minimize the rental cost of the machines. Minimization of makespan does not always lead to minimum rental cost. Further, In most of literature the processing time of jobs is considered to be random. But there are significant situations in which the processing times are not random, but follow some well defined structural conditions. One of the earliest results in flowshop scheduling theory is an algorithm given by Johnson’s (1954) for scheduling jobs in a two, three stage machine flowshop to minimize the time at which all jobs are completed. Gupta, J.N.D (1975) gave an algorithm to find the optimal schedule for specially structured flowshop scheduling. The work was developed by Ignall & Scharge (1965), Bagga (1969), Szware (1977), Maggu & Das (1981), Singh, T.P.(2005), Gupta Deepak (2005) etc. by considering the various parameters. Narain (2005) studied a problem to obtain a sequence which gives minimum possible rental cost while minimize total elapsed time under pre-defined rental policy.

Gupta & Sharma *et al* (2012) studied specially structured two stage flow shop problem to minimize the rental cost of the machines under pre-defined rental policy in which the processing time have been associated with probabilities. The present paper is an attempt to develop a heuristic algorithm to minimize the rental cost for three stages specially structured flow shop scheduling under the specified rental policy.

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II. Practical Situation

Various practical situations occur in real life when one has got the assignments but does not have one’sown machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

III. Notations

S : Sequence of jobs 1, 2, 3,….,n

S_k : Sequence obtained by applying Johnson’s procedure, k = 1, 2, 3, -------

M_j : Machine j, j= 1, 2, 3

a_{ij} : Processing time of i^{th} job on machine M_j

p_{ij} : Probability associated to the processing time a_{ij}

A_{ij} : Expected processing time of i^{th} job on machine M_j

$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j

$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k

$U_j(S_k)$: Utilization time for which machine M_j is required for sequence S_k of jobs

$R(S_k)$: Total rental cost for the sequence S_k of all the machine

C_i : Renal cost of i^{th} machine.

$CT(S_k)$: Total completion time of the jobs for the sequence S_k.

IV. DEFINITION

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$t_{ij} = max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + a_{ij} \times p_{ij}$ for $j \geq 2$.

$= max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + A_{i,j}$, where $A_{i,j}$= Expected processing time of i^{th} job on j^{th} machine.

V. RENTAL POLICY

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine and the 3rd machine is taken on rent when the 2nd job is completed on 2nd machine.

VI. PROBLEM FORMULATION

Let some job i (i = 1,2,……..,n) is to be processed on three machines M_j (j = 1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij}. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine such that either $A_{j2} \leq A_{i1}$ or $A_{j2} \leq A_{i3}$ for all values of i, j. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines.

TABLE I: THE MATHEMATICAL MODEL OF THE PROBLEM IN MATRIX FORM

Jobs	Machine M_1		Machine M_2		Machine M_3	
I	a_{i1}	p_{i1}	a_{i2}	p_{i2}	a_{i3}	p_{i3}
1	a_{11}	p_{11}	a_{12}	p_{12}	a_{13}	p_{13}
2	a_{21}	p_{21}	a_{22}	p_{22}	a_{23}	p_{23}
3	a_{31}	p_{31}	a_{32}	p_{32}	a_{33}	p_{33}
-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	a_{n2}	p_{n2}	a_{n3}	p_{n3}

Mathematically, the problem is stated as: Minimize

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

VII. ALGORITHM

Step 1: Calculate the expected processing times

$A_{ij} = a_{ij} \times p_{ij} \forall i, j.$

Step 2: Check the condition: either $A_{j2} \leq A_{i1}$ or $A_{i2} \leq A_{j3} \forall i, j.$

.i.e. either Max $\{A_{j2}\}$ $\leq$ Min $\{A_{i1}\}$ or Max $\{A_{i2}\} \leq$ Min $\{A_{j3}\}$ $\forall i, j.$

If the conditions are satisfied then go to Step 3, else the data is not in the standard form.

Step 3: Introduce the two fictitious machines G and H with processing times G_i and H_i as

$$G_i = A_{i1} + A_{i2}\, , H_i = A_{i2} + A_{i3}\ \forall i.$$

Step 4: Obtain the sequence S_1(say) by applying Johnson (1954) algorithm on machines G & H.

Step 5: Obtain other sequences by putting 2nd, 3rd… nth jobs of the sequence S_1 in the 1st position and all other jobs of S_1 in the same order. Let these sequences be S_2, S_3, S_4, --------, S_{n-1}.

Step 6: Compute $\sum_{i=1}^{n} A_{i1}$, $U_2(S_k)$, $U_3(S_k)$ and

$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$ for all the possible sequences S_k (k=1, 2, 3,……, n).

