Study of Ladder-Type Price Based on the Utility Function and Time Series

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***Abstract*—As the electricity ladder-type price scheme implemented into the trail stage, studying on it has attracted more and more attention. The present study evaluated the efficiency in energy conservation and timeliness of this scheme based on utility function and time sequence. The results show that the scheme can save energy to certain extend, but it can not work well for long. Taking Beijing as an example, ladder-type scheme can only stabilize 80% of Beijing residents' electricity cost till 2014 and further regulations are required to adapt to the development of the society.**

***Index Terms*—Utility function, ladder-type price, time sequence, exponential smoothing.**

I. Introduction

As the ladder-type price scheme went into trial in China in July 2012, the reform of the electric power industry has drawn a lot of attention. There are many international precedents of implementation of ladder-type price scheme. Japan, South Korea and the United States and other areas executed ladder-type price scheme to the residents after the oil crisis in the 1970s. There are some researches of ladder-type price scheme both at home and abroad in recent years. For example, Taylor SP *et al.* [1] studied the factors which determined the ladder-type price scheme. Koichiro *et al.* [2] came up with the elastic property of residents' power demand. Zhang lizi *et al.* [3] further discussed the application of ladder-type price scheme in China. The implementation of ladder-type price scheme, can not only contribute to the effective use of energy, but also consider the bearing ability of different residents. It can benefit the country's long-term development.

The present study evaluated the ability of the ladder-type price scheme in aspect of energy conservation and timeliness. According to the energy assessment model based on the utility theory of economics, the scheme has a positive effect on energy conservation. In addition, the variation of the consumption of electricity is forecasted and the timeliness is evaluated according to the results. Taking Beijing as an example, it shows that the residents' electricity consumption will increase unceasingly with the development of society. The current scheme can only maintain the residents' effective electricity until 2014, after that new scheme should be implemented. Ladder-type price scheme should be adapted continuously according to the development of the society, in order to achieve the effective utilization of resources and make the residents live and work in peace and contentment.

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II. Assessment of Ladder-Type Price Scheme

A. *Energy Conservation of Ladder-Type Price Scheme E*

The utility theory in microeconomics compares the different grades of the price ladder to different goods. To get the total electricity consumption of the residents, firstly take different levels of electricity as different kinds of goods. Work out the demand of every grades of electricity with the goal that the residents get the largest consumption utility. Estimate energy saving effect of ladder-type price scheme by comparing the residents' total electricity consumption in different ladder-type price scheme and establish the energy-saving evaluation model [4]. This study selected income, consumption expenditure and price as the main influence factors, which will influence consumers' expenditure.

Assume that electricity consumption is divided into grades, the division of power density for *t* (namely *t* unit of electricity for a class). The grades can be seen as kinds of goods, price is p_i , demand is x_i in the *i-th* grade according to the Stone-Geary utility function [5], as follows:

$$\text{U}=a_i\left(\sum_{i=1}^{m}\ln(x_i-\overline{x}_i)+\sum_{i=m+1}^{n}\ln(x_i-\overline{x}_i)\right) \tag{1}$$

where the $\sum_{i=1}^{n}\ln(x_i-\overline{x}_i)$ is ladder-type electricity; the is other commodities; the $\sum_{i=m+1}^{n}\ln(x_i-\overline{x}_i)$ is the demand for i-th commodity in basic standard of living; the is the sum of other types of goods and electricity consumption grade number m; the a_i is the i-th commodity marginal share of the budget, obviously that $\sum_{i=1}^{n}a_i=1$.

The constraint of residents' consumption budget is:

$$\sum_{i=1}^{n}p_i x_i - M = 0 \tag{2}$$

where the M is the total expenditure of households.

Applying the method of Lagrange multiplier [6], based on the optimization of the first-order conditions, making $b_i=\frac{a_i}{\sum_{j=1}^{n}a_j}$, then achieve residents' demand x_i for the i-th commodity:

$$x_i = \overline{x}_i + \frac{b_i}{p_i}[M - \sum_{j=1}^{n} p_j \ \overline{x}_j] \quad i=1, 2, \cdots, n \qquad (3)$$

The total electricity consumption of households is equal to the sum of electricity consumption in various grades, the formula is as follows:

$$X = \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} \overline{x}_i + \sum_{i=1}^{m} [\frac{b_i}{p_i}(M - \sum_{j=1}^{n} p_j \overline{x}_j)] \quad (4)$$

For different ladder-type price scheme, it is able to achieve the effect of energy-saving in different scheme, which is based on the results of formula (4). The smaller total electricity consumption means the better effect of energy-saving about the scheme; on the contrary, the scheme has poor effect of energy-saving.

