A Unified Linear Regression Approach

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***Abstract*—Despite the popularity of least-squares regression in linear regression analysis, it is well known to be sensitive to extreme values. Many researchers have developed a number of alternative estimators. Among the various estimators, least absolute deviationsisone of the most popular alternatives. Some earlier research works attempted to combine least-squares regression and least absolute deviations regression via non-linear programming approaches. Instead of using non-linear programming approaches, this paper introduces a linear programming model combining least-squares regression and least absolute deviations regression in linear regression analysis. The proposed linear programming model is computationally simpler than existing non-linear programming approaches suggested in the literature. Another advantage of the linear programming models is additional constraints and different objective coefficients can be easily added in the formulations. Moreover, the proposed linear programming model can be employed in combining forecasts.**

***Index Terms*—Least absolute deviations, least squares least-squares regression, linear programming.**

I. INTRODUCTION

Least-squares regression (LSR) is sensitive to extreme values. In the literature, many alternative estimators have been explored and among them least absolute deviations (LAD) is one of the most popular alternatives. Since the seminal paper by [1] formulated the LAD regression as a linear programming (LP) problem, numerous research works related to LAD regression have been studied. Among them, [2] introduced a quadratic-programming formulation to solve the convex combination of least-squares regression and LAD regression problems, and [3] proposed an adaptive approach in combining least squares and LAD estimators. However, both of the aforementioned approaches are formulated as non-linear programming problems. Non-linear programming problems usually are difficult to solve and can become problematical when more constraints are added to the problem.

In this paper, instead of using non-linear programming models we introduce a LP model which combines LSR and LAD objectives in linear regression analysis. This unified LP model can be served as an alternative to linear regression. Its regression solution is obtained from combining two of the most popular regression methodologies: LSR and LAD. Another application of the unified model is to apply it in combining forecasts. In the literatures, some studies [4]-[7] suggested that combined forecasts will outperform a single forecast approach.

This paper first proposes a LP model which approximates solutions of least-squares regression. The proposed LP model can produce very close approximations to the solutions of LSR. One of the advantages of using a LP model to solve a LSR problem is that additional linear constraints and different objective coefficients can be added and solved more easily than in traditional LSR problems. Then this paper proposes a second LP model, which combines LSR and LAD regression. Afterward, this paper proposes a LP model minimizes the sum of percentage deviations from the least squares value and the least absolute deviation value.

The remainder of the paper is organized as follows. In the next section, we introduce a LP model, which approximates LSR solutions. This is followed by a model discussion in Section III. In Section IV, we introduce a unified LP model, which combines two criteria: minimizing sum of the squared deviations and minimizing sum of the absolute deviations in a LP formulation. We then propose another LP model which minimizes the sum of percentage deviations in Section V. We present results of a computational example in Section VI. Finally, we provide some conclusions in the last section.

II. A LINEAR PROGRAMING MODEL

In linear regression analysis, let Y be the dependent variable; $X_1, X_2, \ldots, X_J$ be J independent variables; and $\beta_0, \beta_1, \beta_2, \ldots, \beta_J$, be the estimated coefficients. Further, let N be the number of observations; and d_i^+ and d_i^-, for $i = 1, 2, \ldots, N$, be the deviation variables. Then the LAD formulation (LP-LAD) [1] can be stated as follows:

$$Min\ \sum_{i=1}^{N}(d_i^+ + d_i^-) \tag{1}$$

$$s.t.\ y_i - \left(\beta_0 + \sum_{j=1}^{J}\beta_j x_{ij} - d_i^+ + d_i^-\right) = 0, \forall i, \tag{2}$$

where $d_i^+, d_i^- \geq 0, \forall i; \beta_0$ and β_j are unrestricted in sign, $\forall j$.

