Measure of Performance of Queuing Models and Behavior of Customers in Real Life Applications

Priti Bajpai and Maneesha

***Abstract*—The objective of the paper is to determine customer behavior in queues in four different situations in India. The study is limited to Toll Booth-Delhi Gurgaon Highway, Health care facility for Public sector bank's staff and family, Executive check up in Health care organization and Tellers in a Bank. A detailed study of the queues at the Toll Booth on the Delhi Gurgaon expressway was done as the expressway is highly traversed and is a bottlenecked stretch. In spite of the multiple provisions to ease the traffic on this expressway, chronic delays led to a ruling by the court which made the expressway a freeway for two weeks in September 2012. The second case studied is of the work mechanism of a clinic set up for the public sector bank's staff and family in India. The doctor visits the clinic during specified hours. It was observed that one doctor caters to a large number of patients giving rise to long queues. The third case is of the hospital which has an Out Patient Department (OPD) that provides service for executive health checkup. Our study is limited to the executive health checkups, where a patient first registers and then is guided to different departments for the checkups. Lastly the study was conducted for a bank where the major business of this particular bank is retail banking for which the customer is diverted towards the tellers. Due to cost considerations and space constraint the number of tellers in a bank is limited. Inspite of the use of modern technology to streamline the queues, it is observed that during peak business hours the length of the queue increases resulting in increased waiting time for the customers. This paper attempts to do a comparative study for four different models to understand the behavior of customers waiting in queues in India.**

***Index Terms*—Arrival rate, simulation, service rate, waiting time.**

I. Introduction

In 1954, Leslie Edie [1] published a seminal piece on toll design considering traffic delays at tollbooths of homogeneous booths and homogeneous vehicles. Such analysis can be quite difficult if different types of traffic flows are considered [2]. The possible solutions include operations research techniques [3], [4], for example, van Dijk *et al.* [5] presented different queuing models for the simulation of a toll plaza. Pursula [6] has presented an overview on the use of simulation for over forty years and its application in development of traffic systems.

Several articles have been published on Health care facilities as early as N. T. J. Bailey [7], [8]. In the year 1952, N. T. J. Bailey Published his work on Queues and appointment system in an outpatient department. His work was influential in the development of National Health Services of Britain. In the healthcare industry whether it is a specialized hospital or a clinic a queuing system is followed. The patients arrive and wait in a queue for service and depart after being served. It has been seen that queuing theory is a useful tool in studying various processes in the healthcare systems. McClain [9] studied models for evaluating utilization, waiting time, and the probability of turning away patients. Rosenquist [10] studied how an increase in patient arrival rate affects waiting times and queue length for an emergency radiology service. Worthington [11] presented an M (λq)/G/S model and studied service time, arrival rates, queue length and the expected waiting time. Worthington [12] suggested that increasing servers had almost no effect on queue length. A brief history on use of queuing theory in healthcare was given by Preater [13] and Green [14] studied the delays, utilization and the number of servers required using M/M/s model. T. F. Keller and D. J. Laughhunn [15] highlighted the adequacy of physician capacity in an outpatient clinic for medically indigent patients. Brahimi, M. and D. J. Worthington [16] did a similar study on outpatient appointments. R. Hall, D. Belson, P. Murali, and M. Dessouky [17] worked on modeling patient flows in reducing delay in Healthcare delivery. S. Jacobson, S. Hall, and J. Swisher [18] used discrete-event simulation to study patient flow in health care systems.

Ahmed S. A. AL-Jumaily and Huda K. T. AL-Jobori [19] published the work on devising an automatic queuing system based on average waiting time for Banking Applications. Taufemback, Cleiton & Da Silva, Sergio [20] applied queuing theory to bank excess reserves.

II. Queuing Models and Operating Characteristics

A. Queuing Models

In general, a queue is a line of people or things waiting to be handled/served, usually in sequential order starting at the beginning or top of the line or sequence. In computer technology, a queue is a sequence of jobs that are waiting to be processed. When the arrival rate is more than the service rate, queues are formed. The variation in the service rate and the arrival rate determines the length of the queue. The possible factors, arrangements, and processes related to queues is known as queuing theory.