Taking Beijing as an example, analyze the effect of energy- saving about the current scheme, on the basis of the energy-saving evaluation model, and taking two grades of ladder-type price scheme as an example.

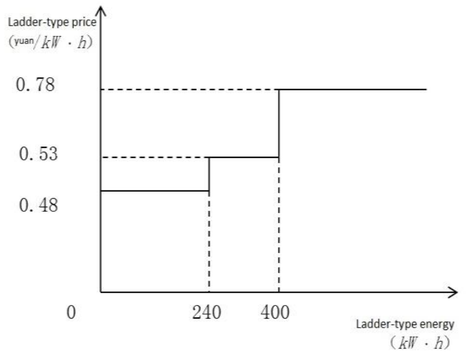

Fig. 1. Ladder-type electrovalent linear relationship.

According to the current trial scheme, the price differential (namely the price ratio of various grades) of scheme is 1:1.104:1.625.

TABLE I: THE ARRANGEMENT OF CHANNELS

scheme	electricity consumption per household (3 people)monthly /kW h			price differential
	first	second	third	
1	240	240	240	1:1:1
2	<240	240~400	>400	1:1.104:1.625

In this example, $m=3$, according to formula (4), electricity consumption per household monthly can be expressed as:

$$X = \sum_{i=1}^{3} x_i = \sum_{i=1}^{3} \overline{x}_i + \sum_{i=1}^{3} [\frac{a_i}{p_i}(M - \sum_{j=1}^{n} p_j \ \overline{x}_j)] \quad (5)$$

Referring to Beijing statistical yearbook [7], the Beijing residents' electricity consumption monthly is 6.749191 billion kW/h, each household electricity consumption monthly is 182.275 kW • h, namely $\sum_{i=1}^{3} x_i = 182.275$. The composition of the population of Beijing family: four and above households accounted for 15.8%, two-person households and three of households accounted for 59.4%. Thus the calculation based on three farmers. Total consumer expenditure of per capita monthly is 1661.167 Yuan, thereby total consumer expenditure of per household is 4983.5 Yuan, namely $M = 4983.5$. Basic living expenses of per capita monthly 838.1667 Yuan, thereby basic living expenses of per household is 2514.5 Yuan, namely $\sum_{j=1}^{n} p_j \overline{x}_j = 2514.5$.

According to previous research [8], we can get $b_1 = 0, \ b_2 = 0.01, \ b_3 = 0.001$ with the use of the method of analogy. Therefore, consumers spend surplus currency on electricity in latter two grades. Electricity consumption per household monthly turn into:

$$X = \sum_{i=1}^{3} x_i = 182.275 + (4983.5 - 2514.5)(\frac{b_1}{p_1} + \frac{b_2}{p_2} + \frac{b_3}{p_3}) \quad (6)$$

When the ladder-type price scheme is executed, $p_1 = p_2 = p_3 = 0.48$, the electricity consumption per household monthly is 239.33kW/h.

When the ladder-type price scheme executed, the price differential is 1:1.104:1.625, $p_1 = 0.48, \ p = 0.53, \ p_3 = 0.78$, the electricity consumption per household monthly is 232.03kW/h.

According to the analysis of the results, it is obvious that when implementing the scheme, the consumption will decrease 7.3 kW/h. Hence, the ladder-type price scheme can help save energy.

B. The Timeliness of Ladder-Type Price Scheme

Select the data of 2500 Beijing residents' electricity consumption and draw the electricity consumption monthly data into frequency histogram.

As can be seen from the Fig. 2, the consumption mean is 183.67, standard deviation is 51.423, and the distribution is similar to normal distribution. Analyzing the data by SPSS, is shows that the distribution is normal distribution. Forecasting the residents' electricity consumption in the next few years by exponential smoothing based on the time series. Once the consumption exceeds limit value of current scheme, it could not keep 80% of Beijing residents' electrovalence stationary, in other words, the current scheme needs to make corresponding adjustment.

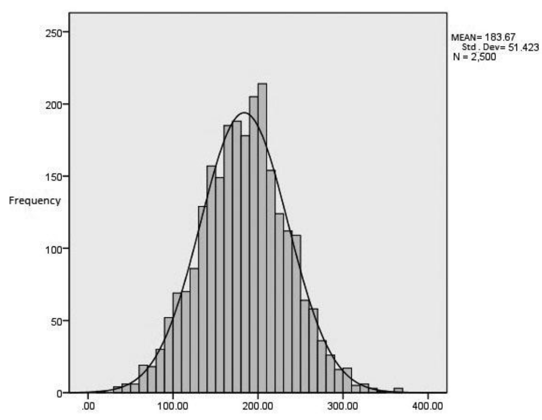

Fig. 2. The electricity consumption monthly.