In order to approximate least-squares regression solutions, piecewise linear segments are used to represent deviations in the LAD formulation. Lam and Moy [8] also used piecewise linear segments to approximate squared values in solving classification problems in discriminant analysis. Let P be the number of piecewise linear segments used for each deviation variable, so that deviation variables in LP-LAD become:

$$d_i^+ - \sum_{k=1}^{P} d_{ik}^+ = 0, \forall i, \tag{3}$$

$$d_{ik}^+ \leq 1, \forall i, k = 1, \ldots, P-1, \tag{4}$$

$$d_i^- - \sum_{k=1}^{P} d_{ik}^- = 0, \forall i, \tag{5}$$

$$d_{ik}^- \leq 1, \forall i, k = 1, \ldots, P-1. \tag{6}$$

For deviation variables with absolute values less than one, we further split d_{i1}^+ and d_{i1}^- as follows:

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$$d_{i1}^{+} - \sum_{m=1}^{10} e_{im}^{+} = 0, \forall i, \quad (7)$$

$$e_{im}^{+} \leq 0.1, \forall (i, m), \quad (8)$$

$$d_{i1}^{-} - \sum_{m=1}^{10} e_{im}^{-} = 0, \forall i, \quad (9)$$

$$e_{im}^{-} \leq 0.1, \forall (i, m). \quad (10)$$

Substituting (7), (8), (9) and (10) into (3), (4), (5) and (6), we obtain:

$$d_{i}^{+} - \sum_{m=1}^{10} e_{im}^{+} - \sum_{k=2}^{P} d_{ik}^{+} = 0, \forall i, \quad (11)$$

$$e_{im}^{+} \leq 0.1, \forall (i, m), \quad (12)$$

$$d_{ik}^{+} \leq 1, \forall i, k = 2, \ldots, P-1, \quad (13)$$

$$d_{i}^{-} - \sum_{m=1}^{10} e_{im}^{-} - \sum_{k=2}^{P} d_{ik}^{-} = 0, \forall i, \quad (14)$$

$$e_{im}^{-} \leq 0.1, \forall (i, m), \quad (15)$$

$$d_{ik}^{-} \leq 1, \forall i, k = 2, \ldots, P-1, \quad (16)$$

Using the piecewise deviation variables, our proposed LP formulation (LP-LSR) can be formulated as follows:

$$\text{Min} \sum_{i=1}^{N} \sum_{m=1}^{10} (0.2m - 0.1)\left(e_{im}^{+} + e_{im}^{-}\right) + \sum_{i=1}^{N} \sum_{k=2}^{P} (2k-1)\left(d_{ik}^{+} + d_{ik}^{-}\right) \quad (17)$$

$$\text{s.t. } y_i - \left(\beta_0 + \sum_{j=1}^{J} \beta_j x_{ij} - \sum_{m=1}^{10} e_{im}^{+} + \sum_{m=1}^{10} e_{im}^{-} - \sum_{k=2}^{P} d_{ik}^{+} + \sum_{k=2}^{P} d_{ik}^{-}\right) = 0, \forall i, \quad (18)$$

$$e_{im}^{+} \leq 0.1, \forall (i, m), \quad (19)$$

$$d_{ik}^{+} \leq 1, \forall i, k = 2, \ldots, P-1, \quad (20)$$

$$e_{im}^{-} \leq 0.1, \forall (i, m), \quad (21)$$

$$d_{ik}^{-} \leq 1, \forall i, k = 2, \ldots, P-1, \quad (22)$$

where e_{im}^{+}, e_{im}^{-}, d_{ik}^{+}, $d_{ik}^{-} \geq 0, \forall i$; β_0 and β_j are unrestricted in sign, $\forall j$.

Notice that each d_{i1}^{+} or d_{i1}^{-} are further divided into 10 segments. Each of the 10 segments, e_{im}^{+} or e_{im}^{-}, for m = 1, 2, …, 10, has a value less than or equal to 0.1. These segments are used to approximate the squared values of those deviations that have absolute values less than one. Furthermore, when $k=P$, the deviation variables, d_{ik}^{+} and d_{ik}^{-}, are not included in constraints (21) and (22), as a result, they are unrestricted in magnitude. The next section discusses how the objective function (17) approximates sum of squared deviations in LP-LSR.

