Multi-Resolution Analysis of Wavelet Like Soliton Solutions of KdV Equations

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Abstract—**Many physical phenomena are described by non-linear partial differential equations. These equations have soliton solutions which exhibit wavelet features called wavelet like solitons. Such wavelet like solitons have expansions in Gaussian family wavelets. In this work, using the fact that the wavelet like soliton has Gaussian representation, multi-resolution analysis which is based on wavelets is carried out to obtain better approximation with the application of wavelet-Galerkin and wavelet-Petrokov-Galerkin methods for soliton solution of Korteweg-de Vries equation which appears in the study of waves in shallow water in the fluid dynamics. In the end, experimental data processing employing Gaussian representation of soliton solution is discussed.**

Index Terms—**Wavelet like solitons, gaussian representation, wavelet decomposition, data processing.**

I. Introduction

Multi-resolution analysis (MRA) uses wavelet functions as basis with an objective to specify the signal as a collection of its successive approximations. This approximations are of different resolutions, whence the name multi-resolution analysis. The term wavelet or the phrase wavelet analysis was first coined by J. Morlet [1]. Earlier, wavelets were used in electrical engineering. The major breakthrough occurred due to D. Gabor [2], who introduced the Windowed Fourier Transform (WFT) for the local spectral analysis of radar signals that actually laid pathway for use of wavelets from electrical engineering to mathematical physics. Due to the limitation of WFT, where the localization is attained due to fast decaying window functions, the scheme called, Wavelet transform (WT) with a wide window for low frequency signals and a narrow window for high frequency signals was introduced and formalized later by Grossman and Morlet [1], Daubechies [3] and many others. Recently, wavelet transform has been emerged as the most effective tool for signal processing and image analysis especially when the signals are random and comprised of fluctuations of different scales. Wavelets have been used in signal processing, problems involving singular potentials in quantum mechanics, in discussions concerning q-algebras, and even in nuclear structure studies [4]. In wavelet inspired approach, the sets of 'wavelets' are employed to approximate signals because of their Gaussian form. The beauty of the wavelet analysis lies in its predominant property of self-similarity that makes wavelet as a powerful tool for analyzing fractal like patterns. Since the soliton-like solutions have infinite extent, it requires rather appropriate compactly supported basis functions to investigate such structures than the traditional nonlinear tools (inverse scattering, group symmetry, functional transforms) that are not always applicable. Such structures/patterns generally have finite space-time extension and a multi-scale structure. Multi-resolution analysis that uses wavelet functions could be therefore a natural useful method for the construction of such nonlinear bases. This motivated us to employ the wavelet methods to analyze wavelet like soliton solutions of .Korteweg-de Vries equation (KdV) that appears in the study of waves in shallow water in the fluid dynamics. We have in [5]-[7], carried out extensively the wavelet analysis of solitons arising as solutions of Non-linear Schrodinger Equation (NLS), Sine-Gordon equation (SG).

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II. Mathematical Pre-Requisites

We need some mathematical formulations that are relevantly useful in the present work.

A. Wavelet Transforms

Practically wavelet transform is a convolution of the signal with a family of functions obtained from a basic wavelet by shifts and dilations. In precise terms and notations, the classical wavelet transform, also called as Continuous Wavelet transform (CWT) [8], is a decomposition of a function, $f(x)$, with respect to a basic wavelet $\psi(x)$, given by the convolution of a function with a scaled and translated version of $\psi(x)$

$$W_\psi(a,\ b)[f] = |a|^{-1/2} \int f(x)\psi^*\left(\frac{x-b}{a}\right)dx \qquad (1)$$

The functions, f and ψ are square integrable functions and ψ satisfies the admissibility condition: $C_\psi = \int \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$. C_ψ is called admissibility constant. The subscript '*' denotes complex conjugation, 'a' is the scale parameter, $a > 0$, 'b' is the translation parameter. The term $1/\sqrt{|a|}$ is the energy conservative term that keeps energy of the scaled mother wavelet equal to the energy of the original wavelet. The function $f(x)$ can be recovered by the reconstruction formula called Inverse transform:

$$f(x) = \frac{1}{C_\psi}\int \int W_\psi f(a,b)\frac{1}{\sqrt{|a|}}\psi\left(\frac{x-b}{a}\right)\frac{dadb}{a^2} \qquad (2)$$

where the admissibility constant, $C_\psi > 0$. Therefore, any function with compact support which satisfies above requirement can be successfully used as a basic wavelet.

