Study of Deconfinement Phase Transition in Heavy Ion Collisions at BNL Energies

M. Ayaz Ahmad, Mir H. Rasool, Shafiq Ahmad, Jamal B. H. Madani, and Rachid Ayad

***Abstract*—The Scaled factorial moments (SFMs) of multiplicity distribution are used to study the deconfinement phase transition in high energy heavy-ion collisions. In the present article we studied the Renyi dimensions and multifractal spectrum in the interaction of ^{28}S-emulsion collisions at 14.6A GeV to investigate non thermal phase transitions during such collisions.**

***Index Terms*—Novel state of matter, quark-gluon plasma (QGP), scaled factorial moments (SFMs).**

I. Introduction

During last couple of years, different nuclei have been accelerated to relativistic energies and brought to collisions with a great variety of target nuclei. An ultimate aim of studying nucleus – nucleus (A-A) collisions is to investigate for a phenomena connecting with large densities obtained in such nuclear collisions.

So due to this one can obtained an opportunity to explore strongly interacting matter at energy densities unprecedented in a laboratory, which eventually gives an evidence for the existence of quark-gluon plasma (QGP). The QGP is a novel state of matter in which quarks and gluons are no longer confined to volumes of hadronic dimensions. In deep inelastic scattering experiments, it has already been revealed that quarks at very short distances move freely, which is referred to as the asymptotic freedom. Quantum Chromo-dynamics (QCD) describes the strong interactions of quarks and gluons [1]. The experimental observation of large rapidity fluctuations in relativistic heavy ion collisions by R. C. Hwa and J. C. Pan [2], [3] using the method of multifractal moments method, G_q, has provided keen interest and excitement in about their nature and origin. Bialas and Peschanski [4], [5] suggested a power law scaling behavior in terms of normalized scaled factorial moments, SFMs ($< F_q > \propto M^{\alpha_q}$) on the bin size and described the phenomenon as "intermittency". The SFMs method cannot only predicts the existence of large non-statistical fluctuations but it could also investigate the pattern of fluctuations and their origin.

The main emphasis of the present experimental / statistical work is to explore the second order phase transition, which take place during the relativistic heavy ion collisions, with the help of Renyi dimension, D_q, and multifractal spectrum, $f(\alpha_q)$.

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M. Ayaz Ahmad, Jamal B. H. Madani and Rachid Ayad are with the Physics Department, College of Science, P.O. Box 741, University of Tabuk, Zip 71491 Saudi Arabia (e-mail: mayaz.alig@gmail.com, jhmadani@ut.edu.sa, rayad@ut.edu.sa).

Mir H. Rasool and Shafiq Ahmad are with the Physics Department, Aligarh Muslim University, Aligarh, 202002, India (e-mail: hrasool1123@gmail.com, sahmad2004amu@yahoo.co.in).

II. Experimental Details and Data Collection

In the present experiment, FUJI nuclear emulsion pellicles were irradiated horizontally with a beam of ^{28}Si nuclei at 14.6A GeV at Alternating Gradient Synchrophasotron (AGS) of Brookhaven National Laboratory (BNL), NewYork, USA have been used for data collection. The method of line scanning has been adopted to scan the stacks, which was carried out carefully using Japan made NIKON (LABOPHOT and Tc-BIOPHOT) high-resolution microscopes with 8 cm movable stage using 40X objectives and 10X eyepieces by two independent observers, so that the bias in the detection, counting and measurements can be minimized. The interactions due to beam tracks making an angle < 2° to the mean direction and lying in emulsion at depths > 35 μm from either surface of the pellicles were included in the final statistics. The other relevant details about the present experiments and target identifications may be seen in our previous work [6]-[8].

In the present analysis, the pseudorapidity (η) and azimuthal angle (ϕ) have been used as the two variables representing phase space. For the study of dynamical fluctuations, the pseudorapidity interval $\Delta\eta$ is taken as -1.25 to 6.75, while the azimuthal angle varies from $\Delta\phi$ = 0 - 2π and it is divided into M_ϕ bins of size, $\delta\phi = 2\pi/M_\phi$.

Only events with $N_S \geq 8$ were considered to maximize the contribution of dynamical fluctuations [3]. We have total 951 data events of relativistic shower charged particle (N_S) with mean multiplicity $\langle N_S \rangle = 21.34 \pm 0.16$. For this purpose we have divided it into three subsets of data with different N_S-intervals: 1) in $8 \leq N_S \leq 15$ with $\langle N_S \rangle = 11.45 \pm 0.19$, 2) $16 \leq N_S \leq 23$ with $\langle N_S \rangle = 19.15 \pm 0.25$ and $N_S \geq 24$ with $\langle N_S \rangle = 34.32 \pm 0.33$ in the collisions ^{28}Si-Em at 14.6A GeV.