According to Kendall's notation, the queuing model used in analyzing the data are of the form $(M/M/s)$: $(FCFS/\infty/\infty)$, $(M/M/s)$:($FCFS/K/K$) and $(M/E_K/1)$:($\infty/FCFS$) where $k = 4$ stages.

where

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M = Markovian interarrival times
M = Markovian service times
s = Number of servers in parallel
FCFS = First come first serve
∞ = number of customers that can be accommodated in a queue
∞ = prospective number of customers that will pass through toll plaza
K = number of customers that can enter in a queue
K = prospective population entitled to visit the system

B. Operating Characteristics

1) The operating characteristics used in the paper [21] for Toll Booth-Delhi Gurgaon Highway and Tellers in a Bank are as follows:

- U: Utilization factor, or the percentage of time that servers are busy

$$U = \lambda / s\mu$$

- P_0: Probability that there is no customer in the system (in queue and being served)

$$P_0 = \left(\sum_{n=0}^{s-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^s}{s!} \left(\frac{s\mu}{s\mu - \lambda}\right)\right)^{-1}$$

- L_q: Average number of customers in the queue

$$L_q = \frac{P_0 (\frac{\lambda}{\mu})^{s+1}}{(s-1)!(s - \frac{\lambda}{\mu})^2}$$

- *L*: Average number of customers in the system (in queue and being served)

$$L = L_1 + \frac{\lambda}{\mu}$$

- *W*: Average time a customer spends in the system (in queue and being served)

$$W = W_q + \frac{1}{\mu}$$

2) The operating characteristics used in the paper [22] for Executive check up in Health care organization are as follows:

$$Lq = \frac{k(k+1)}{2} \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$Wq = \frac{Lq}{\lambda}$$

$$L = Lq + \frac{\lambda}{\mu}$$

$$W = \frac{(k+1)}{2k} \frac{\lambda}{\mu(\mu - \lambda)} + \frac{1}{\mu}$$

3) The operating characteristics used in the paper for Health care facility for Public sector bank's staff and family are as follows:

$$P_0 = \left[\sum_{n=0}^{s-1} \binom{K}{n} \rho^n + \sum_{n=s}^{K} \frac{K!\rho^n}{(K-n)!\,s!s^{n-s}}\right]^{-1}$$

$$P_n = \binom{K}{n} \rho^n P_0, \quad n = 1, 2, \ldots, s-1$$

$$= \frac{K!}{(K-n)!\,s!s^{n-s}} \rho^n P_0, \quad n = s, s+1, \ldots, K$$

$$= 0, \quad n > K$$

$$L_q = \sum_{n=s+1}^{K} (n-s) P_n$$

$$L = \frac{(\rho K + L_q)}{1+\rho}.$$

$$\bar{\lambda} = \lambda(K - L).$$

C. Simulation

Simulation techniques are helpful in analyzing models in which the value to be assumed by one or more independent variables is uncertain. Simulation using a commercial spreadsheet add-in called Crystal Ball was used in this study. The process of assigning values to the cells in the spreadsheet that represent random variables is automated so that the values are assigned in a non-biased way [23].

III. Statistical Analysis

A. Assumptions

The following assumptions have been made in our study:

1) The arrival rate follows Poisson distribution as shown in Fig. 1; inter arrival times therefore are exponentially distributed exhibiting the Markovian property.
2) The service times also follow exponential distribution as shown in Fig. 2.

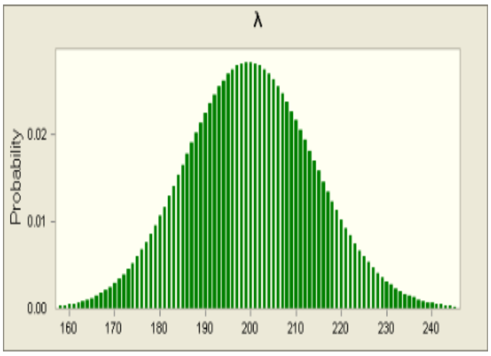

Fig. 1. Poisson distribution of λ.

