Revised Szeged Index and Revised Edge Szeged Index of Certain Special Molecular Graphs

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Abstract: In theoretical chemistry, the revised Szeged index and revised Szeged edge index were introduced to measure the stability of alkanes and the strain energy of cycloalkanes. In this paper, by virtue of mathematical derivation, we determine the revised Szeged index and revised edge Szeged index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their r-corona molecular graphs. These molecular structures are widely used in biology, medical science and pharmaceutical fields. At last, the normalized revised Szeged indexes of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their r-corona molecular graphs are given.

Key words: Chemical graph theory, revised Szeged index, revised edge Szeged index, fan molecular graph, wheel molecular graph, gear fan molecular graph, Gear wheel molecular graph, r-corona molecular graph.

1. Introduction

Wiener index, PI index, revised Szeged index and revised edge Szeged index are introduced to reflect certain structural features of organic molecules. Several papers contributed to determine the distance-based index of special molecular graphs (See Yan *et al.*, [1] and [2], Gao and Shi [3] for more detail). Let P_n and C_n be path and cycle with n vertices. The molecular graph $F_n=\{v\}\vee P_n$ is called a fan molecular graph and the molecular graph $W_n=\{v\}\vee C_n$ is called a wheel molecular graph. Molecular graph $I_r(G)$ is called r- crown molecular graph of G which splicing r hang edges for every vertex in G. By adding one vertex in every two adjacent vertices of the fan path P_n of fan molecular graph F_n, the resulting molecular graph is a subdivision molecular graph called gear fan molecular graph, denote as $\tilde{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle C_n of wheel molecular graph W_n, The resulting molecular graph is a subdivision molecular graph, called gear wheel molecular graph, denoted as $\tilde{W}_n$.

Let $e=uv$ be an edge of the molecular graph G. The number of vertices of G whose distance to the vertex u is smaller than the distance to the vertex v is denoted by $n_u(e)$. Analogously, $n_v(e)$ is the number of vertices of G whose distance to the vertex v is smaller than the distance to the vertex u. Let $n(e)$ be the number of vertices equidistant from both ends of $uv\in E(G)$, The revised Szeged index is defined as

$$Sz^*(G) = \sum_{e=uv}(n_u(e) + \frac{n(e)}{2})(n_v(e) + \frac{n(e)}{2}).$$

The number of edges of G whose distance to the vertex u is smaller than the distance to the vertex v is denoted by $m_u(e)$. Analogously, $m_v(e)$ is the number of edges of G whose distance to the vertex v is smaller than the distance to the vertex u. Let $m(e)$ be the number of edges equidistant from both ends of $uv \in E(G)$. The revised edge Szeged index of G is defined as

$$Sz_e^*(G) = \sum_{e=uv}(m_u(e) + \frac{m(e)}{2})(m_v(e) + \frac{m(e)}{2}).$$

Xing and Zhou [4] determined the n-vertex unicyclic graphs with the smallest, the second-smallest and the third-smallest revised Szeged index. Chen *et al.* [5] studied the differences between the revised Szeged index and the Wiener index. Dong *et al.* [6] considered the revised edge Szeged index of molecular graphs. Faghani and Ashrafi [7] presented new formula for computing molecular descriptor.

In this paper, we present the revised Szeged index and revised edge Szeged index of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$.

2. Revised Szeged Index

Theorem 1. $Sz^*(I_r(F_n)) = r^2(\frac{n^3}{4} + \frac{15}{4}n^2 - \frac{1}{4}n + \frac{11}{4}) + r(\frac{n^3}{2} + \frac{13}{2}n^2 - \frac{7}{2}n + \frac{7}{2}) + (\frac{n^3}{4} + \frac{1}{4}n^2 - \frac{9}{4}n + \frac{7}{4})$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and the r hanging vertices of v_i be v_i^1, v_i^2,…, v_i^r $(1 \le i \le n)$. Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r .Using the definition of revised Szeged index, we have

$$Sz^*(I_r(F_n)) = \sum_{i=1}^{r}\sum_{e=uv}(n_v(vv^i) + \frac{n(vv^i)}{2})(n_{v^i}(vv^i) + \frac{n(vv^i)}{2}) +$$