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Furthermore, with the substitution, $f(x)$ as the inverse Fourier Transform $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(i\omega x)\,\hat{f}(\omega)d\omega$ in the definition of wavelet transform (1), we immediately get the spectral representation as

$$W_\psi[f(x)](a,b) = \frac{1}{2\pi}|a|^{1/2}\int_{-\infty}^{\infty} \exp(i\omega b)\,\overline{\hat{\psi}(a\omega)}\,\hat{f}(\omega)d\omega. \tag{3}$$

B. Discretization

The convenient way for numerical implementation of WT is its discretised version, called Discrete Wavelet transform (DWT). With a and b as scale and translation parameters, taking scale a: $a = a_0^m$ and the translation b: $b = n\, b_0 a_0^m$, where a_0 and b_0 are the discrete scale and translation step sizes, respectively, the DWT is given by [8]

$$W_\psi(m,n)[f] = \frac{1}{\sqrt{a_0^m}}\int_{-\infty}^{\infty} f(x)\psi\left(\frac{x-nb_0a_0^m}{a_0^m}\right) = \frac{1}{\sqrt{a_0^m}}\int_{-\infty}^{\infty} f(x)\psi(a_0^{-m}x - nb_0)\,dx \tag{4}$$

For $a_0 = 1$, the reconstruction of $f(x)$ is given by

$$f(x) \approx k\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[W_\psi f(m,n)\right]a_0^{-\frac{m}{2}}\psi(a_0^{-m} - nb_0) \tag{5}$$

where k is the constant that depends upon the redundancy of the basic wavelet and the lattice combination which is ignored in many applications and $\tilde{\psi}_{m,n}$ is wavelet dual of $\psi_{m,n}$.

III. Korteweg-De Vries Equation

The generalized Korteweg–de Vries equation with time-dependent damping and dispersion [9]:

$$q_t + q^n\, q_x + a(t)q + b(t)q_{xxx} = 0 \tag{6}$$

The first term of the equation is the evolution term, the second term represents the nonlinear term, while the third term is the linear damping with a time-dependent coefficient $a(t)$ while the fourth term is the dispersion term with time-dependent coefficient $b(t)$. In (6), $a, b \in R$ while $n \in Z^+$.

The solitary wave solution to (6) is given by

$$q(x,t) = \frac{A(t)}{\cosh^p[B(t)(x - v(t)t]} \tag{7}$$

where A represents the amplitude of the soliton, while B is the inverse width of the soliton and t represents the velocity of the soliton. Thus, for $p = \frac{2}{n}$, without loss of generality, (7) takes the form

$$q(x,t) = \frac{A}{\cosh^{\frac{2}{n}}[B(x-vt)]} = A\,sech^{\frac{2}{n}}[B(x-vt)] \tag{8}$$

when $n = 1$,

$$q(x,t) = A\,sech^2[B(x-vt)] \tag{9}$$

The same can be written as

$$q(x,t) = q(s)\ with\ s = x - vt \tag{10}$$

IV. Mathematical Analysis

Most often the signals have a Gaussian form and display self-similar fractal like patterns. We have from [4], the soliton-like solution, $u(x,\ t) = u(s)$ with $s = x - vt$ has expansion in a Gaussian family of wavelets $\psi(s) = Ne^{Q(s)}$, where $Q(s)$ is a polynomial and N the normalization constant. In particular, if we choose $Q(s) = -is - \frac{s^2}{2}$, we obtain a very particular wavelet with the support mainly confined in the $(-1,\ 1)$ interval, namely $\psi(s) = exp[-is - \frac{s^2}{2}]\pi^{1/4}$.

We shall use this fact in the development of procedure for approximating the soliton solution through wavelet decomposition and in the further application of experimental data processing.

We consider the most celebrated generalized KdV equation obtained from (6) with $n = 1$

$$q_t + qq_x + \mu q + \nu q_{xxx} = 0. \tag{11}$$

The equation (11) can be written in differential operator form:

$$\hat{L}(q(x,\ t)) = 0, \text{where } \hat{L} \equiv \frac{\partial}{\partial t} + q\frac{\partial}{\partial x} + q + \frac{\partial^3}{\partial x^3} \tag{12}$$

To apply the wavelet method or more appropriately, the wavelet-Galerkin method, the solution is decomposed with respect to the wavelet basis

$$q(s) = q(x,t) = \sum_{j,k} C_{j,k}(t)\psi_{j,k}(x) \tag{13}$$

where $C_{j,k}(t)$ are the time dependent wavelet coefficients and $\psi_{j,k}(x)$ is admissible function/basic wavelet to be taken as

$$\psi_{j,k}(x) = h^{-j/2}\psi(h^{-j}x - k).$$

This is a discrete expansion or wavelet decomposition of soliton solution $q(s)$ in terms of integer translations (k) of ψ which provide the analysis of localization, and in terms of dyadic dilations (h^j) of ψ, which provide the description of different scales.