III. Model Discussion

Objective coefficients of deviation variables in LP-LSR from P=1 to P=11, are listed in Table I. In Table I, the objective coefficients increase monotonically with values of m and k, and since objective function (17) aims to minimize sum of the weighted deviations, deviation variables with smaller objective coefficients will be used as deviations in the LP-LSR solution before those with larger objective coefficients.

We now demonstrate how objective function (17) approximates the squares values of deviations, which we illustrate using the following equation:

$$(a-b)^2 = a^2 + b^2 - 2ab, a \in R, b \in R. \quad (23)$$

Let $a \in R$; then according to (23), $(a-1)^2 = a^2 + 1^2 - 2a$, or

$$a^2 = (a-1)^2 + (2a-1). \quad (24)$$

For example, in equation (24), if a = 9, then 9^2= $(9 – 1)^2$ + (2(9)-1) = 64 + 17 = 81; if a = 13.6, then 13.6^2 = $(13.6 – 1)^2$ + (2(13.6) -1) = 158.76 + 26.2 = 184.96; if a = –17 , then $(–17)^2$ = $((–17) –1)^2$+(2(–17) – 1) = $(–18)^2$ + (–35) = 289. Furthermore, if we let $q \in$ *{integer≥1}*, then from (24), q^2 = $(q-1)^2$ + $(2q-1)$. Similarly, $(q-1)^2$= $(q-2)^2$ + $(2(q-1)-1)$. Substituting the above equation for $(q-1)^2$ into the previous equation,then q^2= $(q-2)^2$ + $(2(q-1)-1)$ + $(2q-1)$. Similarly, since $(q-2)^2$= $(q-3)^2$ + $(2(q-2)-1)$, substituting the above equation for $(q-2)^2$ into the previous equation, then q^2= $(q-3)^2$ + $(2(q-2)-1)$ + $(2(q-1)-1)$ + $(2q-1)$. After substituting consecutively for q number of times, then q^2 = $(q-(q))^2$+ $(2(q-(q-1))-1)$+…+ $(2(q-2)-1)$ + $(2(q-1)-1)$ + $(2q-1)$, or q^2 = $(2(q-(q-1))-1)$+…+ $(2(q-2)-1)$ + $(2(q-1)-1)$ + $(2q-1)$, since $(q-q)^2$=0. Consequently, we obtain the following:

TABLE I: Objective Coefficients of Deviation Variables

e_{im}^{+}, e_{im}^{-}	Objective coefficient	d_{ik}^{+}, d_{ik}^{-}	Objective coefficient
m = 1	0.1	k = 2	3
m = 2	0.3	k = 3	5
m = 3	0.5	k = 4	7
m = 4	0.7	k = 5	9
m = 5	0.9	k = 6	11
m = 6	1.1	k = 7	13
m = 7	1.3	k = 8	15
m = 8	1.5	k = 9	17
m = 9	1.7	k = 10	19
m = 10	1.9	k = 11	21

$$q^2 = \left(2\left(q-(q-1)\right)-1\right) + \ldots + (2(q-2)-1) + (2(q-1)-1) + (2(q-0)-1). \quad (25)$$

For example, if q = 6, then 6^2 = (2(6-5)-1) + (2(6-4)-1) + (2(6-3)-1) + (2(6-2)-1) + (2(6-1)-1) + (2(6-0)-1) = 1 + 3 + 5 + 7 + 9 + 11 = 36. We use equation (25) to approximate square values in (17).

Let us consider cases when deviations in LP-LSR are greater than one. Observe that if the magnitude of an absolutedeviation is greater than or equal to unity, then, $\sum_{m=1}^{10} e_{im}^{+} = 1$, and sum of the objective values of these 10 deviational variables is equal to (0.1 + 0.3 + 0.5 + 0.7 + 0.9 + 1.1 + 1.3 + 1.5 + 1.7 + 1.9) × (0.1) = 1. This is why the summation term of k starts at 2 in (17). Consider the case when a deviation is equal to 7, then the objective value will be (1 + 3 + 5 + 7 + 9 + 11 + 13) × (1) = 49, which is square value of 7.