Step 7: Find $\min\{R(s_k)\}; k = 1,2,3,.....,n$. Let it be minimum for the sequence Sp, and then the sequence Sp will be the optimal sequence with rental cost R(Sp).

VIII. NUMERICAL ILLUSTRATION

Consider 5 jobs, 3 machine flow shop problem with processing time associated with their respective probabilities as given in table 2. The rental cost per unit time for machines M_1, M_2 and M_3 are 4 units, 6 units and 8 units respectively, under the rental policy P. Our objective is to obtain an optimal sequence of the jobs to minimize the rental cost of the machines.

TABLE II: THE PROCESSING TIMES WITH PROBABILITIES

Jobs	Machine M_1		Machine M2		Machine M_3	
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}	a_{i3}	p_{i3}
1	50	0.1	15	0.2	25	0.2
2	25	0.2	10	0.4	40	0.1
3	20	0.3	20	0.1	25	0.2
4	24	0.2	10	0.2	20	0.3
5	20	0.2	30	0.1	20	0.2

Solution: The expected processing times A_{i1}, A_{i2} and A_{i3} for machines M_1, M_2 and M_3 are

TABLE III: EXPECTED PROCESSING TIMES OF THE MACHINES

Jobs	A_{i1}	A_{i2}	A_{i3}
1	5	3	5

2	5	4	4
3	6	2	4
4	4.8	2	6
5	4	3	4

Here, we observe that $A_{i2} \le A_{j3} \;\; \forall i, j.$

The two fictious machines with processing times G_i and H_i are

TABLE IV: THE PROCESSING TIME FOR TWO FICTIOUS MACHINES

Jobs	1	2	3	4	5
G_i	8	9	8	6.8	7
H_i	8	8	7	8	7

Using Johnson (1954) procedure, the sequence with minimum makespan is S_1: 4 – 1 – 3 – 5 – 2.

Other feasible sequences which may correspond to minimum rental cost are S_2: 1 – 4 – 3 – 5 – 2, S_3: 3 – 4 – 1 – 5 – 2, S_4: 5 – 4 – 1– 3 – 2, S_5: 2 – 4 – 1 – 3 – 5.

The In-Out table flow time table for the sequence S_1: 4 – 1 – 3 – 5 – 2 is

TABLE V: THE IN-OUT FLOW TABLE FOR S_1

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In –Out	In – Out	In – Out
4	0.0 – 4.8	4.8 – 6.8	6.8 – 12.8
1	4.8 – 9.8	9.8 – 12.8	12.8 – 17.8
3	9.8 – 15.8	15.8 – 17.8	17.8 – 21.8
5	15.8 – 19.8	19.8 – 22.8	22.8 – 26.8
2	19.8 – 24.8	24.8 – 28.8	28.8 – 32.8

Therefore, the completion time for S_1 = $CT(S_1)$ = 32.8 units

Utilization time of machine M_2 = $U_2(S_1)$ = 24.0 units

Utilization time of machine M_3 = $U_3(S_1)$ = 26.0 units

Hence, the Rental cost of machines for sequence of jobs S_1= 454.4 units.

Similarly, we can have for the sequences

S_2: $CT(S_2)$ = 32.8, $U_2(S_2)$ = 23.8, $U_3(S_2)$ = 24.8, $R(S_2)$ = 443.6 units.

S_3: $CT(S_3)$ = 34.0, $U_2(S_3)$ = 22.8, $U_3(S_3)$ = 26.0, $R(S_3)$ = 447.2 units.

S_4: $CT(S_4)$ = 32.8, $U_2(S_4)$ = 24.8, $U_3(S_4)$ = 25.8, $R(S_4)$ = 457.6 units.

S_5: $CT(S_5)$ = 33.0, $U_2(S_5)$ = 22.8, $U_3(S_4)$ = 24.0, $R(S_5)$ = 431.2 units.

Therefore, Min $\{R(S_k)\}$ = 431.2 units and is for the sequence of jobs S_5.

Hence the sequence S_5: 2 – 4 – 1 – 3 – 5 is the optimal sequence with minimum rental cost 431.2 units although the makespan for S_5 is not minimum.

IX. CONCLUSION

The algorithm proposed in this paper for specially structured three stage flow shop scheduling problem to minimize the rental cost of the machines gives an optimal sequence with minimum total rental cost of machines irrespective of total elapsed time. The algorithm proposed by Johnson (1954) to find an optimal sequence to minimize the makespan/ Total elapsed time is not always corresponds to minimum total rental cost of the machines .Hence proposed algorithm is more efficient to minimize the rental cost of the machines under a specified rental policy(P) as compared with the algorithm proposed by Johnson (1954).

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