On Linkage of Parallel Biserial Servers Linked with a Common Server to a Three Stage Flowshop Scheduling Model

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***Abstract*—The present paper deals with the linkage between a queue network in which a common service server is linked in series with each of two parallel biserial servers and a three stage flowshop scheduling system. The objective of this paper is of two folds, on one hand it finds mean queue length and the total waiting time of jobs and on other hand it minimizes the total elapsed time. The proposed model provides an important tool for manufacturing concern, office management, banking service system, computer networks and in administrative setup etc.**

***Index Terms*—Queue network, flowshop, mean queue length, waiting time, processing time, biserial channels, makespan.**

I. Introduction

A lot of research work has already been done separately in the field of Queueing and Scheduling theory as per literature review. Only some efforts have been made so far to combine these two systems. Johnson (1954) gave a procedure for finding the optimal schedule to minimize the total elapsed time in two and three stage flowshop scheduling. Jackson (1954) studied the behavior of a queuing system containing phase type service. Maggu (1970) studied a network of two queues in biseries to find the total idle time of the jobs/customers which corresponds to a practical situation arises in the production concern. Singh, T.P. *et al.* (2005) studied the different aspects of the various queuing and scheduling models with various parameters. Singh, T.P. and K. Vinod (2009) studied the linkage of queuing system to a flow shop scheduling model.

Recently Gupta and Sharma (2012) made an attempt to link a parallel biserial queue network linked with a common server to a flow shop scheduling model. This paper is further an extension of the study made by Gupta and Sharma (2012) by establishing a linkage between a queuing network involving biserial servers linked with a common server to a flowshop system involving three machines. Therefore the present paper establishes a bridge between a network of queue model given by Singh, T.P. and Kumar Vinod (2009) and the three stage scheduling system given by Johnson (1954).

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II. Practical Situation

Various practical situations of this model arise in manufacturing concern, office management, banking service system, computer networks, administrative setup etc. For example, we can consider a network of such a system in a production concern in which three machines are linked serially with a network of queues comprised of three service servers, one of them is commonly linked in series with each of other two parallel servers in biseries. For example, we can consider two parallel biserial servers one for cutting and other for turning. Some jobs after cutting may go to turning and vice-versa. Both these servers are commonly connected to the server for chroming / polishing. After that the jobs has to passed thought three machine taken as finishing the jobs/goods , inspection of quality of goods produced and third machine for the final packing. After completing the service jobs/goods come out from the network.

III. Mathematical Model

Considered a queue network comprised of three service servers S_1, S_2 and S_3, where the servers S_1, S_2 are parallel biserial servers connected in series with a common server S_3 and which is further linked with a flowshop scheduling model consisting of three machines M_1, M_2 and M_3. The customers/jobs demanding service arrive in Poisson streams at the servers S_1 and S_2 with mean arrival rate λ_1, λ_2 and mean service rate μ_1 and μ_2 respectively. Let μ_3 be the mean service rate for the server S_3. Queues Q_1, Q_2 and Q_3 are said to be formed in front of the channels S_1, S_2 and S_3 respectively, if they are busy.

Customers/Jobs coming at the rate λ_1 either go to the network of servers $S_1 \to S_2 \to S_3$ or $S_1 \to S_3$ with probabilities p_{12} , p_{13} such that $p_{12} + p_{13} = 1$ and those coming at rate λ_2 either goes to the network of the servers $S_2 \to S_1 \to S_3$ or $S_2 \to S_3$ with probabilities p_{21} , p_{23} such that $p_{21} + p_{23} = 1$ .Further the completion time(waiting time + service time) of customers / jobs through Q_1, Q_2 & Q_3 form the setup times for machine M_1.After coming out from the server S_3 .i.e. through Phase I, customers / jobs go to the machines M_1, M_2 and M_3 (in Phase II) for processing with processing time A_{i1}, A_{i2} and A_{i3}. Our objective is to develop a heuristic algorithm to find an optimal sequence of the jobs / customers with minimum makespan in this Queue-Scheduling linkage system.

