A Note on Nonlinear Parametric Interaction of Acoustic Waves in Magnetised Nondegenerate Piezoelectric Semiconductor — A Numerical Approach

Pravat Kr. Mandal[1*]

[1] Dept. of Maths., A.M.College, Purulia, W.B. India.

* Corresponding author. Tel.: -9433468244; email: pravat.mandal1262@gmail.com
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Abstract: The phenomenon of parametric interaction of coupled waves exhibit a vital role in nonlinear acoustics. An attempt has been made to investigate numerically the possible instability and the parametric amplification of acoustic wave when laser beam interacts nonlinearly with a low frequency transverse acoustic wave in presence of transverse magnetostatic field in a heavily doped n-type piezoelectric semiconductor. Using a hydrodynamic model of a homogeneous plasma, the amplification of acoustic waves has been studied when a magnetostatic field is applied perpendicular to the direction of the pump wave propagation. For numerical simulation we use Newton-Raphson method for finding possibility of nonrealor imaginary roots. It is observed that imaginary part of the root becomes positive. So the instability of acoustic wave occurs and hence amplified for the present situation. The values of physical parameters are used for InSb crystal in this article.

Key words: Parametric amplification, magnetostatic plasma medium, nondegenerate semiconductor.

1. Introduction

There have been extensive theoretical and experimental studies on nonlinear parametric interaction in piezoelectric semiconducting material in the last few decades [1]-[3]. The motivation which led to the rapid development in the field of nonlinear acoustics is the possibility of exploiting the nonlinear behavior in various solid state device such as frequency converters, parametric amplifiers and oscillators, optical phase cojugators etc. [4]. The importance of the effect of a d.c magnetic field on parametric interaction of acoustic waves were studied by Cohen [5]. They have shown nonlinearities due to the nonlinear material parameters, plays an important role in the parametric interaction in a strongly piezoelectric material like $LiNbO_3$.Ghosh and Agarwal [6] have discussed the excitation of acoustoheliconwave parametric and modulational interactions in magnetostatic piezoelectric semiconducting media. The parametric amplification of acoustic waves in a piezoelectric semiconductor has been studied by Ghosh and Khan [7]. Parametric amplification of acoustic waves in a thermopiezosemi conducting medium has been discussed by M. Salimullah *et al.* [8], Mandal & Gupta [9]. Ghosh & Khare [10] have studied acoustic electric wave instability in Ion-Implanted semiconductor plasmas. P. K. Mandal [11] has studied the parametric amplification of acoustic wave in presence of transverse magnetostatic field in n-type piezoelectric semiconducting media by numerical approach. The nonlinear propagation of two-dimensional quantum ion-acoustic waves is studied in quantum electron-ion plasma by Misra *et al.* [12]. B. Ghosh *et al.* [13] have studied relativistic effects on the modulational instability of electron plasma waves in quantum plasma. Recently, P. K. Shukla *et al.* [14] have discussed nonlinear propagation of coherent electromagnetic waves in a dense magnetized plasma. In the present paper we investigate the possibility of parametric amplification ofacoustic wave in a nondegenerate n-Insb crystal at 77k when a magnetostatic field is applied perpendicular to the direction of the pump wave propagation.

2. Statement of the Problem and Basic Equations

We consider the well-known hydrodynamical model of a homogeneous *n*-typesemi conductor plasma having both piezoelectric as well as deformation potential couplings and the medium is of infinite extent with electrons as carriers. This model restricts the validity of the analysis to the limit $k_l << 1$, where k is the wave vector and l is the electron mean free path. In order to study the parametric interaction process, the medium is subjected to the magneto static field B_0 (along z-axis) perpendicular to the propagation direction (x-axis) of spatially uniform high frequency pump electric field $E_0 \exp(-i\omega_0 t)$. We apply coupled mode theory to obtain a simplified expression for the acoustic waves via density perturbation.