III. Mathematical Tools

In order to perform a meaningful analysis of intermittency, normalized "cumulative" variables, $X(\eta)$ and $X(\phi)$ were used to reduce the effect of non-uniformity in single charged particle distributions. In terms of new scaled variables, $X(\eta)$ and $X(\phi)$, the single particle density distribution is always uniform in between X = 0 and 1 and both "vertical" and "horizontal" averaging of scaled factorial moments should produce the same result.

The cumulative variable in the phase space (say η) is defined as [9]:

$$X(\eta) = \int_{\eta_{\min}}^{\eta} \rho(\eta')\,d\eta' \Big/ \int_{\eta_{\min}}^{\eta_{\max}} \rho(\eta')\,d\eta' \tag{1}$$

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where, $\rho(\eta)$ is the single particle pseudorapidity density of shower particles and η_{min} (η_{max}) is the minimum (maximum) value of η. Similar relation as Eqn. (1) was used to calculate $X(\phi)$. Though our entire analysis on scaled factorial moments will henceforth be performed taking X_η (X_ϕ) as the basic variable, we shall continue to call the corresponding space $\eta(\phi)$-space.

Various experimental efforts have established the existence of the empirical phenomenon of "intermittency" in multiparticle production using normalized scaled factorial moments (SFMs) towards the deconfinement phase transition [4], [5], [10]-[13]. On the basis of bin averaging the normalized SFMs of the order of q is defined as in its vertical form:

$$F_q^V(\delta\eta) = \frac{1}{M^d} \sum_{m=1}^{M^d} \frac{< n_m^q >}{< n_m >^q} \tag{2}$$

This analysis in a single phase-space dimension in η and ϕ spaces respectively was extended to two dimensions (η, ϕ) space. In order to use above formulism in two dimensions, a rectangle in the (η, ϕ)-space was considered, which was divided into $M_{\eta\phi} = M_\eta \times M_\phi$ bins of each size $\delta\eta\delta\phi = (\Delta\eta/M_\eta)(\Delta\phi/M_\phi)$, where the sum now extends over M^2 bins in Eqn. (2) and n_m is the number of particles in the m^{th} bin in the two dimensions (2D) phase (η, ϕ)-space. The pseudorapidity interval, $\Delta\eta$, is divided into "M" bins of uniform width $\delta\eta = \Delta\eta = \{X(\eta_{max}) - X(\eta_{\min})\}/M$.

The other mathematical description in about the scaled factorial moments (SFMs) and multifractal moments, G_q-moments related to this analysis may be seen in our earlier work in details [10]-[13].

IV. Results and Discussions

Renyi dimension (also known as generalized fractal dimensions), D_q, and multifractal spectrum, $f(\alpha_q)$, are often used to study the presence of multifractal structure. The generalized dimension, D_q, expressed in terms of intermittency index, α_q, given by relation:

$$D_q = 1 - \frac{\alpha_q}{(q-1)} \tag{3}$$

which plays a significant role in fractal theory.

The relation between the spectral function, $f(\alpha_q)$, and the fractal index, τ_q, is obtained through Legendre transformation as follows:

$$f(\alpha_q) = q\alpha_q - \tau_q \tag{4}$$

with

$$\alpha_q = d\tau_q / dq \text{ and } d\, f(\alpha_q) / d\alpha_q = q \tag{5}$$

where, α_a is the Lipschitz-Holder exponent. The following conditions are fulfilled for the existence of multifractal structure in a data provided 1) , $f(\alpha_q)$, are continuous function of α_q, 2) $f(\alpha_q)$ must have an upward convex shape with a distinct peak at $\alpha_q = \alpha_0$ and 3) $f(\alpha_q) < f(\alpha_0)$, for $q \neq 0$. The width of the $f(\alpha_q)$ distribution is a measure of the size of dynamical fluctuations. For a purely statistical system with absolutely no fluctuations, $f(\alpha_q) = \alpha_q = 1$ for all values of q and the function, $f(\alpha_q)$, for all values of q and the function, $f(\alpha_q)$, is a straight line parallel to the y-axis at α_q = 1.