$$\sum_{i=1}^{n}\sum_{e=uv}(n_v(vv_i) + \frac{n(vv_i)}{2})(n_{v_i}(vv_i) + \frac{n(vv_i)}{2}) + \sum_{i=1}^{n-1}\sum_{e=uv}(n_{v_i}(v_iv_{i+1}) + \frac{n(v_iv_{i+1})}{2})(n_{v_{i+1}}(v_iv_{i+1}) + \frac{n(v_iv_{i+1})}{2})$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(n_{v_i}(v_iv_i^j) + \frac{n(v_iv_i^j)}{2})(n_{v_i^j}(v_iv_i^j) + \frac{n(v_iv_i^j)}{2})$$

$$= r(r+n(r+1)) + (2(r+1+\frac{r+1}{2})[(n-1)(r+1)+\frac{r+1}{2}] + (n-2)2(r+1)[(n-1)(r+1)]) +$$

$$2(r+1+\frac{(n-2)(r+1)}{2})[2(r+1)+\frac{(n-2)(r+1)}{2}] + (n-3)(2(r+1)+\frac{(n-3)(r+1)}{2})(2(r+1)+\frac{(n-3)(r+1)}{2}) +$$

$$nr(r+n(r+1))$$

$$= r^2(\frac{n^3}{4} + \frac{15}{4}n^2 - \frac{1}{4}n + \frac{11}{4}) + r(\frac{n^3}{2} + \frac{13}{2}n^2 - \frac{7}{2}n + \frac{7}{2}) + (\frac{n^3}{4} + \frac{1}{4}n^2 - \frac{9}{4}n + \frac{7}{4}).$$

Corollary 1. $Sz^*(F_n) = \frac{n^3}{4} + \frac{1}{4}n^2 - \frac{9}{4}n + \frac{7}{4}$.

Theorem 2. $Sz^*(I_r(W_n)) = r^2(\frac{n^3}{4}+\frac{7}{2}n^2+\frac{1}{4}n+1)+r(\frac{n^3}{2}+6n^2-\frac{5}{2}n)+(\frac{n^3}{4}+\frac{5}{2}n^2-\frac{7}{4}n)$.

Proof. Let $C_n=v_1v_2\ldots v_n$ and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of $v_i(1\le i\le n)$. Let v be a vertex in W_n beside C_n, and v^1, v^2, …, v^r be the r hanging vertices of v. We denote $v_nv_{n+1}=v_nv_1$. In view of the definition of revised Szeged index, we infer

$$Sz^*(I_r(W_n)) = \sum_{i=1}^{r}\sum_{e=uv}(n_v(vv^i)+\frac{n(vv^i)}{2})(n_{v^i}(vv^i)+\frac{n(vv^i)}{2}) +$$

$$\sum_{i=1}^{n}\sum_{e=uv}(n_v(vv_i)+\frac{n(vv_i)}{2})(n_{v_i}(vv_i)+\frac{n(vv_i)}{2})+\sum_{i=1}^{n}\sum_{e=uv}(n_{v_i}(v_iv_{i+1})+\frac{n(v_iv_{i+1})}{2})(n_{v_{i+1}}(v_iv_{i+1})+\frac{n(v_iv_{i+1})}{2})$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(n_{v_i}(v_iv_i^j)+\frac{n(v_iv_i^j)}{2})(n_{v_i^j}(v_iv_i^j)+\frac{n(v_iv_i^j)}{2})$$

$$= r(r+n(r+1)) + n[(n-1)(r+1)][2(r+1)]$$

$$+n[2(1+r)+\frac{(n-3)(r+1)}{2}][2(1+r)+\frac{(n-3)(r+1)}{2}] + nr(r+n(r+1))$$

$$= r^2(\frac{n^3}{4}+\frac{7}{2}n^2+\frac{1}{4}n+1)+r(\frac{n^3}{2}+6n^2-\frac{5}{2}n)+(\frac{n^3}{4}+\frac{5}{2}n^2-\frac{7}{4}n).$$

Corollary 2. $Sz^*(W_n) = \frac{n^3}{4}+\frac{5}{2}n^2-\frac{7}{4}n$.

Theorem 3. $Sz^*(I_r(\tilde{F}_n)) = r^2(22n^2-43n+28)+r(40n^2-88n+56)+(18n^2-43n+28)$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of $v_i(1\le i\le n)$. Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ $(1\le i\le n\text{-}1)$. Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r.

