# The Interfacial Layer Effect on Thermal Conductivity of Nano-Colloidal Dispersions

Pasrija R. and Srivastava S.

***Abstract*—A mathematical model has been proposed for calculating the effective Thermal Conductivity of nano colloidal dispersions by incorporating a thermal conductivity profile for the interfacial layer. The present model has been successfully applied to the colloidal dispersions containing rod shaped nanoparticles as well as those containing spherical nanoparticles. The results have also been compared with other existing models.**

***Index Terms*—Thermal conductivity, nanofluids, colloidal dispersion and interfacial layer.**

## I. Introduction

About a decade ago Choi at Argonne National Laboratory coined a word 'nanofluid' for fluid containing dispersed nanoparticles. Since then nanofluids have attracted a worldwide attention from researchers across the world due to their wide range of potential applications. They are a two phase system with a sold phase and a liquid phase. These find applications in the field of electronics, transportation, space and defense, drug delivery and the areas where energy saving and cooling is the key issue [1]. The major reasons attributed are to their showing intensified heat transfer properties than the conventional solid liquid suspensions are the higher specific surface area, higher stability of dispersion, reduced pumping power and lesser particle clogging. Due to the above stated reasons, some of the thermophysical properties of nanofluids have been found to be enhanced in comparison to the conventionally used base fluids like water, ethylene glycol etc. These include thermal conductivity, thermal diffusivity, viscosity and the heat transfer coefficients. These enhanced properties have been the main driving force for instigating interest amongst the research community towards these nano colloidal dispersions.

There has been a large scale research to study the anomalous enhancement in effective thermal conductivity of nanofluids both theoretically as well as experimentally [2]-[8]. The most commonly employed experimental methods have been Temperature oscillation technique [4], [5] and a Transient hot-wire method [6]-[9] to determine thermal conductivity of the nanofluids. All the experiments have reported unusually higher increase in the effective thermal conductivity values of the nanofluids. About 4% volume fraction of $Al_2O_3$ nanoparticles when dispersed in water shows an increase of 30% in thermal conductivity as reported by Masuda *et al.* [10]. Similarly a 4 volume percent of CuO nanoparticles with average diameter of 35nm in ethylene glycol dispersion show a 20% increase in thermal conductivity of the dispersion obtained as shown by Eastman *et al.* [11]. More interestingly, above 100% enhancement in thermal conductivity in case of CNT based nanofluids. There has been a wide variety of experimental works with a dispersed range of values for different nanoparticle volume fraction which has tempted the research community to look for theoretical physical mechanisms or models that can explain such an anomalous enhancement in effective thermal conductivity of dispersions with nano-sized particles.

A number of theoretical mechanisms proposed so far [12], [13] have been unable to account for one or the other important factors affecting the effective thermal conductivity of a nanofluid These include thermal conductivity of the fluid medium, thermal conductivity of the nanoparticle dispersed, nanoparticle size and its volume fraction, formation of layer around the nanoparticle etc. Yu and Choi [14] have improved the Maxwell model to incorporate the concept of interfacial layer as a more ordered layer is found to have major impact on the nanofluids thermal conductivity. But this model remained silent about the interaction between nanoparticles and the fluid medium. Many interfacial layer models have come up so far which presented empirical formulae for describing the profile of thermal conductivity of the Interfacial nanolayer. Some of these include Tillman *et al.* [15], Nsofor *et al.* [16] and Xie *et al.* [17] among others.

In the present theoretical formulation, a general empirical form of thermal conductivity profile inside the interfacial layer has been proposed. This is then used to develop a model for calculating the effective thermal conductivity of the colloidal dispersion containing nanoparticles. The parameters involved in the present study are the effect of Nanoparticle size, the volume fraction of the nanoparticles, thermal conductivity of the base fluid and the thermal conductivity of the nanoparticle itself.

## II. Theoretical Formulation

### *A. The Nanoparticle Structure and the Thermal Conductivity Profile*

The equivalent nanoparticle consists of nanoparticle and interfacial nanolayer consisting of the orderly arrangement of base fluid molecules around the nanoparticles. This nanoparticle is dispersed in the base fluid forming a colloidal suspension called 'nanofluid' or nano colloidal dispersion. This orderly arrangement is expected to have intermediate thermal conductivity profile between the base fluid and the dispersant nanoparticle. The nanoparticle structure is

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R. Pasrija is with DAV College, Abohar (Punjab), India (e-mail: goodluckritu@gmail.com).