The basic equations used are as follows :

$$\frac{\partial \boldsymbol{v}_0}{\partial t} + \nu\, v_0 = -\frac{e}{m}[E_0 + v_0 \times B_1] \tag{1}$$

$$\frac{\partial \boldsymbol{v}_1}{\partial t} + \nu\, \boldsymbol{v}_1 + (\boldsymbol{v}_0 \frac{\partial}{\partial x})\boldsymbol{v}_1 = -\frac{e}{m}[E_1 + v_0 \times B_1 + v_1 \times B_0] \tag{2}$$

$$v_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial \boldsymbol{v}_1}{\partial x} = -\frac{\partial n_1}{\partial t} \tag{3}$$

$$\frac{\partial E_1}{\partial x} + \frac{\beta}{\epsilon}\frac{\partial^2 u}{\partial x^2} - \frac{\boldsymbol{C}_d}{e}\frac{\partial^3 u}{\partial x^3} = -\frac{n_1 e}{M} \tag{4}$$

$$\rho \frac{\partial^2 u}{\partial t^2} + 2\, v_s \rho \frac{\beta}{\epsilon}\frac{\partial n}{\partial x} + \beta \frac{\partial E_s}{\partial x} + \frac{\boldsymbol{C}_d \epsilon}{e}\frac{\partial^2 E_s}{\partial x^2} = C\, \frac{\partial^2 u}{\partial x^2} \tag{5}$$

Equations (1) & (2) represent the zero-th and first order momentum transfer equations, respectively in which v_0 and v_1 are the zeroth and first order oscillatory fluid velocities having effective mass m and charge e and ν is the phenomenological electron collision frequency. Equation (3) represents the continuity equation for electrons, where$n_0$andn_1 are the equilibrium and purterbed electron densities respectively. The Poisson equation (4) gives the space charge field E_1 in which the second and third terms on the left side give the piezoelectric and deformation potential contribution to polarization respectively. ϵ, β, C_d are the scalar dielectric, piezoelectric and deformation potential constants of the semiconductor respectively, equation (5) describes the motion of the lattice in a crystal having piezoelectric and deformation couplings both. In this equationρ ,u, v_s and C having the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode and crystal elastic constant respectively. In equation (2) we neglect the effect due to $v_0 \times B_1$ by assuming that the shear acoustic wave is propagating along such a direction of the crystal that it produces a longitudinal electric field e.g in *n*-Insb, if k is taken along (011) and the lattice displacement u is along (100) the electric field induced by the wave is a longitudinal field [10].

Neglecting the doppler shift under the assumption that $\omega_0 >> \nu > kv_0$ and the effect of deformation potential we obtain from equations (1) to (4) as

$$\frac{\partial^2 n_1}{\partial t^2} + \nu \frac{\partial n_1}{\partial t} + n_1 \omega_p^2 + \frac{n_0 e\ \beta}{m\epsilon} \frac{\partial^2 u}{\partial x^2} = -E\ \frac{\partial n_s}{\partial x} \tag{6}$$

Combining equations (4) and (5), we get

$$\rho \frac{\partial^2 u}{\partial t^2} + 2\ \nu_s \rho \frac{\partial u}{\partial t} - (\frac{\beta^2}{\epsilon} + C) \frac{\partial^2 u}{\partial x^2} = \frac{n_s e\ \beta}{\epsilon} \tag{7}$$

where

$$\omega_p^2 = \frac{n_0 e^2}{m\epsilon},$$

$$\omega_p^2 = \omega_p^2 \frac{\nu^2}{(\nu^2 + \omega_c^2)}, \qquad \omega_c = \frac{e\ \beta_0}{m}$$

$$E = -[\left(\frac{e}{m}\right) E_0 + \omega_c v_{0y})]$$

3. Numerical Solution

To study the parametric amplification process in a highly doped semiconductor the low frequency mode (ω_s) as well as the pump electromagnetic mode (ω_0) produce a density perturbation (n_1) at the respective frequencies in the medium on assuming the low-frequency perturbations are proportional to exp[i(kx- ωt)]. The density perturbations associated with the phonon mode (n_s) and the scattered electromagnetic waves (n_f) arising due to the three wave parametric interaction will propagate at the generated frequencies ω_s and the $\omega_0 \pm \omega_s$ respectively. For these modes the phase matching condition $\omega_0 = \omega_f + \omega_s$ and $k_0 = k_f + k_s$ i.e the energy and momentum conservation relations should be satisfied. Now for spatially uniform laser irradiation $|k_0| \approx 0$. In the present study we have restricted ourselves only to the Stokes component of $(\omega_0 - \omega_s)$ of the scattered electromagnetic waves.

