Global Exponential Stability of Impulsive Cohen-Grossberg-Type BAM Neural Networks with Time-Varying and Distributed Delays

Haydar Akça, Jamal Benbourenane, and Valéry Covachev

***Abstract*—The purpose of this paper is to investigate the global exponential stability of a class of impulsive bidirectional associative memories (BAM) neural networks that possesses Cohen-Grossberg dynamics. By constructing and using some inequality techniques and a fixed point theorem sufficient conditions are obtained to ensure the existence and global exponential stability of the solutions for impulsive Cohen-Grossberg neural networks with time delays and distributed delays.**

***Index Terms*—Cohen-grossberg neural networks, impulses, globally exponential stability, time delay, distributed delay.**

I. Introduction

The Cohen-Grossberg neural network models proposed by Cohen and Grossberg [1] have been widely applied to various problems in scientific and engineering fields [1]. BAM neural networks are useful in many fields such as pattern recognition and automatic control, the stability properties and applications of BAM's models have been researched by many scholars see [2]-[6] and references are given therein. Most widely used neural networks are neither of purely continuous-time nor of purely discrete-time type [7]-[10]. Also there has been a new category of neural networks called impulsive neural networks, which display a combination of characteristics of both continuous-time and discrete-time systems. The bidirectional associative memory (BAM) neural network model known as an extension of the unidirectional auto-associator of Hopfield [11], and references cited therein. The method was introduced first by Kosko [12].

To the best of our knowledge, there are few results on the stability of impulsive Cohen-Grossberg neural networks with both time-varying and distributed delays [9]-[10]. We study the stability problem of BAM impulsive Cohen-Grossberg neural networks with time-varying and distributed delays and derived to guarantee the global asymptotic stability of the solution by using some inequality techniques, fixed point theorem and some analysis techniques. We consider impulsive Cohen-Grossberg-type BAM neural networks with time-varying and distributed delays which are described by the following functional integro-differential equations:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = a_i(x_i(t))\left[-\alpha_i(x_i(t)) + \sum\limits_{j=1}^{m} p_{ji} f_j(y_j(t-\tau_{ji}(t))) \right. \\ \left. + \sum\limits_{j=1}^{m} \int_0^{\infty} r_{ji}(s) f_j\left(y_j(t-s)\right) ds + I_i \right], \\ \qquad t>0,\ t \neq t_k,\ i=1,2,\dots,n, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = \bar{I}_k(x_i(t_k)), \\ \qquad i=1,2,\dots,n,\ k=1,2,\dots, \\ \dfrac{dy_j(t)}{dt} = b_j(y_j(t))\left[-\beta_j(y_j(t)) + \sum\limits_{i=1}^{n} q_{ij} g_i(x_i(t-\sigma_{ij}(t))) \right. \\ \left. + \sum\limits_{i=1}^{n} \int_0^{\infty} s_{ij}(s) g_i(x_i(t-s))\, ds + J_j \right], \\ \qquad t>0,\ t \neq t_k,\ j=1,2,\dots,m, \\ \Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-) = \bar{J}_k(y_j(t_k)), \\ \qquad j=1,2,\dots,m,\ k=1,2,\dots, \end{cases} \tag{1}$$

$$\begin{cases} x_i(s) = \varphi(s), & s \in (-\infty, 0],\ i=1,2,\dots,n, \\ y_j(s) = \phi(s), & s \in (-\infty, 0],\ j=1,2,\dots,m, \end{cases} \tag{2}$$

where n and m correspond to the number of neurons in the $X-$ layer and $Y-$layer, respectively, $\varphi(s)$ and $\phi(s)$ are bounded continuous functions on $(-\infty,0]$, and $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$ are the impulses at the moments t_k and $0 < t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{t\to\infty} t_k = +\infty$, and $x_i(t), y_j(t)$ are the activations of the i th neuron in F_X and the j th neuron in F_Y, respectively. The functions a_i, b_j are abstract amplification functions and α_i, β_j are self-excitation rate functions. $\tau_{ji}(t)$, $\sigma_{ij}(t)$ are positive time delays corresponding to the finite speed of the axonal signal transmission. f_j, g_i present the activation functions of the neuron. The functions p_{ji}, q_{ij}, $r_{ji}(t)$, $s_{ij}(t)$ are the connection weights, they denote the strengths of connectivity between the cell j in F_Y and cell i in F_X at time $t, t-\tau_{ji}(t), t-\sigma_{ij}(t)$, and I_i, J_j denote the i th and j th component of an external input source introduced from