Substituting the decomposition (13) into (12), it yields the system of equations

$$\sum_{j,k} C_{j,k}(t)\hat{L}\psi_{j,k}(x) = 0. \tag{14}$$

By scalar multiplication $\int dx\bar{\psi}_{l,m}$, where $\bar{\psi}_{l,m}$ is dual wavelet, we obtain the orthogonal system of compactly supported wavelets $\Omega_{mk}^{lj} \equiv \int dx\bar{\psi}_{l,m}\,\hat{L}\psi_{j,k}(x)$.

The system of (14) becomes

$$\sum_{j,k}\Omega_{mk}^{lj}C_{j,k} = 0. \tag{15}$$

This is a system of ordinary differential equations in the wavelet coefficients $C_{j,\ k}$.

For the orthogonal Daubechies wavelets with compact support, only the matrix elements Ω_{mk}^{lj} with the basic functions of the same scale $l = j$ are different from zero. Thus, (15) provides a sparse structure of non-linear system suitable for numerical implementation.

The main component of the wavelet-Galerkin solution is the evaluation of the matrix elements Ω_{mk}^{lj} of the differential operators in wavelet basis $\psi_{j,k}$.

For this purpose, the analytically determined wavelets such as Mexican hat, Morlet wavelets are employed.

The wavelet –Galerkin scheme for (11) consists in substitution of discrete wavelet decomposition of the solution $q(x,t) = \sum_{j,k} C_{j,k}(t)\psi_{j,k}(x)$ in to (11), followed by the projection of the result onto orthogonal basis of $\psi_{l.m}$

$$\int dx\bar{\psi}_{l,m}(x)[\,\dot{C}_{j,k} - C_{s,r}\psi_{s,r}(x)C_{j,k}\frac{d}{dx} + \mu C_{j,k} + \nu C_{j,k}\frac{d^3}{dx^3}]\,\psi_{j,k}(x) = 0 \tag{16}$$

For the orthogonal Daubechies wavelets with compact support, this gives a system of nonlinear ordinary differential equations with unknown wavelet coefficients $C_{j,k}$ depending on time t only

$$\dot{C}_{l,m} - \Omega_{mrk}^{lsj}C_{j,k} + \mu\Omega_{1,mk}^{lj}C_{j,k} + \nu\Omega_{mk}^{lj}C_{j,k} = 0 \tag{17}$$

where the matrix elements are

$$\Omega_{mrk}^{lsj} = \int dx\bar{\psi}_{l,m}(x)\,\psi_{s,r}(x)\frac{d}{dx}\psi_{j,k}(x),$$

$$\Omega_{1,mk}^{lj} = \int dx\bar{\psi}_{l,m}(x)\,\psi_{j,k}(x)$$

$$\Omega_{mk}^{lj} = \int dx\bar{\psi}_{l,m}(x)\,\psi_{s,r}(x)\frac{d^3}{dx^3}\psi_{j,k}(x). \tag{18}$$

The direct integration in the matrix elements is numerically unstable for the irregularity of the basic functions $\psi(x)$. However, they can be evaluated analytically. When all coefficients of (18) are known, the system of ordinary differential equations (17) can be solved numerically by an implicit or explicit method.

In the simplest case of an explicit scheme we have $C_{l,m}(t+\tau) = C_{l,m}(t) + \tau[\Omega_{mrk}^{lsj}C_{j,k}(t)C_{r,s}(t) + \mu\Omega_{1,mk}^{lj}C_{j,k} + \nu\Omega_{mk}^{lj}C_{j,k}(t)]$, where τ is a time step of integration.

The evaluation of the matrix elements of all differential operators is provided by the knowledge of connection coefficients- the matrix elements of those operators in the basis of wavelet scaling function $\varphi(x)$ $\Lambda_{k_1\ldots k_n}^{(d_1\ldots d_n)} = \int dx\varphi_{k_1}^{(d_1)}\ldots\varphi_{k_n}^{(d_n)}$, where the superscripts of the parentheses stand for the order of differentiation. Then all the terms with the wavelet basic functions ψ are evaluated by the substitution $\psi(x) = \sqrt{2}\sum g_n\varphi(2x-1)$. The general method of evaluation of connection coefficients is presented in [10].