For resolving the equations (6) and (7) into two components (slow and fast), we denote as $n_1 = n_f + n_s$ where

$$\left.\begin{aligned} n_f &= n_f(t)\ e^{i(k_f\, x - \omega_f\, t)} + c.c \\ n_s &= n_s(t)\ e^{i(k_s\, x - \omega_s\, t)} + c.c \\ u_s &= u_s(t)\ e^{i(k_s\, x - \omega_s\, t)} + c.c \end{aligned}\right\} \tag{8}$$

c.c – conjugate complex.

Substituting (8) in (6) and (7) and equating the terms of equal frequencies, we may obtain

$$A_1 \frac{\partial n_f}{\partial t} + B_1 n_f + E \frac{\partial n_s^*}{\partial t} + C_1 n_s^* = 0 \tag{9}$$

$$D_1 \frac{\partial n_s}{\partial t} + F_1 n_s + G_1 u_s + E\frac{\partial n_f^*}{\partial t} + H_1 n_f^* = 0 \tag{10}$$

$$K_1 \frac{\partial u_s}{\partial t} + L_1 u_s - M_1 n_s = 0 \tag{11}$$

where

$$A_1 = \nu - 2i\omega_f$$

$$B_1 = \omega_p^2 - \omega_f^2 - i\nu\omega_f$$

$$C_1 = i\omega_s^* E$$

$$D_1 = \nu - 2i\omega_s$$

$$F_1 = \omega_p^2 - \omega_s^2 - i\nu\omega_s$$

$$G_1 = \frac{n_0 e\beta}{m\epsilon} k_s^2$$

$$H_1 = i\omega_f E$$

$$K_1 = 2\nu_s\rho - 2i\omega_s\rho$$

$$L_1 = \left(\frac{\beta^2}{\epsilon} + C\right) k_s^2 - \rho\omega_s^2 - 2i\nu_s\omega_s\rho$$

$$M_1 = \frac{e\,\beta_0}{\epsilon}$$

To study the parametric amplification in a magnetized semiconducting medium, we consider

$$\left.\begin{aligned} u_s(t) &= u_s\, e^{-i\omega_s \alpha t} & u_s^*(t) &= u_s\, e^{i\omega_s \alpha t} \\ n_s(t) &= n_s\, e^{-i\omega_s \alpha t} & n_s^*(t) &= n_s\, e^{i\omega_s \alpha t} \\ n_f(t) &= n_f\, e^{i\omega_s \alpha t} & n_f^*(t) &= n_f\, e^{-i\omega_s \alpha t} \end{aligned}\right\} \tag{12}$$

where u_s , n_s , n_f etc. are real constants (independent of t) and α is the fractional change in ω_s due to nonlinear interaction.

Using (12) in equations (9), (10), (11) we get

$$(C_1 + Ei\omega_s\alpha)n_s + (B_1 + A_1 i\omega_s\alpha)n_f = 0 \tag{13}$$

$$G_1 u_s + (F_1 - D_1 i\omega_s\alpha)\, n_s + (H_1 - Ei\omega_s\alpha)n_f = 0 \tag{14}$$

$$(L_1 - K_1 i\omega_s\alpha)u_s - M_1 n_s = 0 \tag{15}$$

where

$$P = C_1 + Ei\omega_s\,\alpha$$

$$Q = B_1 + A_1 i\omega_s\,\alpha$$

$$R = G_1$$

$$S = F_1 - D_1 i\omega_s\,\alpha$$

$$T = H_1 - Ei\omega_s\,\alpha$$

$$U = L_1 - K_1 i\omega_s\,\alpha$$

$$V = -M_1$$

Elimination of u_s , n_s and n_f from (13), (14) & (15) gives

$$\begin{vmatrix} 0 & C_1 + Ei\omega_s\,\alpha & B_1 + A_1 i\omega_s\,\alpha \\ G_1 & F_1 - D_1 i\omega_s\,\alpha & H_1 - Ei\omega_s\,\alpha \\ L_1 - K_1 i\omega_s\,\alpha & -M_1 & 0 \end{vmatrix} = 0 \qquad (16)$$