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outside the network to the cell i in F_X and the cell j in F_Y, respectively (see [8] and references cited therein). As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution

$$t \mapsto (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T$$

We assume that

$$x_i(t_k) = x_i(t_k - 0),\ \ y_j(t_k) = y_j(t_k - 0),$$

$$x_i'(t_k) = x_i'(t_k - 0),\ \ y_j'(t_k) = y_j'(t_k - 0),$$

where $i = 1, 2, \ldots, n,\ \ j = 1, 2, \ldots, m$. The vector function

$$(x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T$$

is said to be a solution of the system (1)–(1)(2).

Throughout this paper we assume the following conditions are satisfied:

H1: $a_i(x_i) > 0,\ b_j(y_j) > 0$ and a_i, b_j are bounded, that is, there exist positive constants $\underline{a}_i$, $\overline{a}_i$, and $\underline{b}_j$, $\overline{b}_j$ such that

$$\underline{a}_i \le a_i(x_i) \le \overline{a}_i,\ \ i=1,2,\ldots,n,$$
$$\overline{a}=max\{\overline{a}_i\},\ \ \underline{a}=min\{\underline{a}_i\},$$

$$\underline{b}_j \le b_j(y_j) \le \overline{b}_j,\ \ j=1,2,\ldots,m,$$
$$\overline{b}=max\{\overline{b}_j\},\ \ \underline{b}=min\{\underline{b}_j\},$$
$$a=max\{\overline{a},\overline{b}\},\quad b=min\{\underline{a},\underline{b}\}.$$

H2: The delayed feedback functions r_{ji}, s_{ij} are real-valued nonnegative continuous functions defined on $[0, \infty)$ with

$$\int_0^\infty r_{ji}(s)\, ds \le r_{ji},\quad \int_0^\infty s_{ij}(s)\, ds \le s_{ij}.$$

Here r_{ji}, s_{ij} are nonnegative constants. There exists a positive constant number, such that

$$\int_0^\infty e^{\varepsilon s} r_{ji}(s)\, ds<\infty, \int_0^\infty e^{\varepsilon s} s_{ij}(s)\, ds<\infty$$

and there must be constants λ_1, λ_2 satisfying

$$\left\| \int_0^\infty R(s)\, ds \right\|_2 \le \left\| \int_0^\infty e^{\varepsilon s} R(s)\, ds \right\|_2 = \lambda_1,$$

$$\left\| \int_0^\infty S(s)\, ds \right\|_2 \le \left\| \int_0^\infty e^{\varepsilon s} S(s)\, ds \right\|_2 = \lambda_2,$$

where $R(s)$ and $S(s)$ are matrix-valued functions with entries respectively $r_{ji}(s)$ and $s_{ij}(s)$ (see below).

H3: There exist positive constant numbers L_j, M_i such that

$$|f_j(u) - f_j(v)| \le L_j |u - v|,\quad j = 1, 2, \ldots, m,$$

$$|g_i(u) - g_i(v)| \le M_i |u - v|,\quad i = 1, 2, \ldots, n,$$

for any $u, v \in \mathbb{R}$ and there exist positive constants γ_i, ξ_j such that

$$\frac{\alpha_i(u) - \alpha_i(v)}{u - v} \ge \gamma_i > 0,\quad \frac{\beta_j(u) - \beta_j(v)}{u - v} \ge \xi_j > 0$$

for any $u, v \in \mathbb{R},\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m$ and $u \ne v$.