In order to calculate the value the fractal index, τ_q, we have used the following relations:

$$\tau_q = q - 1 - \alpha_q \tag{6}$$

where q is order of moments and α_q, is intermittency index, so first we have obtained the values α_q, by plotting the graphs ln<F_q> versus lnM by getting their slopes. These figures have not been shown here then finally we get the values of τ_q by above mentioned relation and did the following analysis for the η and ϕ phase space in one dimensions and also $\eta\phi$ phase space together in two dimensions.

We have avoided calculating τ_q from G_q-moment method, because G_q-moments are dominated by statistical fluctuations, whereas F_q-moments are free from statistical fluctuations.

Now the Renyi dimension, D_q, and multifractal spectrum, $f(\alpha_q)$, are calculated in η space using the Eqns. (4) and (5) with the order of q = – 0.8 to 6.6 in step of 0.2. The variations of D_q versus q and $f(\alpha_q)$ as a function of α_q for different N_S intervals are shown in Figs. 1 (a)-(c) and Fig. 2 (a)-(c) for η and ϕ (1D) phase spaces and $\eta\phi$ (2D) phase space respectively. It is obvious from Fig. 1 (a)-(c) that the values of D_q decrease from 1.153 ± 0.023 to 0.278 ± 0.061 in η-space, 1.160 ± 0.029 to 0.293 ± 0.064 in ϕ-space and 1.158 ± 0.026 to 0.301 ± 0.057 in $\eta\phi$-space as q increases from – 0.8 to 6.6 for η, ϕ and $\eta\phi$ phase-spaces respectively. Thus the decreasing behaviour of the generalized fractal dimensions, D_q, with increasing order of moments, q for all N_S intervals in ^{28}Si-Em collisions at 14.6A GeV clearly indicates the presence of multifractality for the present experimental data. From Fig. 2 (a)-(c), it may be seen that the $f(\alpha_q)$ are represented by continuous curves in one and two dimensional phase spaces. The figure also shows a distinct peak at $\alpha_q = \alpha_0$ for all N_S samples and solid line represents tangent at an angle of 45° at $\alpha_1 = f(\alpha_1)$. The left hand sides ($q > 0$) of the spectra $f(\alpha_q)$ are sensitive to peaks and the right hand sides ($q < 0$) describe the valleys of single particle η-distribution [14], which might be responsible for producing relativistic particles in nuclear collisions. The most basic property of any fractal measure is its dimensions and a set of conventional dimensions for q = 0, 1 and 2 are the fractal dimension, $D_0 = f(\alpha_0)$, the information dimension, $D_1 = f(\alpha_1)$, and correlation dimension, $D_2 = 2\alpha_2 - f(\alpha_2)$. The values of these dimensions in η, ϕ and $\eta\phi$ phase spaces are reported in Table I. The values of D_0, D_1 and D_2 also calculated using Eqn. (3) with intermittency indices are also depicted in Table II. In both the Tables the associated errors with their values are pure statistical. A consistency is found in the two values obtained by the multifractal spectrum and intermittency indices. From the discussion of the Renyi dimension, D_q, and the spectral function, $f(\alpha_q)$, it may be said that no phase transition is taking place. Other high energy physicists also reported similar results in heavy ion collisions at BNL and CERN SPS energies [15]. Very similar results were also found in Proton-Emulsion (P-Em) interactions at 800GeV by N. Parashar *et al.* [16].

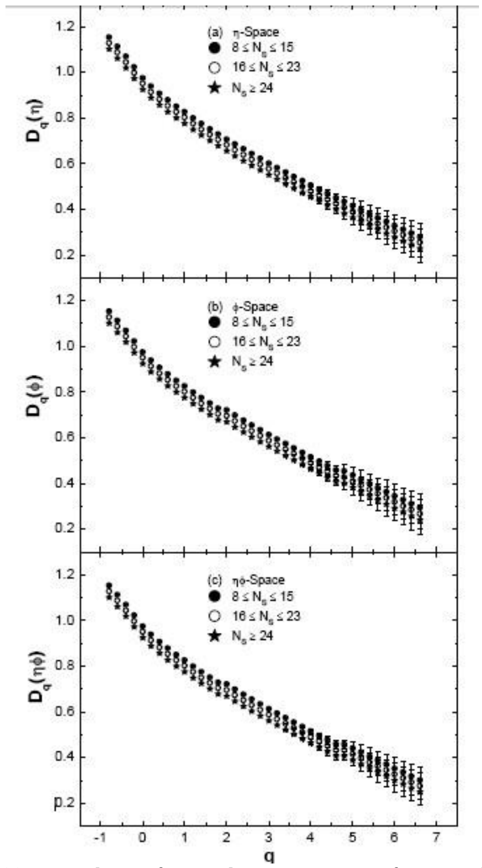

Fig. 1 (a)-(c). Dependence of Renyi dimension, Dq as a function of q for: (a) η-phase space, (b) ϕ-phase space and (c) for $\eta\phi$-phase space in different N_S -intervals in the interactions of 28Si-Em at 14.6A GeV.