By virtue of the definition of revised Szeged index, we yield

$$Sz^*(I_r(\tilde{F}_n)) = \sum_{i=1}^{r}\sum_{e=uv}(n_v(vv^i)+\frac{n(vv^i)}{2})(n_{v^i}(vv^i)+\frac{n(vv^i)}{2}) +$$

$$\sum_{i=1}^{n}\sum_{e=uv}(n_v(vv_i)+\frac{n(vv_i)}{2})(n_{v_i}(vv_i)+\frac{n(vv_i)}{2})+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(n_{v_i}(v_iv_i^j)+\frac{n(v_iv_i^j)}{2})(n_{v_i^j}(v_iv_i^j)+\frac{n(v_iv_i^j)}{2})$$

$$+\sum_{i=1}^{n-1}(n_{v_i}(v_iv_{i,i+1})+\frac{n(v_iv_{i,i+1})}{2})(n_{v_{i,i+1}}(v_iv_{i,i+1})+\frac{n(v_iv_{i,i+1})}{2})$$

$$+\sum_{i=1}^{n-1}(n_{v_{i,i+1}}(v_{i,i+1}v_{i+1})+\frac{n(v_{i,i+1}v_{i+1})}{2})(n_{v_{i+1}}(v_{i,i+1}v_{i+1})+\frac{n(v_{i,i+1}v_{i+1})}{2})$$

$$+\sum_{i=1}^{n-1}\sum_{j=1}^{r}(n_{v_{i,i+1}}(v_{i,i+1}v_{i,i+1}^j)+\frac{n(v_{i,i+1}v_{i,i+1}^j)}{2})(n_{v_{i,i+1}^j}(v_{i,i+1}v_{i,i+1}^j)+\frac{n(v_{i,i+1}v_{i,i+1}^j)}{2})$$

$$= r(r+(r+1)(2n-1)) + 2(2n-2)(r+1)2(r+1) + (n-2)(2n-3)(r+1)3(r+1)$$

$$+nr(r+(r+1)(2n-1)) + (n-1)(2n-3)(r+1)3(r+1) + (n-1)(2n-3)(r+1)3(r+1)$$

$$+(n-1)r(r+(r+1)(2n-1)) = r^2(22n^2-43n+28)+r(40n^2-88n+56)+(18n^2-43n+28).$$

Corollary 3. $Sz^*(\tilde{F}_n) = 18n^2 - 43n + 28$.

Theorem 4. $Sz^*(I_r(\tilde{W}_n)) = r^2(22n^2 - 14n + 1) + r(40n^2 - 34n) + (18n^2 - 18n)$.

Proof. Let $C_n = v_1 v_2 \ldots v_n$ and v be a vertex in W_n beside C_n, and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v^1, v^2, …, v^r be the r hanging vertices of v and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of $v_i (1 \le i \le n)$. Let $v_{n,n+1} = v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ $(1 \le i \le n)$. Let $v_{n.n+1} = v_{n.1}$, $v_{n+1} = v_1$. In view of the definition of revised Szeged index, we deduce

$$
\begin{aligned}
Sz^*(I_r(\tilde{W}_n)) &= \sum_{i=1}^{r}\sum_{e=uv}(n_v(vv^i) + \frac{n(vv^i)}{2})(n_{v^i}(vv^i) + \frac{n(vv^i)}{2}) + \\
&\sum_{i=1}^{n}\sum_{e=uv}(n_v(vv_i) + \frac{n(vv_i)}{2})(n_{v_i}(vv_i) + \frac{n(vv_i)}{2}) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(n_{v_i}(v_i v_i^j) + \frac{n(v_i v_i^j)}{2})(n_{v_i^j}(v_i v_i^j) + \frac{n(v_i v_i^j)}{2}) \\
&+ \sum_{i=1}^{n}(n_{v_i}(v_i v_{i,i+1}) + \frac{n(v_i v_{i,i+1})}{2})(n_{v_{i,i+1}}(v_i v_{i,i+1}) + \frac{n(v_i v_{i,i+1})}{2}) \\
&+ \sum_{i=1}^{n}(n_{v_{i,i+1}}(v_{i,i+1} v_{i+1}) + \frac{n(v_{i,i+1} v_{i+1})}{2})(n_{v_{i+1}}(v_{i,i+1} v_{i+1}) + \frac{n(v_{i,i+1} v_{i+1})}{2}) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{r}(n_{v_{i,i+1}}(v_{i,i+1} v_{i,i+1}^j) + \frac{n(v_{i,i+1} v_{i,i+1}^j)}{2})(n_{v_{i,i+1}^j}(v_{i,i+1} v_{i,i+1}^j) + \frac{n(v_{i,i+1} v_{i,i+1}^j)}{2}) \\
&= r(r + 2n(r+1)) + n(2n-2)(r+1)3(r+1) + nr(r + 2n(r+1)) + n(2n-2)(r+1)3(r+1) \\
&\quad + n(2n-2)(r+1)3(r+1) + nr(r + 2n(r+1)) \\
&= r^2(22n^2 - 14n + 1) + r(40n^2 - 34n) + (18n^2 - 18n).
\end{aligned}
$$

Corollary 4. $Sz^*(\tilde{W}_n) = 18n^2 - 18n$.