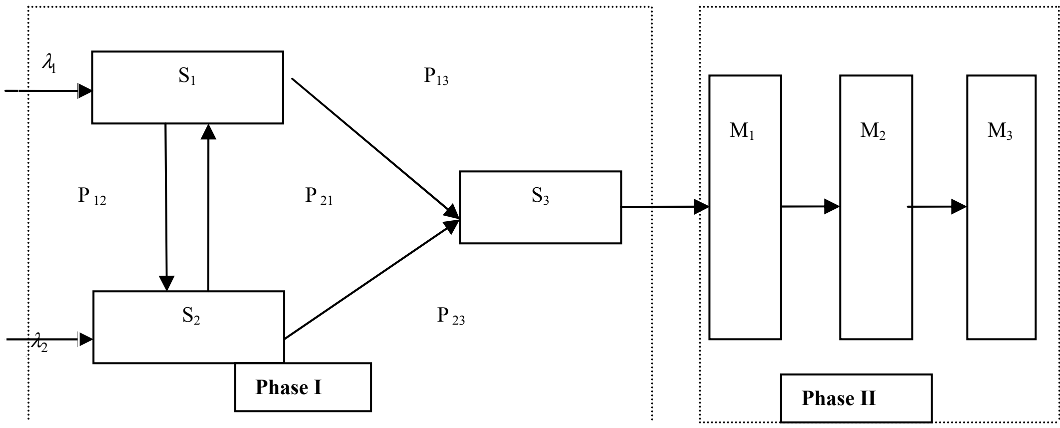

Fig. 1. Queuing network model.

IV. Assumptions

1. We assume that the arrival rate in the queue network follows Poisson distribution.
2. Each job / customer is processed on all the machines M_1, M_2 and M_3 in the same order and pre-emission is not allowed, .i.e. once a job is started on a machine, the process on that machine can not be stopped unless job is completed.
3. For the existence of the steady state behaviour the following conditions hold good:

(i) $\rho_1 = \frac{(\lambda_1 + \lambda_2 p_{21})}{\mu_1 (1 - p_{12} p_{21})} < 1$,

(ii) $\rho_2 = \frac{(\lambda_2 + \lambda_1 p_{12})}{\mu_2 (1 - p_{12} p_{21})} < 1$

(iii) $\rho_3 = \frac{p_{13}(\lambda_1 + \lambda_2 p_{21}) + p_{23}(\lambda_2 + \lambda_1 p_{12})}{\mu_2 (1 - p_{12} p_{21})} < 1$.

V. Algorithm

The following algorithm provides the procedure to determine the optimal sequence of the jobs to minimize the flow time of machines M_1 , M_2 and M_3 when the completion time (waiting time + service time) of the jobs coming out of Phase I is the setup times for the machine M_1.

Step 1: Find the mean queue length on the lines of Singh & Kumar (2005) using the formula

$$L = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3}$$

Here $\rho_1 = \frac{(\lambda_1 + \lambda_2 p_{21})}{\mu_1 (1 - p_{12} p_{21})}$,

$$\rho_2 = \frac{(\lambda_2 + \lambda_1 p_{12})}{\mu_2 (1 - p_{12} p_{21})}$$

$$\rho_3 = \frac{p_{13}(\lambda_1 + \lambda_2 p_{21}) + p_{23}(\lambda_2 + \lambda_1 p_{12})}{\mu_2 (1 - p_{12} p_{21})};$$

λ_i is the mean arrival rate, μ_i is the mean service rate, p_{ij} are the probabilities.

Step 2: Find the average waiting time of the customers on the line of Little's (1961) using relation $E(w) = \frac{L}{\lambda}$, where $\lambda = \lambda_1 + \lambda_2$.

Step 3: Find the completion time(C) of jobs / customers coming out of Phase I, .i.e. when processed thought the network of queues Q_1, Q_2 and Q_3 by using the formula $C = E(W) + \frac{1}{\mu_1 p_{12} + \mu_1 p_{13} + \mu_2 p_{21} + \mu_2 p_{23} + \mu_3}$.

Step 4: The completion time C of the customers / jobs through the network of queues Q_1, Q_2 and Q_3 will be the setup time for machine M_1. Define the three machines M_1, M_2 and M_3 with processing time $A'_{i1} = A_{i1} + C$, A_{i2} and A_{i3}.

Step 5: Check the condition: either Min $A'_{i1} \geq$ Max A_{i2} or Min $A_{i3} \geq$ Max A_{i2} or Both for all i.

If the conditions are satisfied then go to Step 6, else the data is not in the standard form.

Step 6: Introduce the two fictitious machines G and H with processing times G_i and H_i as

$$G_i = A'_{i1} + A_{i2} \text{ and } H_i = A_{i2} + A_{i3} \text{ for all } i.$$

Step 7: Apply Johnson's (1954) procedure to find the optimal sequence(s) with minimum elapsed time.

Step 8: Prepare In-Out tables for the optimal sequence(s) obtained in step 7. The sequence S_k having minimum total elapsed time will be the optimal sequence for the given problem.