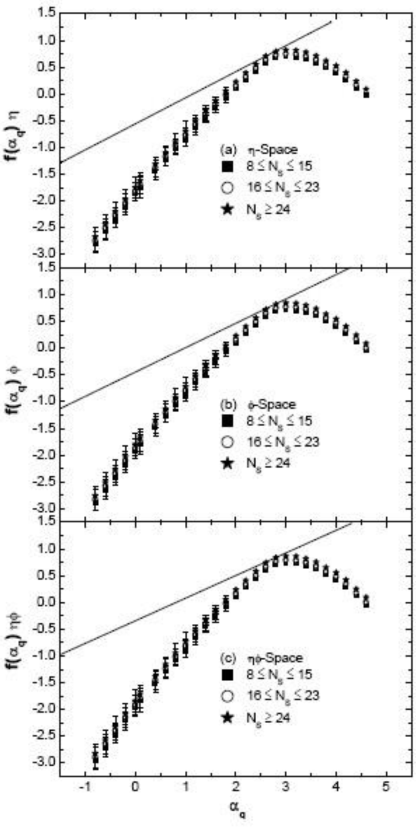

Fig. 2 (a)-(c). Dependence of spectral function, $f(\propto_q)$ as a function of q for: (a) η-phase space, (b) ϕ-phase space and (c) for $\eta\phi$-phase space in different N_S -intervals in the interactions of ^{28}Si-Em at 14.6A GeV.

TABLE I: THE VALUES OF VARIOUS DIMENSIONS D_q OBTAINED FROM THE SPECTRAL FUNCTION, $f(\alpha_q)$, FOR DIFFERENT NS-INTERVALS IN THE INTERACTIONS OF ^{28}SI-EM AT 14.6A GEV

Multiplicity intervals	Phase space	Fractal dimension	Information dimension	Correlation dimension
$8 \le N_S \le 15$	η	0.951±0.045	0.923±0.043	0.896±0.042
	ϕ	0.905±0.031	0.879±0.030	0.853±0.029
	$\eta\phi$	0.875±0.036	0.850±0.035	0.825±0.034
$16 \le N_S \le 23$	η	0.856±0.047	0.831±0.045	0.806±0.044
	ϕ	0.814±0.057	0.791±0.056	0.768±0.054
	$\eta\phi$	0.788±0.046	0.765±0.044	0.743±0.043
$16 \le N_S \le 23$	η	0.770±0.030	0.748±0.029	0.726±0.028
	ϕ	0.733±0.060	0.712±0.059	0.691±0.057
	$\eta\phi$	0.709±0.045	0.688±0.043	0.688±0.042

TABLE II: THE VALUES OF VARIOUS DIMENSIONS D_q OBTAINED FROM THE INTERMITTENCY INDEX, α_q, FOR DIFFERENT NS-INTERVALS IN THE INTERACTIONS OF ^{28}SI-EM AT 14.6A GEV

Multiplicity intervals	Phase space	Fractal dimension D_0	Information dimension D_1	Correlation dimension D_2
$8 \le N_S \le 15$	η	0.908±0.064	0.883±0.062	0.825±0.058
	ϕ	0.898±0.062	0.873±0.060	0.816±0.056
	$\eta\phi$	0.892±0.056	0.868±0.055	0.811±0.051
$16 \le N_S \le 23$	η	0.890±0.063	0.866±0.061	0.809±0.057
	ϕ	0.889±0.072	0.865±0.070	0.808±0.065
	$\eta\phi$	0.884±0.057	0.860±0.056	0.804±0.052
$16 \le N_S \le 23$	η	0.0882±0.064	0.858±0.062	0.802±0.056
	ϕ	0.879±0.070	0.855±0.068	0.799±0.064
	$\eta\phi$	0.877±0.057	0.853±0.053	0.797±0.052

V. CONCLUSION

The present study also gives a strong evidence of self-similar structure in multiparticle production in such collisions at 14.6A GeV for two-dimensional $\eta\phi$-phase space rather than one-dimensional η and ϕ-phase spaces.