3. Revised Edge Szeged Index

Theorem 5. $Sz_e^*(I_r(F_n)) = r^2(\frac{n^3}{4} + \frac{7}{2}n^2 + \frac{3}{4}n + 3) + r(n^3 + \frac{19}{2}n^2 - \frac{27}{2}n + \frac{9}{2}) + (n^3 + 5n^2 - \frac{63}{4}n + 13)$.

Proof. Using the definition of revised edge Szeged index, we have

$$
\begin{aligned}
Sz_e^*(I_r(F_n)) &= \sum_{i=1}^{r}\sum_{e=uv}(m_v(vv^i) + \frac{m(vv^i)}{2})(m_{v^i}(vv^i) + \frac{m(vv^i)}{2}) + \\
&\sum_{i=1}^{n}\sum_{e=uv}(m_v(vv_i) + \frac{m(vv_i)}{2})(m_{v_i}(vv_i) + \frac{m(vv_i)}{2}) + \sum_{i=1}^{n-1}\sum_{e=uv}(m_{v_i}(v_i v_{i+1}) + \frac{m(v_i v_{i+1})}{2})(m_{v_{i+1}}(v_i v_{i+1}) + \frac{m(v_i v_{i+1})}{2}) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(m_{v_i}(v_i v_i^j) + \frac{m(v_i v_i^j)}{2})(m_{v_i^j}(v_i v_i^j) + \frac{m(v_i v_i^j)}{2}) \\
&= r(2n + r + nr - 2) + (2(2n + nr - r - 4 + \frac{r+2}{2})(r + 1 + \frac{r+2}{2}) + 2(2n + nr - 2r - 4 + \frac{2r+1}{2})(r + 2 + \frac{2r+1}{2}) + \\
&(n-4)(2n + nr - 2r - 5 + r + 1)(r + 2 + r + 1)) + (2(r + 1 + \frac{nr + 2n - 2r - 5}{2})(2r + 3 + \frac{nr + 2n - 2r - 5}{2}) \\
&+ 2(2r + 2 + \frac{nr + 2n - 3r - 6}{2})(2r + 3 + \frac{nr + 2n - 3r - 6}{2}) +
\end{aligned}
$$

$$(n-4)(2r+3+\frac{nr+2n-3r-7}{2})(2r+3+\frac{nr+2n-3r-7}{2}))+nr(2n+r+nr-2)$$

$$=r^2(\frac{n^3}{4}+\frac{7}{2}n^2+\frac{3}{4}n+3)+r(n^3+\frac{19}{2}n^2-\frac{27}{2}n+\frac{9}{2})+(n^3+5n^2-\frac{63}{4}n+13).$$

Corollary 5. $Sz_e^*(F_n)=n^3+5n^2-\frac{63}{4}n+13$.

Theorem 6. $Sz_e^*(I_r(W_n))=r^2(\frac{n^3}{4}+\frac{7}{2}n^2+\frac{1}{4}n+1)+r(n^3+\frac{21}{2}n^2-\frac{19}{2}n-1)+(n^3+7n^2-\frac{49}{4}n)$.

Proof. In view of the definition of revised edge Szeged index, we infer

$$Sz_e^*(I_r(W_n))=\sum_{i=1}^{r}\sum_{e=uv}(m_v(vv^i)+\frac{m(vv^i)}{2})(m_{v^i}(vv^i)+\frac{m(vv^i)}{2})+$$

$$\sum_{i=1}^{n}\sum_{e=uv}(m_v(vv_i)+\frac{m(vv_i)}{2})(m_{v_i}(vv_i)+\frac{m(vv_i)}{2})+\sum_{i=1}^{n}\sum_{e=uv}(m_{v_i}(v_iv_{i+1})+\frac{m(v_iv_{i+1})}{2})(m_{v_{i+1}}(v_iv_{i+1})+\frac{m(v_iv_{i+1})}{2})$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(m_{v_i}(v_iv_i^j)+\frac{m(v_iv_i^j)}{2})(m_{v_i^j}(v_iv_i^j)+\frac{m(v_iv_i^j)}{2})$$

$$=r(2n+r+nr-1)+n(r+2+\frac{2r+3}{2})(2n+nr-2r-5+\frac{2r+3}{2})+$$

$$n(2r+3+\frac{nr+2n-3r-6}{2})(2r+3+\frac{nr+2n-3r-6}{2})+nr(2n+r+nr-1)$$

$$r^2(\frac{n^3}{4}+\frac{7}{2}n^2+\frac{1}{4}n+1)+r(n^3+\frac{21}{2}n^2-\frac{19}{2}n-1)+(n^3+7n^2-\frac{49}{4}n).$$

Corollary6. $Sz_e^*(W_n)=n^3+7n^2-\frac{49}{4}n$.