Let us consider cases when absolute deviations in LP-LSR are less than one. Consider a deviation equals 0.3, then according to constraints (17) to (20) in LP-LSR $e_{i1}^{+} = e_{i2}^{+} = e_{i3}^{+} = 0.1$. Since these three deviation variables have smaller objective coefficients than the other deviation variables in LP-LSR, they will be used as deviations in LP-LSR solutions prior to those with larger objective coefficients. Then the objective value will be equal to $0.1e_{i1}^{+} + 0.3e_{i2}^{+} + 0.5e_{i3}^{+} = 0.1(0.1) + 0.3(0.1) + 0.5(0.1) = 0.09$, which is the square value of 0.3. If the deviation is equal to 0.8, then the objective value equals (0.1 + 0.3 + 0.5 + 0.7 + 0.9 + 1.1 + 1.3 + 1.5) × (0.1) = 0.64, which is square value of 0.8. Consider the case when the deviation is 0.59, then, the objective value equals (0.1+ 0.3 + 0.5 + 0.7 + 0.9) ×(0.1) + (1.1) × (0.09) = 0.3490, which is very close to $0.59^2 = 0.3481$. Negative deviations(e_{im}^{-}) work similar to positive deviations (e_{im}^{+}).

If the deviation is a non-integer, then the approximation will not be exact, but will, however, remain very close to the square value. For instance, if the deviation is equal to 13.2, then the objective value will be (1+ 3+ 5 + … + 25) × (1) + (27) × (0.2) = 174.4, whichis close to 13.2^2=174.24. Negative deviations (d_{ik}^{-}) work similar to positive deviations (d_{ik}^{+}).

TABLE II: VALUES OF OBJECTIVE FUNCTION IN LP-LSR (P=9)

Absolute deviations (d_i^+ or d_i^-)	Value of objective function (17)	Square term of (d_i^+ or d_i^-)
0.1	(0.1)(1)(0.1) = 0.01	0.01
0.2	0.01 + (0.1)(3)(0.1) = 0.04	0.04
0.3	0.04 + (0.1)(5)(0.1) = 0.09	0.09
0.4	0.09 + (0.1)(7)(0.1) = 0.16	0.16
0.5	0.16 + (0.1)(9)(0.1) = 0.25	0.25
0.6	0.25 + (0.1)(11)(0.1) = 0.36	0.36
0.7	0.36 + (0.1)(13)(0.1) = 0.49	0.49
0.8	0.49 + (0.1)(15)(0.1) = 0.64	0.64
0.9	0.64 + (0.1)(17)(0.1) = 0.81	0.81
1.0	0.81 + (0.1)(19)(0.1) = 1.00	1.00
2	1 + (3) = 4	4
3	4 + (5) = 9	9
4	9 + (7) = 16	16
5	16 + (9) = 25	25
6	25 + (11) = 36	36
7	36 + (13) = 49	49
8	49 + (15) = 64	64
9	64 + (17) = 81	81

Table II provides a summary of the values of the objective function of LP-LSR in relation to the absolute deviations. It can be observed from Table II that, (17) closely approximatesthe squares values of the absolute deviations in LP-LSR.

Since LP-LSR is a LP model, additional constraints and different objective coefficients can be added more easily into the model than that of the least-squares regression model. For example, in the next section, we introduce a LP model that combines LSR and LAD objectives in a single LP formulation.