H4: The impulsive operators

$\overline{I}_k(x(t_k)),\ \overline{J}_k(y(t_k))$ satisfy [10]

$$\overline{I}_k(x_i(t_k)) = -l_1(x_i(t_k) - x_i^*),\quad 0 < l_1 < 2,\ \ i = 1, 2, \ldots, n,$$

$$\overline{J}_k(y_j(t_k)) = -l_2(y_j(t_k) - y_j^*),\quad 0 < l_2 < 2,\ \ j = 1, 2, \ldots, m.$$

For the sake of convenience, we can rewrite system (1) in the form

$$\begin{cases} \dfrac{dx(t)}{dt} = \mathcal{A}(x(t))\Big(-A(x(t)) + Pf(y(t)) + \int_0^\infty R(s) f(y(t-s))\, ds + I\Big), \\ \quad where \quad t > 0,\ t \ne t_k, \\ \Delta x(t_k) = \overline{I}_k(x(t_k)),\quad k = 1, 2, \ldots, \\ \dfrac{dy(t)}{dt} = \mathcal{B}(y(t))\Big(-B(y(t)) + Qg(x(t)) + \int_0^\infty S(s) g(x(t-s))\, ds + J\Big), \\ \quad where \quad t > 0,\ t \ne t_k, \\ \Delta y(t_k) = \overline{J}_k(y_k(t_k)),\quad k = 1, 2, \ldots \end{cases} \tag{3}$$

where

$$x(t) = col\{x_i(t)\},\quad y(t) = col\{y_j(t)\},$$
$$\mathcal{A}(x) = diag\{a_i(x_i)\},$$

$$\mathcal{B}(y) = diag\{b_j(y_j)\},\quad A(x) = diag\{\alpha_i(x_i)\},$$
$$B(y) = diag\{\beta_j(y_j)\},$$

$$P = (p_{ji})_{m\times n},\quad R(s) = (r_{ji}(s))_{m\times n},$$
$$f(t) = col\{f_j(t)\},\quad I = col\{I_i\},$$

$$Q = (q_{ij})_{n\times m},\quad S(s) = (s_{ij}(s))_{n\times m},$$
$$g(t) = col\{g_i(t)\},\quad J = col\{J_j\}.$$

Assume that system (3) has a unique equilibrium point (x^*, y^*), then let

$$u(t) = x(t) - x^*,\ \ v(t) = y(t) - y^*,\ \ \widetilde{\mathcal{A}}(u) = \mathcal{A}(u + x^*),$$

$$\widetilde{\mathcal{B}}(v = \mathcal{B}(v + y^*),$$

$$\tilde{A}(u) = A(u + x^*) - A(x^*),\quad \tilde{B}(v) = B(v + y^*) - B(y^*),$$

$$\tilde{f}(v) = f(v + y^*) - f(y^*),$$

$$\tilde{g}(u) = g(u + x^*) - g(x^*),$$
$$\Psi(t) = \varphi(t) - x^*,\ \Phi(t) = \phi(t) - y^*,$$

then system (3) can be written as

$$\begin{cases} \frac{du(t)}{dt} = \widetilde{\mathcal{A}}(u(t))\Big(-\tilde{A}(u(t)) + P\tilde{f}(v(t)) + \int_0^\infty R(s)\tilde{f}(v(t-s))\, ds\Big), \\ \quad where \quad t>0,\ t \neq t_k, \\ \Delta u(t_k) = -l_1 u(t_k), \quad k=1,2,\ldots, \\ \frac{dv(t)}{dt} = \widetilde{\mathcal{B}}(v(t))\Big(-\tilde{B}(v(t)) + Q\tilde{g}(u(t)) + \int_0^\infty S(s)\tilde{g}(u(t-s))\, ds\Big), \\ \quad where \quad t>0,\ t \neq t_k, \\ \Delta v(t_k) = -l_2 v(t_k), \quad k=1,2,\ldots \end{cases} \tag{4}$$

The initial conditions associated with system (4) can be defined as

$$u(s) = \Psi(s), \quad v(s) = \Phi(s), \quad s \in (-\infty, 0].$$

For

$$u=(u_1, u_2, \ldots, u_{n+m})^T \in \mathbb{R}^{m+n}, \ \phi: (-\infty, 0] \to \mathbb{R}^{m+n}$$

we define norms by

$$\| u \| = \sum_{k=1}^{n+m} |u_k|,$$
$$\| \phi \| = \sup_{s\in(-\infty,0]} \| \phi(s) \|.$$

Let us denote

$$\tau = \max_{1\le i\le n, 1\le j\le m}\{\tau_{ji}(t), \sigma_{ij}(t)\}$$

and let $\varphi_i(\cdot), \psi_j(\cdot)$ be real-valued continuous functions defined on $(-\infty, 0]$. The model introduced (1)–(1)(2) is studied [7], [8], [10] with delays and impulses.