Alternative scheme of evaluating the system of differential equations is provided by modified method- *Wavelet-Petrokov-Galerkin method (WPG)* [11], where we make the substitution $\psi_{j,k}(x) = h^{-j/2}\psi(h^{-j}x - k)$ in the expression (16) to write

$$\sum_{j,k}\int dxh^{-j/2}\psi(h^{-j}x-m)[\tfrac{\partial}{\partial t}C_{j,k} - C_{s,r}h^{-j/2}\psi(h^{-j}x - r)C_{j,k}\tfrac{d}{dx} + \mu C_{j,k}\tfrac{d}{dx} + \nu C_{j,k}\tfrac{d^3}{dx^3}]\,h^{-j/2}\psi(h^{-j}x-k) = \tag{19}$$

Introducing the change of variable $y = h^{-j}x - k$, the expression (19) becomes

$$\sum_{j,k}a(k)\frac{dC_{j,k}}{dt} + h^{-3j/2}\sum_{s,r}\,\sum_{j,k}b(l,k)\,C_{s,r}C_{j,k} + \mu h^{-j/2}\sum_{j,k}a(k)\,C_{j,k} + \nu h^{-3j}\sum_{j,k}d(k)\,C_{j,k} = 0 \tag{20}$$

where $a(k) = \int dy\psi(y)\bar{\psi}(y-k)$,

$$b(l,k) = \int dy\frac{d\psi(y)}{dy}\bar{\psi}(y-k),$$

$$c(k) = (-1)^\beta\int dy\frac{d^\alpha\psi(y)}{dy}\frac{d^\beta\bar{\psi}(y-k)}{dy}.$$

The unknown coefficients $C_{j,k}$ are determined from the system of ordinary differential equations written in matrix form:

$$\frac{d}{dt}LC + C^TMC + NC + TC = 0 \tag{21}$$

where

$$C = C_{j,k},\, L(l,k) = a(l-k),$$

$$M(l,k,s) = h^{-3j/2}b(l-k,l-s),$$

$$N(l,k) = \mu h^{-j/2}a(l-k), T(l,k) = \nu(h^{-3j}c(l-k).$$

Note that the unknown coefficients are only the time dependent. By Trapezoidal rule $\frac{dC}{dt} = \frac{C^{n+1}-C^n}{\Delta t}$, where $\Delta t = t_{n+1} - t_n$, the time interval.

The equation (21) becomes

$$L\left(\frac{C^{n+1}-C^n}{\Delta t}\right) + C^TMC + NC + TC = 0. \tag{22}$$

Now setting $G(C) = C^TMC + NC + TC$, we have from (22)

$$L(C^{n+1} - C^n) + \frac{G(C^{n+1}) + G(C^n)}{2}\Delta t = 0. \tag{23}$$

This algebraic equation can be finally solved by Newton's iterative method using the recursive construct

$$U^{n+1} = U^n - \frac{f(U^n)}{f'(U^n)}, n = 0,1,2,\ldots$$

The solution thus obtained by approximation process can be eventually compared with the exact solution obtained from (9) $q(x,t) = A\,sech^2[B(x - vt)]$ computed at different positions depending on time.

V. Experimental Data Processing

In experimental data processing, the central problem either in one dimensional or multi-dimensional set up is the separation of two or more signals from a noisy background. Most often these signals have a Gaussian form which itself is a wavelet. Therefore, the wavelet analysis provides robust method in the presence of noise especially if taken as wavelet image of the Gaussian $exp(-s^2/2)$ with vanishing momenta wavelets $\frac{d^n}{ds^n}exp\,(-s^2/2)$ known analytically. The central idea here is to assume the Gaussian distribution representing the soliton solution as the best fit for the experimental data and take the wavelet image of this Gaussian as testing wavelet with appropriate analytically tested function as analyzing wavelet say.

Recalling that the wavelet like soliton solution of KdV has the Gaussian form, we can therefore assume that Gaussian function is the 'best fit' to describe the experimental data set to be processed as testing wavelet function. Then, the problem of fitting the distribution of Gaussian sources is to find the parameter set $(N^k, \sigma_k, s_k^m)_{k=1}^M$ that minimizes the difference

$$F(N,\sigma,x^m) = f_{exp}(x) - \sum_{k=1}^{M} \frac{N_k}{\sqrt{2\pi\sigma_k^2}} exp\left(-\frac{(x-x_k^m)^2}{2\sigma_k^2}\right) \tag{24}$$

Applying wavelet transform to (24) with some analytically tested basic wavelet, say, Mexican hat or Morlet wavelet, one can precisely locate the position of the sources s_k^m.