S. Srivastava is with Department of Physics, Panjab University, Chandigarh, India (e-mail: Sunita@pu.ac.in).

depicted in the Fig. 1.

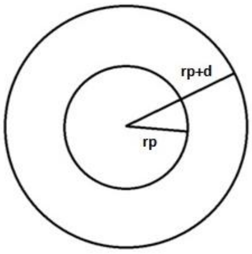


Fig. 1. The nanoparticle structure.

The thermal conductivity of the nanolayer is supposed to vary with distance r from inner radius $r_p$ of the nanoparticle to the entire thickness of the interfacial layer around the nanoparticle before it becomes equal to the $k_f$ at the interface. Here, the empirical formulation is introduced based on the assumptions that thermal conductivity equals the thermal conductivity of the nanoparticle $k_p$ at the inner interface and that at the outer interface it is equal to base fluid's thermal conductivity i.e $k_f$ so that

$$k(r)=\begin{cases} k_p & size \le r_p \\ f(r) & r_p \le r \le r_p + d \\ k_f & r_p + d \le size \end{cases} \quad (1)$$

Size in the equation (1) refers to radius of the nanoparticle Here, the following tangential variation of $f(r)$ is proposed

$$f(r) = k_p + \frac{(k_f - k_p)}{\tanh m} \tanh\left( m \frac{(r - r_p)}{d} \right) \quad (2)$$

where, $m$ is a real positive integer. The corresponding thermal conductivity profile within the interfacial layer is displayed in Fig. 2.

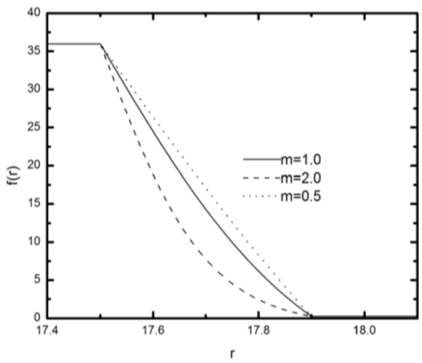


Fig. 2. The thermal conductivity profile versus radius of the nanoparticle.

The above graph shows the thermal conductivity profile of the Interfacial nanolayer for $Al_2O_3$ nanoparticles in Ethylene Glycol with size of 35nm. The graph is plotted for different values of m ($m$=1, 2, 0.5). The value of m as decreased from 1.0 shift towards a linear profile while m if increased from 1.0 show a fast decrease moving towards being a step function profile. In the present model the value of m is taken as 1 in each type of nanofluid system.

### B. Dispersions with Spherical Nanoparticles

It is assumed that the $Al_2O_3$ and CuO nanoparticles are spherical when dispersed in base fluids, ethylene Glycol and water respectively. The thermal conductivity of the interfacial layer is a continuous function of r within the nanolayer. The thermal resistance offered by the nanolayer is given by

$$R_L = \frac{1}{4\pi k_l} \left( \frac{1}{r_p} - \frac{1}{r_p + d} \right) \quad (3)$$

Also the resistance is related to the thermal conductivity distribution between $r_p$ and $r_p$+$d$ as

$$R_L = \int_{rp}^{rp+d} \frac{dr}{4\pi r^2 f(r)} \quad (4)$$

Combining the preceding two equations, we get

$$k_l = \frac{d}{r_p (r_p + d) \int_{r_p}^{r_p+d} \frac{dr}{r^2 f(r)}} \quad (5)$$

### C. Dispersions with Rod Shaped Nanoparticle

In case the nanoparticles are assumed to be cylindrical like rod shaped nanoparticles then the thermal resistance offered by the nanolayer will be given by

$$R_L = \frac{1}{\pi K_l} \left[ \frac{l}{r_p^2} - \frac{(l+2d)}{(r_p+d)^2} \right] \quad (6)$$

where $l$ $X$ $r_p$ are the dimensions of the nanoparticle

And the resistance is related to the thermal conductivity distribution between $r_p$ and $r_p$+$d$ as

$$R_L = \int_{r_p}^{r_p+d} \frac{dr}{2\pi r_p (r_p + l) f(r)} \quad (7)$$

Using the two preceding equations we get

$$k_l = \frac{\left[ \frac{l}{r_p^2} - \frac{(l+2d)}{(r_p+d)^2} \right]}{\int_{r_p}^{r_p+d} \frac{dr}{2 r_p (r_p + l) f(r)}} \quad (8)$$