IV. A UNIFIED LINEAR PROGRAMMING FORMULATION

Arthanari and Dodge [2] introduced a quadraticprogramming formulation to solve the problem on convex combination of LSR and LAD regression. We also apply a convex combination of LSR and LAD regression in our next proposed formulation. However, a significant difference in our work is that we formulate the problem as a LP problem instead of a quadratic-programming problem. Let $0 \le \delta \le 1$, where δ is used to maintain the degree of minimizing absolute deviations and squared deviations in the objective function. Then our proposed model (LP- δ) can be stated as follows:

$$Min \sum_{i=1}^{N}\sum_{m=1}^{10}\big((1-\delta)(0.2m-0.1)+\delta\big)\,(e_{im}^{+} + \quad eim{-}{+}i{=}1Nk{=}2P(1{-}\delta 2k{-}1{+}\delta)dik{+}{+}dik{-} \tag{26}$$

$$\text{s.t.}\ (18), (19), (20), (21), (22),$$

where $e_{im}^{+},\ e_{im}^{-},\ d_{ik}^{+},\ d_{ik}^{-}\ \ge 0, \forall i$; β_0 and β_j are unrestricted in sign, $\forall j$.

When δ = 0, LP-δ becomes LP-LSR, which approximates least-squares regression solutions. When δ = 1, then LP-δ becomes the LAD minimization problem. Applying different values of δ to LP-δ, a spectrum of solutions to the linear regression model reflecting different degrees of minimizing least squares and least absolute deviations can be generated.

V. MINIMIZING PERCENTAGE DEVIATIONS

LP-δ requires one to provide a δ value prior to solving the problem. While it allows flexibility for decision makers to put different weights on minimizing least-squares regression and LAD regression, however, sometimes it may be difficult to determine the 'best' δ value. Consequently, in this section we introduce a LP model which does not require one to provide a priori δ value. The objective function of the LP model is to minimize the sum of percentage deviations from the least squares value (LS) and the least absolute deviation value (LAD). We call this LP model Minimum Sum of Percentage Deviations, or LP-MSPD. Prior to solving LP-MSPD, the sum of squares error, SSE(LP-LSR) is obtained from solving LP-LSR and the sum of absolute error, SAE(LP-LAD) is obtained from solving LP-LAD. Then, LP-MSPD can be stated as follows:

$$\text{Min} \sum_{i=1}^{N}\sum_{m=1}^{10}\left(\begin{matrix}\left(\dfrac{(0.2m-0.1)}{SSE(LP-LSR)}\right)+\\ \left(\dfrac{1}{SAE(LP-LAD)}\right)\end{matrix}\right)\left(\begin{matrix}e_{im}^{+}+\\ e_{im}^{-}\end{matrix}\right)+$$

$$\sum_{i=1}^{N}\sum_{k=2}^{P}\left(\begin{matrix}\left(\frac{(2k-1)}{SSE(LP-LSR)}\right)+\\ \left(\frac{1}{SAE(LP-LAD)}\right)\end{matrix}\right)(d_{ik}^{+}+d_{ik}^{-}) \tag{27}$$

$$\text{s.t.}\ (18), (19), (20), (21), (22),$$

where $e_{im}^{+},\ e_{im}^{-},\ d_{ik}^{+},\ d_{ik}^{-}\ \ge 0, \forall i$; β_0 and β_j are unrestricted in sign, $\forall j$.

The objective function (27) can be viewed as minimizing the sum of two ratios. The first ratio, $\frac{\sum_{i=1}^{N}\sum_{m=1}^{10}(0.2m-0.1)(e_{im}^{+}+e_{im}^{-})+\sum_{i=1}^{N}\sum_{k=2}^{P}(2k-1)(d_{ik}^{+}+d_{ik}^{-})}{SSE(LP-LSR)}$, actually is $\frac{SSE(LP-MSPD)}{SSE(LP-LSR)}$. The second ratio, $\frac{\sum_{i=1}^{N}\sum_{m=1}^{10}(e_{im}^{+}+e_{im}^{-})+\sum_{i=1}^{N}\sum_{k=2}^{P}(d_{ik}^{+}+d_{ik}^{-})}{SAE(LP-LAD)}$, actually is

$\frac{SAE(LP-MSPD)}{SAE(LP-LAD)}$. Each ratio has a lower bound equal to one. When one ratio is greater than one, it implies that the solution deviates from the minimum value, and the ratio represents the deviation percentage. For example, since the percentage deviation from SSE(LP-LSR) is equal to $\frac{SSE(LP-MSPD)-SSE(LP-LSR)}{SSE(LP-LSR)} = \frac{SSE(LP-MSPD)}{SSE(LP-LSR)} - 1$, consequently $\frac{SSE(LP-MSPD)}{SSE(LP-LSR)}$ measures the percentage deviation from SSE(LP-LSR).