Let us start with the wavelet image of a single Gaussian representing the soliton, located without loss of generality at $s^m = 0$.

$$q_{gauss}(s) = \frac{N}{\sqrt{2\pi\sigma_k^2}} exp\left(-\frac{s^2}{2\sigma_k^2}\right), s = x - vt. \tag{25}$$

We need the wavelet images of the Gaussian with different vanishing momenta wavelets as analyzing wavelet where the first m family of vanishing momenta wavelets [12], of basic wavelets ψ:

$$g_n(s) = -1^{n+1}\frac{d^n}{dx^n}exp(-s^2/2), n > 0$$

which satisfies the condition

$$\int ds\, s^m\, \psi(s) = 0, \forall m, 0 \le m < n, n \in Z.$$

The wavelet images of the Gaussian with vanishing momenta wavelet are therefore given by:

$$W_{g_n}(a,b)[q_{gauss}] = \int \overline{\frac{1}{\sqrt{a}} g_n\left(\frac{s-b}{a}\right)}\, q_{gauss}(s)ds. \tag{26}$$

The integrals in (26) can be evaluated if the Fourier representation (3)

$$W_\psi(a,b)[q_{gauss}] = \frac{1}{2\pi}|a|^{1/2}\int_{-\infty}^{\infty} \exp(ikb)\,\overline{\tilde{g}_n(ak)}\,\tilde{q}_{gauss}(k)dk, \tag{27}$$

where $\tilde{g}_n(k) = \sqrt{2\pi}(ik)^n exp(-k^2/2)$.

Instead of evaluating integrals for each n separately, we can evaluate it once for the Morlet wavelet

$$\tilde{g}(\tau,k) = \sqrt{2\pi} exp(ik\tau - k^2/2) \tag{28}$$

and then take the nth derivative of n with respect to the formal parameter τ at $\tau = 0$ to obtain the wavelet image of g_n family:

$$\tilde{g}_n(k) = \left(\frac{d}{d\tau}\right)^n\Big|_{\tau=0} \tilde{g}(\tau,k) \tag{29}$$

$$W_{g_n}(a,b) = \left(\frac{d}{d\tau}\right)^n\Big|_{\tau=0} W_{g(\tau)}(a,b) \tag{30}$$

Substituting (28) instead of $\tilde{g}_n(k)$ into (29) and taking into account the Fourier image of Gaussian (25) $\tilde{q}_{gauss}(k) = \quad N\,exp\left(-\frac{k^2\sigma^2}{2}\right)$, we arrive at

$$W_{g_n}(a,b)[q_{gauss}] = N\sqrt{\frac{a}{2\pi}}\int dk\, exp\left(ik(b-a\tau) - \frac{k^2}{2}(a^2+\sigma^2)\right) = N\sqrt{a}\frac{exp\left(-\frac{(b-a\tau)^2}{2(a^2+\sigma^2)}\right)}{\sqrt{a^2+\sigma^2}} \tag{31}$$

for the wavelet image of a single Gaussian with respect to the analyzing wavelet, vanishing momenta wavelet g_n in instant case (30).

To find the distribution parameters for the case of single Gaussian source we use the coefficients of its g_2 decomposition.

Equation (30) for $n = 2$ for example leads to

$$W_{g_2}(a,b)[q_{gauss}] = Na\left(\frac{a}{a^2+\sigma^2}\right)^{3/2}\left[1-\frac{b^2}{a^2+\sigma^2}\right]exp\left(-\frac{b^2}{2(a^2+\sigma^2)}\right) \tag{32}$$

Taking the derivative $\partial/\partial a$ of (32) at the central point $b = 0$, we find the extremum of the g_2 coefficient at a scale $a_m = \sqrt{5}\sigma$.

The value of the wavelet coefficient at the extremal point is therefore

$$W_{g_2}(a_m,0)[q_{gauss}] = \frac{N}{\sqrt{\sigma}}5^{5/4}6^{-3/2} = \frac{N}{\sqrt{a_m}}\left(\frac{5}{6}\right)^{3/2}$$

Thus, performing the convolution (1) or for numerical implementation (4) with $g_n(s) \equiv \psi_{j,k}(x) = h^{-j/2}\psi(h^{-j}x - k)$ numerically and finding the maximum of the g_2 wavelet coefficient we obtain the dispersion and amplitude of the original distribution q_{gauss} such that

$$q_{gauss} = \sum_{jk}\tilde{\psi}_{j,k} < \psi_{j,k}, q_{gauss} >.$$

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