II. Main Results

If $(x_1^*, x_2^*, \ldots x_n^*, y_1^*, y_2, \ldots, y_m^*)^T$ is an equilibrium point of system (1), then it satisfies

$$\begin{cases} A(x^*) = Pf(y^*) + \int_0^\infty R(s)\, ds\ f(y^*) + I, \\ \\ B(y^*) = Qg(x^*) + \int_0^\infty S(s)\, ds\ g(x^*) + J, \end{cases} \tag{5}$$

where the impulses $\bar{I}_k(\cdot),\ \bar{J}_k(\cdot)$ satisfy

$$\bar{I}_k(x^*) = 0, \ \bar{J}_k(y^*) = 0.$$

The equilibrium point (x^*, y^*) of the system (3) is said to be globally exponentially stable, if there exist constants $\varepsilon > 0$ and $E \ge 1$ such that

$$\| x(t) - x^* \|_2 + \| y(t) - y^* \|_2 \le E(\| \varphi - x^* \| + \| \phi - y^* \|)e^{-\varepsilon t}$$

for all $t > 0$, and $x(t), y(t)$ is any solution of the system (3) with initial values $(\varphi(s), \phi(s))$.

Theorem 1 [10] Under the assumptions $H1, H2$ and $H3$ the solutions of the system (4) are uniformly bounded.

Proof. For $t \neq t_k$, on the first partition interval $0 < t < t_1$ there exists a sufficiently large number M such that if

$$\| \Psi \| < \frac{M}{2}, \quad \| \Phi \| < \frac{M}{2},$$

then

$$\| u(t) \| + \| v(t) \| < M.$$

Then without loss of generality, integrating system (4) on the interval $[0, t_1]$ and using the assumptions $H1 - H4$ we get

$$\begin{aligned} \| u(t_1) \| + \| v(t_1) \| &\le e^{-a_0 t_1} \| \Psi \| \\ &+ \int_0^{t_1} e^{-a_0(t_1-\theta)} \| P \| \xi \| v(\theta) \| d\theta \\ &+ \int_0^{t_1} e^{-a_0(t_1-\theta)} \xi \| \int_0^\infty R(s) v(\theta - s) \| d\theta \\ &+ e^{-b_0 t_1} \| \Phi \| + \int_0^{t_1} e^{-b_0(t_1-\theta)} \| Q \| \eta \| u(\theta \| d\theta \\ &+ \int_0^{t_1} e^{-b_0(t_1-\theta)} \eta \| \int_0^\infty S(s) u(\theta - s) \| d\theta \\ &\le e^{-a_0 t_1} \frac{M}{2} + \int_0^{t_1} e^{-a_0(t_1-\theta)} \| P \| \xi \frac{M}{2} d\theta \\ &+ \int_0^{t_1} e^{-a_0(t_1-\theta)} \xi \| \int_0^\infty e^{\varepsilon s} R(s)\, ds \| \frac{M}{2} d\theta \\ &+ e^{-b_0 t_1} \frac{M}{2} + \int_0^{t_1} e^{-b_0(t_1-\theta)} \| Q \| \eta \frac{M}{2} d\theta \\ &+ \int_0^{t_1} e^{-b_0(t_1-\theta)} \eta \| \int_0^\infty e^{\varepsilon s} S(s)\, ds \| \frac{M}{2} d\theta. \end{aligned} \tag{6}$$