Theorem 7. $Sz_e^*(I_r(\tilde{F}_n))=r^2(20n^2-31n+16)+r(44n^2-\frac{221}{2}n+\frac{147}{2})+(21n^2-58n+\frac{167}{4})$.

Proof. By virtue of the definition of revised edge Szeged index, we yield

$$Sz_e^*(I_r(\tilde{F}_n))=\sum_{i=1}^{r}\sum_{e=uv}(m_v(vv^i)+\frac{m(vv^i)}{2})(m_{v^i}(vv^i)+\frac{m(vv^i)}{2})+\sum_{i=1}^{n}\sum_{e=uv}(m_v(vv_i)+\frac{m(vv_i)}{2})(m_{v_i}(vv_i)+\frac{m(vv_i)}{2})$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(m_{v_i}(v_iv_i^j)+\frac{m(v_iv_i^j)}{2})(m_{v_i^j}(v_iv_i^j)+\frac{m(v_iv_i^j)}{2})$$

$$+\sum_{i=1}^{n-1}(m_{v_i}(v_iv_{i,i+1})+\frac{m(v_iv_{i,i+1})}{2})(m_{v_{i,i+1}}(v_iv_{i,i+1})+\frac{m(v_iv_{i,i+1})}{2})$$

$$+\sum_{i=1}^{n-1}(m_{v_{i,i+1}}(v_{i,i+1}v_{i+1})+\frac{m(v_{i,i+1}v_{i+1})}{2})(m_{v_{i+1}}(v_{i,i+1}v_{i+1})+\frac{m(v_{i,i+1}v_{i+1})}{2})$$

$$+\sum_{i=1}^{n-1}\sum_{j=1}^{r}(m_{v_{i,i+1}}(v_{i,i+1}v_{i,i+1}^j)+\frac{m(v_{i,i+1}v_{i,i+1}^j)}{2})(m_{v_{i,i+1}^j}(v_{i,i+1}v_{i,i+1}^j)+\frac{m(v_{i,i+1}v_{i,i+1}^j)}{2})$$

$$=r(3n+2nr-3)+(2(2r+2)(2nr+3n-2r-4)+(n-2)(3r+2+\frac{3}{2})(2nr+3n-3r-7+\frac{3}{2}))+nr(3n+2nr-3)+$$

$$(n-1)(3r+2+\frac{3}{2})(2nr-3r+3n-7+\frac{3}{2})+(n-1)(3r+2+\frac{3}{2})(2nr-3r+3n-7+\frac{3}{2})+(n-1)r(3n+2nr-3)$$
$$=r^2(20n^2-31n+16)+r(44n^2-\frac{221}{2}n+\frac{147}{2})+(21n^2-58n+\frac{167}{4}).$$

Corollary 7. $Sz_e^*(\tilde{F}_n)=21n^2-58n+\frac{167}{4}$.

Theorem 8. $Sz_e^*(I_r(\tilde{W}_n))=r^2(22n^2-14n+1)+r(54n^2-\frac{103}{2}n-1)+(\frac{63}{2}n^2-\frac{105}{2}n)$.