### D. The Expression for the Effective Thermal Conductivity

The effective thermal conductivity is defined using Fourier's law of heat conduction as

$$\langle \vec{q} \rangle = -k_{eff} \langle \nabla T \rangle \quad (9)$$

Here $<\vec{q}>$ is the average heat flux and $\nabla T$ is the temperature gradient vector. The mixture is assumed to be statistically homogeneous and isotropic. The effective thermal conductivity $k_{eff}$ of the nanoparticle containing dispersion can be obtained by combining the contributions from the fluid, the nanoparticles and the nanolayer as

$$<\vec{q}> = -k_f <\nabla T> + <\vec{q}_p> + <\vec{q}_l> \quad (10)$$

Lu and Song [18] used equilibrium hard sphere fluid model and determined the effective thermal conductivity by considering the inclusion which is the dispersed nanoparticle to be coated and debonded for two interacting particles. They considered temperature field to be linear and applied the ambient temperature gradient as unity along a specified direction to the system. The effective thermal conductivity is computed to be given by

$$k_{eff} = k_f \left[ 1 + 3F\phi_T + \frac{3F^2\phi_T^2}{1 - F\phi_T} \right] \quad (11)$$

with

$$F = \frac{\alpha_1[(1+\beta)^3 - \frac{\alpha_2}{\alpha_3}]}{(1+\beta)^3 + 2(\alpha_1)(\alpha_2)} \quad (12)$$

and

$$\alpha_1 = \frac{k_p - k_l}{k_p + 2k_l}, \alpha_2 = \frac{k_l - k_f}{k_l + 2k_f}, \alpha_3 = \frac{k_f - k_l}{k_f + 2k_l} \quad (13)$$

The total volume fraction of the equivalent nanoparticle which is formed from the original nanoparticle of radius $r_p$ and the nanolayer of thickness d is given by

$$\phi_T = \frac{4}{3}\pi(r_p + d)^3 n = \phi(1+\beta)^3 \quad (14)$$

where

$$\phi = \frac{4}{3}\pi r_p^3 n \quad (15)$$

is the original volume fraction of the nanoparticle without including the nanolayer and n defines the number density of particles and $\beta = d / r_p$.

## III. Graphs and Discission

Equation (11) is used to study the effective thermal conductivity ratio ($k_{eff}/k_f$) of the nanoparticle dispersion and its relationship with the volume fraction and the effect of size on effective thermal conductivity is also studied using the same expression for the nanofluid. Fig. 3 below shows the variation of thermal conductivity ratio at different interfacial layer thickness with the size of the nanoparticle. The ratio decreases abruptly with the increase in size of the nanoparticle as well as with the decrease in interfacial layer thickness. The graph indicates that the nanofluid system shows higher thermal conductivity at smaller radius of the nanoparticle and higher interfacial layer thickness. The systems studied for spherical shaped nanoparticle dispersions are Alumina in ethylene glycol and copper oxide in water taking the size of nanoparticles to be 35nm and 23.6nm respectively.

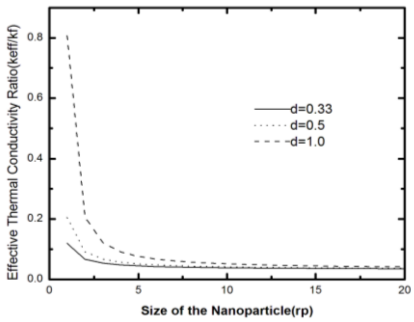


Fig. 3. The effective thermal conductivity ratio versus size of the nanoparticle.

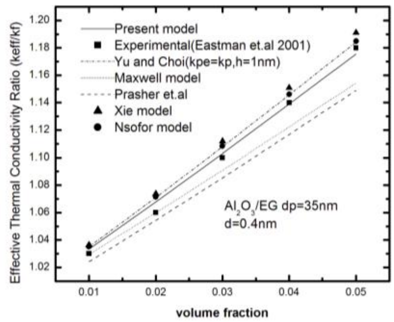


Fig. 4. The variation of effective thermal conductivity ratio with volume fraction for $Al_2O_3$/EG nano-dispersion.