VI. A Computational Example

The first data set, containing values of expenditure on education, is from [9]. The data set contains values representing expenditure on education (Y), personal income (X_1), youth percentage (X_2) and urban percentage (X_3) in 50 states of the United States of America. In the linear regression model, the expenditure on education is the dependent variable while the other three variables are independent variables, $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + e$, where e is error term. We run least-squares regression, LP-LSR, LP-LAD, and LP-MSPD using the dataset, and the resulting *SSE* and *SAE* are reported in Table III. We then solve LP-δ for various values of δ, and the corresponding values of *SSE* and *SAE* are reported in Table IV.

TABLE III: Regression Coefficients by Different Methods

	Regression coefficients			
	Least-Squares Regression	LP-LSR	LP-MSPD	LP-LAD
β_0	-556.8200	-556.2367	-433.1811	-358.0441
β_1	0.07228	0.07226	0.062998	0.06474
β_2	15.52700	15.51776	12.484118	9.65902
β_3	-0.03499	-0.03697	0.231198	0.28535
SSE	75351.4	75351.5	77842.8	84567.9
SAE	1586.83	1586.92	1547.16	1528.21

TABLE IV: SSE and SAE Obtained from LP-δ

δ	0	0.9	0.99	0.995	0.998	1
SSE	75351	75411	77639	79692	80956	84568
SAE	1587	1577	1548	1538	1535	1528

According to the computational results obtained above, we observe that the model LP-LSR is quite accurate in approximating solutions obtained from least-squares regression. Their *SSE* measures are almost identical. We can conclude that LP-LSR can produce very close approximations to least-squares regression solutions. Applying different values of δ to LP-δ, one can generate a spectrum of solutions to the linear regression model reflecting trade-offs between LSR and LAD in linear regression analysis. Among this spectrum of solutions, LP-MSPD can determine a solution which minimizes the sum of the percentage deviations from the minimum least-squares regression and LAD regression solutions.

VII. Conclusion

This paper introduces three LP models. The first model, LP-LSR, utilizes piecewise LP to approximate square errors in least-squares regression. The results of a computational example confirm that LP-LSR can produce good approximations to least squares solutions. Furthermore, LP-LSR can add additional constraints and use different objective coefficients in the formulation. This is an advantage over the traditional least-squares regression method. For example, in the second LP model, LP-δ, which combines LSR and LAD objectives in linear regression analysis, is a modified formulation of LP-LSR. Its objective function is to minimize the convex combination of LSR and LAD in linear regression. If the δ weight equals one, the LP formulation will yield a LAD solution; however, if the δ weight equals zero, it will yield a LSR solution to an approximation. Applying different values of δ where $0 \leq \delta \leq 1$, to LP-δ, one can generate a spectrum of solutions to the linear regression model reflecting different degrees of least squares and least absolute deviations characteristics. Furthermore, the third LP model, LP-MSPD determines a solution which minimizes the percentage deviations from the solutions of LSR and LAD regressions. An advantage of LP-MSPD is it does not require any input of subjective weights. Both LP-δ and LP-MSPD can be used as alternative approaches to least-squares regression. In future research works, it may be interesting to examine regression solutions obtained from combining the two most popular regression methodologies namely LSR and LAD regression. In addition, the proposed unified LP models can be employed in combining forecasts. Other objectives like minimize maximum deviation can be added to the combining forecasts models.

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