Let

$$max\{\lambda_1, \lambda_2\} = \lambda, \ max\{\| P \|, \| Q \|\} = \Gamma, \ max\{\xi, \eta\} = \zeta$$

and $a_0, b_0 > 1$ and $\Xi > 0$ is a positive constant with the assumptions $H1 - H4$ and after simplification of the system (6), we get

$$\begin{aligned} &\| u(t_1) \| + \| v(t_1) \| \le \\ &e^{-a_0 t_1} \frac{M}{2} + [\frac{1 - e^{-a_0 t_1}}{a_0} + \frac{1 - e^{-a_0 t_1}}{a_0}]\Gamma \lambda \zeta \frac{M}{2} \\ &+ e^{-b_0 t_1} \frac{M}{2} + [\frac{1 - e^{-b_0 t_1}}{b_0} + \frac{1 - e^{-b_0 t_1}}{b_0}]\Gamma \lambda \zeta \frac{M}{2} \\ &= \left[\frac{e^{-a_0 t_1} + e^{-b_0 t_1}}{2} + \left(\frac{1 - e^{-a_0 t_1}}{a_0} + \frac{1 - e^{-b_0 t_1}}{b_0} \right) \Gamma \lambda \zeta \right] M \\ &= \Gamma \lambda \zeta \frac{M}{2} \left[e^{-a_0 t_1} + e^{-b_0 t_1} + 1 - e^{-a_0 t_1} + 1 - e^{-b_0 t_1} \right] \\ &= \Gamma \lambda \zeta M = \Xi \end{aligned}$$

Repeating the above procedure on the successive interval we can easily conclude that the solutions of the system (4) are uniformly bounded. The proof is completed.

Theorem 2. [10] Assume that $H1-H4$ are satisfied, then the system (3) has a unique equilibrium point which is a solution of the system (5).

Proof. We proved in Theorem 1 that all solutions of system (4) are bounded, that means for any initial value, if

$$\bar{I}_k(x_i^*) = 0,\ \bar{J}_k(y_j^*) = 0,\ i=1,2,\dots,n,\ j=1,2,\dots,m,$$

then system (3) has a unique equilibrium point. Then we have

$$\begin{aligned}&\left\|Pf(y(t))+\int_0^\infty R(s)f(y(t-s))\,ds+I\right\|\\ &\le \|P\|\,\xi\,\|y(t)\|+\xi\left\|\int_0^\infty R(s)y(t-s)\,ds\right\|+\|I\|\\ &<\|P\|\,\xi\,\|y(t)\|+\xi\,\|y(t)\|\,\lambda_1+\|I\|\\ &<\|y(t)\|(\Gamma\zeta+\zeta\lambda)+\|I\|\\ &\le \pi_1.\end{aligned} \tag{7}$$

Similarly we can have

$$\begin{aligned}&\left\|Qg(x(t))+\int_0^\infty S(s)g(x(t-s))\,ds+J\right\|\\ &\le \|Q\|\,\eta\,\|x(t)\|+\eta\left\|\int_0^\infty S(s)x(t-s)\,ds\right\|+\|J\|\\ &<\|Q\|\,\eta\,\|x(t)\|+\eta\,\|x(t)\|\,\lambda_2+\|J\|\\ &<\|x(t)\|(\Gamma\zeta+\zeta\lambda)+\|J\|\le\pi_2.\end{aligned} \tag{8}$$

Let us define $T(t)=A(x(t))$ and $W(t)=B(y(t))$ in such a way that

$$\begin{aligned}T(t)&=Pf(B^{-1}(W(t)))+\int_0^\infty R(\tau)f(B^{-1}(W(\tau)))\,d\tau+I,\\ W(t)&=Qg(A^{-1}(T(t)))+\int_0^\infty S(\tau)g(A^{-1}(T(\tau)))\,d\tau+J.\end{aligned} \tag{9}$$