Proof. In view of the definition of revised edge Szeged index, we deduce

$$Sz_e^*(I_r(\tilde{W}_n))=\sum_{i=1}^{r}\sum_{e=uv}(m_v(vv^i)+\frac{m(vv^i)}{2})(m_{v^i}(vv^i)+\frac{m(vv^i)}{2})+$$
$$\sum_{i=1}^{n}\sum_{e=uv}(m_v(vv_i)+\frac{m(vv_i)}{2})(m_{v_i}(vv_i)+\frac{m(vv_i)}{2})$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{e=uv}(m_{v_i}(v_iv_i^j)+\frac{m(v_iv_i^j)}{2})(m_{v_i^j}(v_iv_i^j)+\frac{m(v_iv_i^j)}{2})$$
$$+\sum_{i=1}^{n}(m_{v_i}(v_iv_{i,i+1})+\frac{m(v_iv_{i,i+1})}{2})(m_{v_{i,i+1}}(v_iv_{i,i+1})+\frac{m(v_iv_{i,i+1})}{2})$$
$$+\sum_{i=1}^{n}(m_{v_{i,i+1}}(v_{i,i+1}v_{i+1})+\frac{m(v_{i,i+1}v_{i+1})}{2})(m_{v_{i+1}}(v_{i,i+1}v_{i+1})+\frac{m(v_{i,i+1}v_{i+1})}{2})$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}(m_{v_{i,i+1}}(v_{i,i+1}v_{i,i+1}^j)+\frac{m(v_{i,i+1}v_{i,i+1}^j)}{2})(m_{v_{i,i+1}^j}(v_{i,i+1}v_{i,i+1}^j)+\frac{m(v_{i,i+1}v_{i,i+1}^j)}{2})$$
$$=r(2nr+3n+r-1)+n(3r+2+\frac{3}{2})(2nr+3n-2r-5+\frac{3}{2})+nr(2nr+3n+r-1)+$$
$$n(3r+2+\frac{3}{2})(2nr+3n-2r-5+\frac{3}{2})+n(3r+2+\frac{3}{2})(2nr+3n-2r-5+\frac{3}{2})+nr(2nr+3n+r-1)$$
$$=r^2(22n^2-14n+1)+r(54n^2-\frac{103}{2}n-1)+(\frac{63}{2}n^2-\frac{105}{2}n).$$

Corollary 8. $Sz_e^*(\tilde{W}_n)=\frac{63}{2}n^2-\frac{105}{2}n$.

4. Normalized Revised Szeged Index

Let m be the edge number of molecular graph G. Aouchiche and Hansen [8] defined the normalized revised Szeged index by dividing $Szs^*(G)$ by m and then taking the square root, i.e.,

$$Szs^*(G)=\sqrt{\frac{Sz^*(G)}{m}}.$$

Using the conclusions raised in Section 2, we infer the following results concern normalized revised Szeged index.

Theorem 9.

$$Szs^{*}(I_r(F_n))=\sqrt{\frac{r^2(\frac{n^3}{4}+\frac{15}{4}n^2-\frac{1}{4}n+\frac{11}{4})+r(\frac{n^3}{2}+\frac{13}{2}n^2-\frac{7}{2}n+\frac{7}{2})+(\frac{n^3}{4}+\frac{1}{4}n^2-\frac{9}{4}n+\frac{7}{4})}{nr+2n+r-1}}.$$

Corollary 9.

$$Szs^{*}(F_n)=\sqrt{\frac{\frac{n^3}{4}+\frac{1}{4}n^2-\frac{9}{4}n+\frac{7}{4}}{2n-1}}.$$

Theorem 10.

$$Szs^{*}(I_r(W_n))=\sqrt{\frac{r^2(\frac{n^3}{4}+\frac{7}{2}n^2+\frac{1}{4}n+1)+r(\frac{n^3}{2}+6n^2-\frac{5}{2}n)+(\frac{n^3}{4}+\frac{5}{2}n^2-\frac{7}{4}n)}{(n+1)r+2n}}.$$

Corollary 10.

$$Szs^{*}(W_n)=\sqrt{\frac{\frac{n^3}{4}+\frac{5}{2}n^2-\frac{7}{4}n}{2n}}.$$

Theorem 11.

$$Szs^{*}(I_r(\tilde{F}_n))=\sqrt{\frac{r^2(22n^2-43n+28)+r(40n^2-88n+56)+(18n^2-43n+28)}{2nr+3n-2}}.$$

Corollary 11.

$$Szs^{*}(\tilde{F}_n)=\sqrt{\frac{18n^2-43n+28}{3n-2}}.$$

Theorem 12.

$$Szs^{*}(I_r(\tilde{W}_n))=\sqrt{\frac{r^2(22n^2-14n+1)+r(40n^2-34n)+(18n^2-18n)}{(2n+1)r+3n}}.$$

Corollary 12.

$$Szs^{*}(\tilde{W}_n)=\sqrt{\frac{18n^2-18n}{3n}}.$$

5. Conclusion

In this paper, we conduct the revised Szeged index and revised edge Szeged index and third geometric-arithmetic index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their *r*-corona molecular graphs. Furthermore, the normalized revised Szeged indexes of these molecular graphs are manifested. The result achieved in our paper illustrates the promising application prospects for chemical engineering.

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