Furthermore, the decreasing trend of the Renyi dimensions, D_q, with increasing q gives an evidence of self-similar process in η, $\phi$and $\eta\phi$ phase spaces respectively.

A smooth and an upward convex shape of multifractal spectral function, $f(\alpha_q)$, in one and two dimension phase space may predict the presence of non-statistical fluctuation for our data.

In this study there are not any direct evidences in about the deconfinement phase transition in such collisions and energies, but it can be hunt at much more high energies.

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M. Ayaz Ahmad completed Ph.D. in experimental high energy physics in 2010 and M. Phil. (physics) in 2005 from the Physics Department, Aligarh Muslim University, Aligarh, India, under the supervision of Prof. Shafiq Ahmad. He worked as a guest lecturer for B. Sc. Laboratory Classes in the same Department, Aligarh Muslim University, Aligarh w.e.f. 10th Oct. 2002 to 10th Oct. 2008 and also as a Lecturer at Senior Secondary College (Boy's) of Aligarh Muslim University, Aligarh w.e.f. 11th Oct. 2008 to 15th Dec. 2010.

Presently, He is working as an assistant professor at Physics Department, University of Tabuk, Saudi Arabia w.e.f. 16th Dec. 2010. He is involved in teaching and research more than ten years. Besides the undergraduate courses He is teaching/taught courses of Nuclear Physics, Particle Physics and Electrodynamics to graduate / postgraduate students. For the past several years, He is working in the field of Experimental High Energy Heavy Ion Collisions Physics and has published research papers in various refereed journals, like Journal of Physics G (IOP Journal), Nuclear Physics A (Journal of Science Direct/ Elsevier Journals), Journal of Physical Society Japan, Internal National Journal of Mod. Physics E, Ukrainian Journal of Physics, e.t.c.

Recently, he also started some research work on Carbon Nano Tube (CNT) in partial Collaboration with INFN (Istituto Nazionale di Fisica Nucleare), Italy and University of Tabuk Saudi Arabia, and Polymer Science. He is also an active member of Belle 2 experiment at KEK Laboratory, Japan since 20th April, 2012 and working in its hardware using some polarizing effect on behalf of a faculty member of University of Tabuk, Saudi Arabia.

Mir H. Rasool is working as a research scholar under the Supervision of Prof. Shafiq Ahmad in Experimental High Energy Physics Laboratory at Physics Department, Aligarh Muslim University Aligarh, India. He is working in high energy heavy ion collisions Physics. During his academic schedule / training, he attended various National / International Conferences / Symposia and summer Schools and also presented various research papers.

Shafiq Ahmad completed Ph. D. in Experimental High Energy Physics in 1980 and M. Phil in 1977 from Aligarh Muslim University, Aligarh. He is working as a Professor at the same University. His working field is relativistic heavy-ion collisions. He is a distinguish personality in his field or academic. He has completed various research projects in collaboration with TIFR and DST, India.

The current objective of his research is to study the existence of dynamical fluctuations**.** The existence of large fluctuations may also indicate a phase transition. The dynamical fluctuations in the distributions of relativistic shower particles produced in high energy collision may be studied by the method of scaled factorial moments (SFMs). The power law behavior of SFMs is known as Intermittency.

Jamal B. H. Madani completed Ph. D. from Durham University, Durham, U.K. He is a very good academician. He is working as a senior Associate Professor and Vice Dean Faculty of Science at University of Tabuk and also he is in Belle II collaboration, KEK center, Japan. University of the Tabuk, Kingdom of Saudi Arabia is full member of the Belle II collaboration since April 2012.

He has a long experience in teaching Physics at the Undergraduate and postgraduate for General Physics, Astro-Physics, Quantum Mechanics and Classical Mechanics e.tc. and he has a lot of publications in various refereed journals.

Rachid Ayad is working as a professor (physics) at University of Tabuk, Saudi Arabia. Academically, He has a long experience in teaching Physics at the Undergraduate (General Physics, Modern Physics, Analytical Mechanics) and Graduate level (Particle Physics and Analytical Mechanics). He contributed in commissioning a proton dosimeter in collaboration with the Harvard Medical Center at Boston. He preferable working environment is in the Research and Development, Teaching, and Software development (C/C++) within Unix/Linux platform.

His main tasks at the University of Tabuk are: teaching, research and academic issues. In teaching area He taught modern physics, special relativity, and optics, while in research He is the leader of the University of Tabuk group at the Belle II collaboration, KEK center, Japan. Belle II will take data at the SuperKEKB starting on 2015. University of the Tabuk is full member of the Belle II collaboration since April 2012.