VI. Numerical Illustration

Consider eight customers/jobs are processed through the network of three queues Q_1, Q_2 and Q_3 with the servers S_1, S_2 and S_3, where S_3 is commonly linked in series with each of the two parallel biserial servers S_1 and S_2. Let the number of the customers, mean arrival rate, mean service rate and associated probabilities are given as follows:

After completing the service at Phase-I, jobs / customers go to the machines M_1, M_2 and M_3 with processing time A_{i1}, A_{i2} and A_{i3} respectively given as follows:

TABLE I: NO. OF CUSTOMERS, MEAN ARRIVAL RATE, MEAN SERVICE RATE WITH PROBABILITIES

S. No	No. of Customers	Mean Arrival Rate	Mean Service Rate	Probabilities
1	n_1= 5	$\lambda_1 = 5$	μ_1 =12	p12 = 0.4
2	n_2= 3	λ_2 =4	μ_2 = 9	p13 = 0.6
			μ_3 = 10	p21 = 0.5
				p23 = 0.5

TABLE II: PROCESSING TIMES FOR THE MACHINES M_1, M_2 AND M_3

Jobs	1	2	3	4	5	6	7	8
M_1 (A_{i1})	5			10		2		
M_2 (A_{i2})	8							
M_3 (A_{i3})	10					0		

The objective is to find an optimal sequence of the jobs / customers to minimize the makespan in this Queue Scheduling linkage system by considering the first phase service into account.

Solution: We have

$$\rho_1 = \frac{\lambda_1 + \lambda_2 p_{21}}{(1 - p_{12}p_{21})\mu_1} = \frac{7}{9.6},$$

$$\rho_2 = \frac{\lambda_2 + \lambda_1 p_{12}}{(1 - p_{12}p_{21})\mu_2} = \frac{6}{7.2},$$

$$\rho_3 = \left[\frac{(\lambda_1 + \lambda_2 p_{21})p_{13} + (\lambda_2 + \lambda_1 p_{12})p_{23}}{\mu_3(1 - p_{12}p_{21})}\right] = \frac{7.2}{8.8}.$$

Mean Queue Length = Average number of Jobs / Customers = $L = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3}$ = 12.2692

Average waiting time of the jobs / customers = $E(w) = \frac{L}{\lambda} = \frac{12.2692}{9} = 1.3632$.

The total completion time of Jobs / Customers when processed through queue network in Phase I =

$$C = E(W) + \frac{1}{\mu_1 p_{12} + \mu_1 p_{13} + \mu_2 p_{21} + \mu_2 p_{23} + \mu_3}$$

$$= 1.3632 + \frac{1}{4.8 + 7.2 + 4.5 + 4.5 + 11} = 1.39445 \approx 1.39.$$

On taking the completion time C = 1.39 as the setup time, when jobs / customers came for processing with machine M_1. The new reduced two machine problem with processing time $A'_{i1} = A_{i1} + C$, A_{i2} and A_{i3} on machine M_1, M_2 and M_3 is as shown :

TABLE III: MODIFIED PROCESSING TIMES OF THE MACHINES

Jobs	1	2	3	4	5	6	7	8
M_1 (A_{i1})	6.39	8.39	9.39	11.39	7.39	13.39	10.39	8.39
M_2 (A_{i2})	8	7	8	6	5	6	4	7
M_3 (A_{i3})	10	8	9	8	9	10	9	8

The two fictitious machines G and H with processing times G_i and H_i are as follows

TABLE IV: TWO FICTIOUS MACHINES WITH PROCESSING TIMES G_i AND H_i

Jobs	1	2	3	4	5	6	7	8
Gi	14.39	15.39	17.39	17.39	12.39	19.39	14.39	15.39
Hi	18	15	17	14	14	16	13	15

By Johnson's (1954) procedure the optimal sequence is S = 5 – 1 – 2 – 8 – 3 – 6 – 4 – 7 .

The In-Out Table for the optimal sequence S is

TABLE V: IN-OUT FLOW TABLE

Jobs	M1	M2	M3
5	0.0 – 7.39	7.39 – 12.39	12. 39 – 21.39
1	7.39 – 13.78	13.78 – 21.78	21.78 – 31.78
2	13.78 – 20.78	21.78 – 28.78	31.78 – 39.78
8	20.78 – 29.17	29.17 – 36.17	39.78 – 37.78
3	29.17 – 38.56	38.56 – 46.56	46.56 – 55.56
6	38.56 – 51.92	51.92 – 57.92	57.92 – 67.92
4	51.92 – 63.31	63.31 – 69.31	69.31 – 77.31
7	63.31 – 73.3	73.30 – 77.3	77.31 – 86.31

Hence total elapsed time is 86.31 units and Mean queue length is 12.2692 units.

VII. CONCLUSION

The present paper establishes a bridge between a queuing network consisting of parallel biserial servers linked in series with a common server and a three stage flow shop scheduling system. The model discussed here provides an important tool for the decision makers in industrial/production concern, in banking services, in computer networks etc where we have to minimize the average waiting time and flow time of the jobs/customers simultaneously. The study can further be extended by introducing more complex queueing networks and various other constraints in the flow shop system of machines.

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