The systems of inequalities (7)–(7)(8) suggests us to define a set $\Omega\subset\mathbb{R}^{n+m}$ by

$$\Omega=\{(T,W),\ such\ \ that\ \ \|T\|\le\pi_1,\|W\|\le\pi_2\}.$$

If $(T,W),(\bar{T},\bar{W})$ are any two different points of Ω, then

$$\begin{aligned}&\|F(T,W)-F(\bar{T},\bar{W})\|\\ &=\|P(f(B^{-1}(W(t)))-f(B^{-1}(\bar{W}(t))))\\ &+\int_0^\infty R(\tau)f(B^{-1}(W(\tau)))\,d\tau\\ &-\int_0^\infty R(\tau)f(B^{-1}(\bar{W}(\tau)))\,d\tau\|\\ &+\|Q(g(A^{-1}(T(t)))-Qg(A^{-1}(\bar{T}(t))))\\ &+\int_0^\infty S(\tau)g(A^{-1}(T(\tau)))\,d\tau\\ &-\int_0^\infty S(\tau)g(A^{-1}\bar{T}(\tau))\,d\tau\|\\ &\le\|P\|\,\|f(B^{-1}(W(t)))-f(B^{-1}(\bar{W}(t)))\|\\ &+\left\|\int_0^\infty R(\tau)(f(B^{-1}(\bar{W}(\tau)))-f(B^{-1}(W(\tau))))\,d\tau\right\|\\ &+\|Q\|\,\|g(A^{-1}(T(t)))-g(A^{-1}(\bar{T}(t)))\|\\ &+\left\|\int_0^\infty S(\tau)(g(A^{-1}(\bar{T}(\tau)))-g(A^{-1}(T(\tau))))\,d\tau\right\|.\end{aligned} \tag{10}$$

Using the inequalities (7)–(7)(8) it can be easily concluded that

$$\begin{aligned}&\|F(T,W)-F(\bar{T},\bar{W})\|\\ &\le\Gamma\zeta\,\|B^{-1}\|\,\|W(t)-\bar{W}(t)\|+\lambda\zeta\,\|B^{-1}\|\,\|\bar{W}(t)-W(t)\|\\ &+\Gamma\zeta\,\|A^{-1}\|\,\|T(t)-\bar{T}(t)\|\,\lambda\zeta\,\|A^{-1}\|\,\|\bar{T}(t)-T(t)\|\\ &=\zeta(\Gamma-\lambda)(\|B^{-1}\|\,\|W(t)-\bar{W}(t)\|+\|A^{-1}\|\,\|T(t)-\bar{T}(t)\|)\\ &\le\gamma(\|W(t)-\bar{W}(t)\|+\|T(t)-\bar{T}(t)\|)=\gamma\,\|(W,T)-(\bar{W},\bar{T})\|\end{aligned} \tag{11}$$

Provided that $0<\gamma<1$ where

$$\gamma=max\{\zeta(\Gamma-\lambda)\|B^{-1}\|,\zeta(\Gamma-\lambda)\|A^{-1}\|\}.$$

Because of the contraction mapping principle the mapping F has a unique fixed point (W^*,T^*) and this result completes the proof.

Theorem 3. Suppose that the assumptions $H1-H4$ are satisfied, then the equilibrium point of the system (4) is globally exponentially stable.

Proof. From $H1-H2$ define $0<\varepsilon<a$ and

$$\Theta=max\{\frac{\zeta(\Gamma-\lambda)\|B^{-1}\|}{a-\varepsilon},\frac{\zeta(\Gamma-\lambda)\|A^{-1}\|}{a-\varepsilon}\}<1 \tag{12}$$

To prove the assertion for $t \neq t_k$ for any $\Psi, \Phi \in C$ and $\beta > 1$ it can be shown that the following inequality holds:

$$\| u(t) \| + \| v(t) \| < \beta E(\| \Psi \| + \| \Phi \|) e^{-\varepsilon t} \tag{13}$$

for all $t>0$ and $t \neq t_k$

Assume that (13) is not true and there must be some $t_2 > 0$ such that

$$\| u(t_2) \| + \| v(t_2) \| = \beta E(\| \Psi \| + \| \Phi \|) e^{-\varepsilon t_2} \tag{14}$$

for $t_2 > 0$, and $t_2 \neq t_k$.

Following the similar procedure presented in the reference [10], one can easily reach a contradiction and that result completes the proof of the theorem.

III. Conclusion

We study the stability problem of BAM impulsive Cohen-Grossberg neural networks with time-varying and distributed delays by constructing and using some inequality techniques and fixed point theorem. We obtained sufficient conditions to ensure the existence and global exponential stability of the solutions for impulsive Cohen-Grossberg neural networks with time-varying and distributed delays.

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