TABLE I: COMPARATIVE OPERATING CHARACTERISTICS RESULTS FOR FOUR STUDIES IN INDIA

STUDY	Toll Booth-Delhi Gurgaon Highway	Health care clinic	Executive check up in Health care organization		Tellers in Bank
MODEL	(M/M/s) :(FCFS/∞/∞)	(M/M/s) :(FCFS/K/K)	$(M/E_K/1)$:(∞/FCFS) k = 4 stages		(M/M/s) :(FCFS/∞/∞)
Λ	200 vehicles/hr/server	0.018 Arrival rate /patient	3 patients per hour		100 customers/hour
μ	72 vehicles/hr/server	7 patients/hour	4 patients per hour		35 customers/hour
Existing number of servers	11(besides the lanes for smartag and 2& 3vheelers)	1 doctor	1server	1server	3 tellers
Recommended number of servers	3 times present capacity	2 doctors	2servers at each stage	2 servers at ECG stage only	4 tellers
U	92.59%	64.01%	-	-	71.43%
Lq	9	0.8717	6.75	13.5	1.1277
L	13	2.1519	7.5	14.25	3.9848
Wq	0.0540	0.0973	2.25 hours	4.5 hours	0.0113
W	0.0678	0.2401	2.5 hours	4.75 hours	0.0398

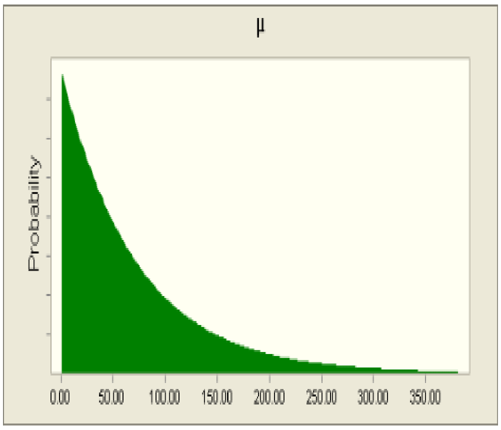

Fig. 2. Exponential distribution of μ.

B. Analysis

Table I shows comparative study of four cases namely- Toll Booth-Delhi Gurgaon Highway, Health care facility for Public sector bank's staff and family, Executive check up in Health care organization and Tellers in Bank [24], [25]. Fig 3 depicts the patterns observed as follows :

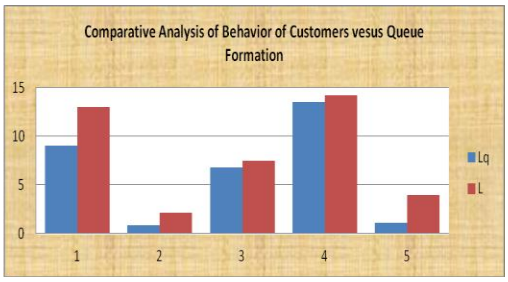

Fig. 3. Comparative analysis of behavior of customers versus queue formation.

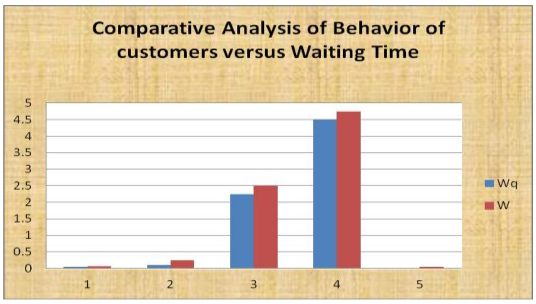

Fig. 4. Comparative analysis of behavior of customers versus waiting time.

Inference 1: It is observed from Fig. 3, the highest Lq and L are found in Executive check up in health care organization followed by Delhi-Gurgaon toll booth.

Inference 2: In case of comparison of waiting time in the queue and system, it is observed as shown in Fig. 4, the highest Wq and W are found in health care organization again.