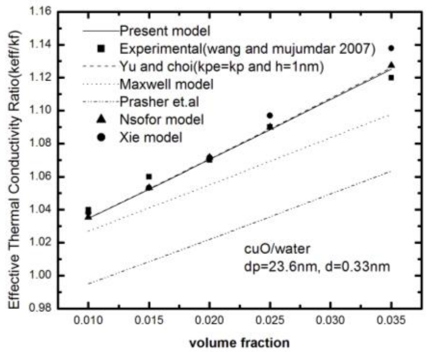


Fig. 5. The variation of effective thermal conductivity ratio with volume fraction for CuO/Water nano-dispersion.

The above graphs show that the effective thermal conductivity ratio increases with an increase in the volume fraction of the nano-dispersants in the base fluid. The results from this model have been compared with other theoretical models like Yu and Choi [14], Maxwell model [12] and Prasher model [19], [20].

The maximum deviation in case of $Al_2O_3$/EG nanodispersion is 0.75% in the present model whereas it is 1.3% in case of Xie model [17] and it is 1.07% in case of Nsofor model [16]. The maximum deviation in case of

CuO/Water nanodispersion is 0.7% in the present model and the maximum deviation in case of Xie model is 1.6% and in case of Nsofor model is 0.6%. In case of TiO2/EG our maximum deviation from experimental data is 0.5%. Also in case of CuO/water system, the present model overlaps the model by Yu and Choi when the thermal conductivity of the equivalent particle is taken to be equal to the thermal conductivity of the nanoparticle with an interfacial layer thickness of 1nm. The value for the interfacial layer thickness in each type of nanofluid system has been calculated using least square fitting method.

The system studied for rod shaped nanoparticle dispersion is $TiO_2$/EG and the available experimental data by Murshed *et al.* [9] is used for comparison.

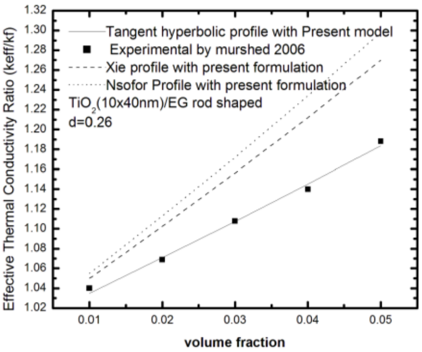


Fig. 6. The variation of effective thermal conductivity ratio with volume fraction for $TiO_2$/EG system.

It is evident from the Fig. 6 that the proposed thermal conductivity profile (tangent hyperbolic) fits well with the avaliable experimental values by Murshed [9]. It shows the comparison with the thermal conductivity profiles given by Xie *et al*. ( linear Profile) and Nsofor *et al*. (Quadratic Profile).

## IV. Conclusion

The present model successfully explains the anomalous behavior of effective thermal conductivity of nanofluids containing spherical as well as cylindrical nanoparticles. The maximum deviation from the experimental result is 0.75% in case of present proposed semi empirical model.

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**R. Pasrija** has done her masters in science from Panjab University Chandigarh, India in 2003. She is presently teaching as an assistant professor at P.G. Department of Physics, DAV College, Abohar, Punjab, India.

She is presently working for her PhD from Department of Physics, Panjab University Chandigarh.

She is a life member of Indian Assosiation of Physics Teachers.

**S. Srivastava** has obtained her doctor's degree from Panjab University, Chandigarh in 1992 in the field of general theoretical Physics particularly on non linear Oscillators. She completed her M.Phil in 1987 and Masters in Science in 1985 from Panjab University. Her major field of study is Condensed Matter Theory.

She is at present an assistant professor at Department of Physics at Panjab University, Chandigarh. She has about 50 research papers to her credit with 36 papers in Journals of International repute. She has authored a chapter "Restricted flow in Nano-Channels" in a book "*Nanotechnology Research Progress*", edited by Julian F. Vogel and Felix T. Jung (Nova Publishers, New York, 2009). Her present research interest includes confinement of fluids in nanogeometries and nanofluids

Dr. Srivastava is a life member of Indian Association of Physics Teachers and Indian Physics Association.