Thus the equation (16) reduces to a cubic equation of α such as

$$P_1\,\alpha^3 + Q_1\,\alpha^2 + R_1\alpha + S_1 = 0 \qquad (17)$$

where

$$P_1 = \omega_s^3 K_1(A_1 D_1 - E^2)i$$

$$Q_1 = [E(E\,L_1 + H_1 K_1) - A_1(F_1 K_1 + L_1 D_1) + B_1 D_1 K_1 - C_1 E\,K_1]\omega_s^2$$

$$R_1 = [L_1 E\,H_1 + B_1(F_1 K_1 + L_1 D_1) - G(E\,L_1 + H_1 K_1) - A_1(G_1 M_1 + L_1 F_1)]i\omega_s$$

$$S_1 = -B_1(G_1 M_1 + L_1 F_1) + C_1 L_1 H_1$$

To simplify the equation (17) the set of values for InSb crystal at 77K are used, which are stated below:

$$\nu_s = 10^{-11} s$$

$$\rho = 5.8 \times 10^3 kg\ m^{-3}$$

$$\omega_c = 2\times 10^2 s^{-1}$$

$$m = 0.13664 \times 10^{-31} kg$$

$$v_s = 4\times 10^3 ms^{-1}$$

$$n_0 = 2\times 10^{24} m^{-3}$$

$$\epsilon = 18 \times 8.85 \times 10^{-12}$$

$$K = 2 \times 10^{6} s^{-1}$$

$$\omega_0 = 1.78 \times 10^{14} s^{-1}$$

$$\omega_f = 1.778 \times 10^{14} s^{-1}$$

$$\nu = 4 \times 10^{11} s^{-1}$$

$$\epsilon_0 = 8.85 \times 10^{-12} F/m$$

$$\omega_s = 2 \times 10^{11} s^{-1}$$

$$E_0 \ 10^{7} V m^{-1}$$

$$\beta = 0.054 \ C m^{-2}$$

$$e = 1.6 \times 10^{-19} C$$

$$k_s = 5 \times 10^{7}$$

Using the above set of values, we have

$$P_1 = -0.43987201 - 3.41920191 \times 10^{-4} i$$

$$Q_1 = -0.24301678 + 1.05172392 \times 10^{-2} i$$

$$R_1 = 0.27409418 + 2.49240962 \, i$$

$$S_1 = 2.09291801 - 1.69495185 \times 10^{-4} i$$

Therefore, the equation (17) is a cubic equation inα with complex coefficients and the roots are in general complex quantities. The possibility of amplification in a magnetizednondegenerate semiconducting medium due to nonlinear interaction, occurs if one of the roots of the equation (17) has a positive imaginary part. We solve the above equation numerically by Newton-Raphson method following computer programming language FORTRAN-77 and run in PC, we then obtain

$$\alpha = -2.59137536 \times 10^{-3} + 6.1955094 \times 10^{-3} i$$

as one of the roots.

4. Conclusion

We see that the imaginary part of α is positive. So the amplification of acoustic wave occurs in the present situation. In our analysis the effect of deformation potential coupling with piezoelectric interaction is not considered here. The effect of magnetostatic field B_0 is applied perpendicular to the direction of the pump wave propagation. We have considered here that electron frequency ω_p is nearly equal to the pump frequency ω_0 and very much large incomparison with electron collision frequency ν (i.e $\omega_0 \approx \omega_p \gg \nu \gg \omega$). In the present analysis, our discussion is confined only in highly doped crystal Insb crystals only. The study can be extended for various cases and different materials.

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Pravat Kumar Mandalis was born in West Bengal, India and resides permanently at Kolkata, India. He completed his M.Sc degree in mathematics from Jadavpur University in the year 1987 and then completed M.Phil degree from the same University in 1991. He obtained his Ph.D degree in the topic “Numerical approach to study the nonlinear effects in theproblems of wave propagation in piezoelectric semiconductors” under the supervision of Prof. M. Gupta, Department of Mathematics, Jadavpur University, Kolkata, India in the year 2002.

At present he has been serving as an assistant professor of Mathematics permanently in Achhruram Memorial College, Purulia, West Bengal, India since 2001. He has some national and few international published papers relating to above mentioned field.

Dr. Mandal is a member of Journal of Indian Academy of Mathematics, India and also a member of International Journal of Mathematics “Acta Ciencia India”, Meerut, India and other academic and administrative bodies in the different institutes so far.