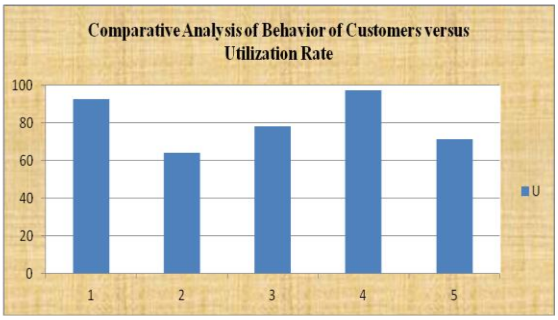

Fig. 5. Comparative analysis of behavior of customers versus utilization rate.

Inference 3: The results related to the utilization rate (U) of servers revealed as is evident from Fig. 5 that the highest U is again in case of Executive check up in health care organization followed by Delhi-Gurgaon toll booth.

IV. CONCLUSION AND RECOMMENDATIONS

A. Behavior of Customers in Various Situations and Queuing Models

Customers arrive for service in general one by one and form a queue whereby the queue discipline followed in the above mentioned cases is First come First serve basis. Sometimes, arrival may be in bulk. In this case, service rate cannot keep up the pace with the arrival rate which results in long queues and more waiting time. In both situations it is interesting to study the behavior of the customers. Customers may resort to balking or reneging if the customer finds the queue too long. Jockeying queue by the customers may occur due to limited options of service.

The behavioral patterns as observed are:

1) Queuing Models with servers in series versus parallel: In the Executive check up in Health care organization, the queuing model is $(M/E_K/1)$ involving four stages.

However, in the other three cases, the queuing models have servers in parallel. It is revealed from a comparative analysis of performance parameters that the queue length is high resulting in long waiting time for the customers where queuing model applicable is ($M/E_K/1$). In case of toll booth, health care facility of bank's staff & family and tellers in bank, an increase in number of servers by three times, one and one respectively reduced the waiting time substantially.

2) Queuing Models with servers in parallel where prospective population is finite versus infinite i.e.(M/M/s):(FCFS/∞/∞) versus (M/M/s) :(FCFS/K/K). When the prospective population is finite, a slight increase in the number of servers reduces the waiting time of customers substantially as evident in the study of health care facility of bank's staff & family. However, in case of Delhi-Gurgaon toll booth with infinite prospective population, it is recommended that servers be increased three times the present capacity then the waiting time reduces to 3 minutes.
3) Services as a necessity versus alternatives available: In case of Executive check up in Health care organization, it a requirement by a prospective employer to get a fitness report or individuals want it as a precautionary assessment of their health. People in India are increasingly becoming conscious about their health. The number of service providers is not able to keep pace with the number of people availing the health care facility. Many a times, it is a necessity for the customer (patients) to avail the facility. Apart from visiting the health care organization, the service is not available elsewhere. It therefore results in long waiting time for customers to avail the facility in this situation. From the perspective of service provider, the constraint in increasing the number of servers is the cost consideration. However, when the number of servers is doubled at each of the four stages, waiting time still continues to be substantially high i.e 2.5 hours.
4) In case of toll booth, the alternatives available to commuters on Delhi Gurgaon Highway apart from crossing the toll booth by making cash payment are use of smart card, smart tags and alternative routes. Due to the sheer volume of traffic, an increase in number of servers to almost three times the present capacity substantially reduces the crossing time to just 3 minutes.
5) The alternative available in case of health care facility for Public sector bank's staff and family is to visit any other doctor in a private or public health care organization for a payment. An increase of one doctor in the facility reduced the waiting time of patients to almost one-fourth of the original time.
6) Banking sector provides customers many alternatives to tellers in a bank. Online banking, use of ATMs , mobile banking to name a few. To avoid queue, customer can always resort to any of these options. Thus, by increasing just one teller more, the waiting time for the customer reduced from 13 minutes to 3 minutes per customer.
7) If the service is a necessity, then it is observed from the performance parameters, that customers will have a longer waiting time inspite of increased number of servers as provided by the organization. Number of alternatives to the service is inversely proportional to the waiting time of the customer.

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REFERENCES

[1] G. B. Dantzig, "Letter to the Editor—A Comment on Edie's "Traffic Delays at Toll Booths"," *Operations Research*, vol. 2, pp. 339-341, August 1, 1954.
[2] S. Algers *et al.*, *Review of Micro-Simulation Models*, 1997.
[3] B. Render *et al.*, *Quantitative Analysis for Management*, 9th Ed., PHI/Pearson 2008.
[4] H. A. Taha, *Operations Research: An Introduction*, 8th Ed., Prentice-Hall, Inc., 2006.
[5] P. A. Farrington *et al.*, *Designing The Westerschelde Tunnel Toll Plaza Using A Combination Of Queueing And Simulation*, 2009.
[6] M. Pursula, "Simulation of Traffic Systems-An Overview," *Journal of Geographic Information and Decision Analysis*, vol. 3, pp. 1-8, 1999 .
[7] N. T. J. Bailey, "A study of queues and appointment systems in hospital out-patient departments, with special reference to waiting times," *Journal of the Royal Statistical Society*, vol. 14, pp. 185-199, 1952.
[8] N. T. J. Bailey, "Queuing for medical care," *Applied Statistics*, vol. 3, pp. 137-145, 1954.
[9] J. O. McClain, "Bed planning using queuing theory models of hospital occupancy: a sensitivity analysis," *Inquiry*, vol. 13, pp. 167-176, 1976.
[10] C. J. Rosenquist, "Queueing analysis: a useful planning and management technique for Radiology," *Journal of Medical Systems*, vol. 11, pp. 413-419, 1987.
[11] D. J. Worthington, "Queueing Models for Hospital Waiting Lists," The *Journal of the Operation Research Society*, vol. 38, pp. 413-422, 1987.
[12] D. Worthington, "Hospital waiting list management models," *The Journal of the Operational Research Society*, vol. 42, pp. 833-843, 1991.
[13] J. Preater, "Queues in health," *Health Care Management Science*, vol. 5, pp. 283, 2002.
[14] L. Green, S. Savin, and B. Wang, "Managing Patient Demand in a Diagnostic Medical Facility,"*Operations Research*, vol. 54, pp. 11-25, 2006.
[15] T. F. Keller and D. J. Laughhunn, "An Application of Queing theory to a congestion problem in an outpatient Clinic," *Decision Sciences*, vol. 4, pp. 379-394, 1973.
[16] M. Brahimi and D. J. Worthington, "Queueing models for out-patient appointment systems – a case study," *The Journal of the Operational Research Society*, vol. 42, pp. 733-746, 1991.
[17] R. Hall, D. Belson, P. Murali, and M. Dessouky, *Modeling patient flows through the healthcare system, in Patient Flow: Reducing Delay in Healthcare Delivery*, R. W. Hall, ed., New York: Springer, pp. 1-44, 2006.
[18] S. Jacobson, S. Hall, and J. Swisher, *Discrete-event simulation of health care systems, in Patient Flow: Reducing Delay in Healthcare Delivery*, New York: Springer, pp. 211-252, 2006.
[19] A. S. A. AL-Jumaily and H. K. T. AL-Jobori "Automatic Queuing Model for Banking Applications," *International Journal of Advanced Computer Science and Applications*, vol. 2, no. 7, pp. 11-15, 2011.
[20] C. Taufemback and D. S. Sergio, "Queuing theory applied to the optimal management of bank excess reserves," *Physica A: Statistical Mechanics and its Applications*, Elsevier, vol. 391, no. 4, pp. 1381-1387, 2012.
[21] S. Venkateswaran and B. Singh, *Operations Research EDD Notes Part Three: BITS*, Pilani (India), August 1997.
[22] N. H. Shah, R. M. Gor, and H. Soni, *Operations Research*, New Delhi: PHI Learning Pvt Limited, pp. 335-371, 2012.
[23] C. Ragsdale, *Spreadsheet Modeling and Decision Analysis (with CD-ROM and Microsoft Project 2003 120 day version)*, South-Western College Publishing, 2006.
[24] M. Bhagchandani and P. Bajpai, "A Study of Traffic Management at Toll Plaza on the Delhi-Gurgaon Expressway," *American Journal of Mathematics and Sciences*, vol. 2, no. 1, pp. 63-69.

[25] P. Bajpai and M. Bhagchandani, "A Study of Healthcare Facility for Public Sector Banks Staff and Family," in *Proc. International Conference: Mathematical Science & Applications*, 2012.

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