According to the critical state of normal distribution, to ensure that 80% of residents` electrovalence stationary, electricity consumption should satisfy the upper limit as follow:

$$\Phi(\frac{240-\mu}{51.423})=0.8 \quad (7)$$

Check normal distribution table get, namely the upper threshold is 197.76 kW/h.

Refer to the statistical yearbook, nearly 8 years, the Beijing residents' electricity consumption as the Table II:

TABLE II: IN RECENT 8 YEARS RESIDENTS' CONSUMPTION (KW/H)

year	2004	2005	2006	2007
consumption	136.55	146.7	152.7	162.775
year	2008	2009	2010	2011
Consumption	168.7	177.35	182.275	181.8

The time series exponential smoothing method [9] is applied to forecast of Beijing residents' electricity consumption monthly.

The single exponential smoothing, set time series as $y_1, y_2, ...y_t$, α is weighting efficient, namely smoothness index $0<\alpha<1$, in that way, formula for single exponential smoothing is:

$$S_t^{(1)} = \alpha y_t + (1-\alpha)\ S_{t-1}^{(1)} \quad (8)$$

When changes of time series appear linear tendency, the forecast has a noticeable lag deviation by single exponential smoothing method. Therefore, after amending the single exponential smoothing method, it could be the second and triple exponential smoothing methods. In this paper triple exponential smoothing method is used to forecast.

Calculating formula for three exponential smoothing methods is:

$$\begin{cases} S_t^{(1)} = \alpha y_t + (1-\alpha)\ S_{t-1}^{(1)} \\ S_t^{(2)} = \alpha S_t^{(1)} + (1-\alpha)\ S_{t-1}^{(2)} \\ S_t^{(3)} = \alpha S_t^{(2)} + (1-\alpha)\ S_{t-1}^{(3)} \end{cases} \quad (9)$$

In the light of the single and second exponential smoothing methods, the model of triple exponential smoothing method can be derived:

$$\hat{y}_{t+m} = a_t + b_t m + c_t m^2, m = 1,\ 2, ... \quad (10)$$

Therein:

$$a_t = 3S_t^{(1)} - 3S_t^{(2)} + S_t^{(3)} \quad (11)$$

$$b_t = \frac{\alpha}{2(1-\alpha)^2}\ [(6-5\alpha)\ S_t^{(1)} - 2(5-4\alpha)\ S_t^{(2)} + (4-3\alpha)\ S_t^{(3)}] \quad (12)$$

$$c_t = \frac{\alpha^2}{2(1-\alpha)^2}\ [S_t^{(1)} - 2S_t^{(2)} + S_t^{(3)}] \quad (13)$$

So far, the basic forecast model of the exponential smoothing of time series is establishment.

It is important to choose a weighting coefficient in the exponential smoothing method, the size of the determined the proportion of new predictive value and the original data in the predictions. The size of reflects the amplitude of correction, the value of α is greater, the amplitude correction is larger. According to the theoretical calculation method [10], smoothness index approximation is determined by the formula: $\alpha = 2/(n+1)$.

With the improvement of living standards, the residents' electricity consumption increases speedily, it can't be ignored, especially the major cities like Beijing. Take $\alpha = 0.22$ get the predicted value of Beijing residents' electricity consumption monthly in the following four years:

TABLE III: THE ELECTRICITY CONSUMPTION IN 2012-2015(KW/H)

year	2012	2013	2014	2015
consumption	188.889	195.513	204.812	219.015

Considering the trial scheme can play a role in saving energy, then removed the savings on the predictions, the savings as 7.3 kW/h. So the electricity consumption in 2012-2015:

TABLE IV: CORRECTION OF THE ELECTRICITY CONSUMPTION IN 2012-2015

year	2012	2013	2014	2015
consumption	181.589	188.213	197.512	211.715

According to Table IV, the residents' electricity consumption monthly is 211.715 kW/h in 2015, exceed the critical value of 197.76 kW/h. the first grade 240 kW/h of the scheme can meet the 80% of Beijing residents. Therefore, the ladder-type price scheme of Beijing can only be applicable to 2014 years, after that the scheme needs to make corresponding adjustment.

III. CONCLUSION

This paper evaluates the electricity ladder-type price scheme in terms of energy conservation and timeliness based on the supply theory in economics and the time series theory.

In the paper the problem is simplified by converting evaluation problem into economics problem using utility function theory.

The implementation of the exponential smoothing method not only simplified the forecast of residents' electricity consumption, but also maintained the veracity of prediction. However, as the data is the mean, it has certain deviation. But the prediction of the total trend is true. The evaluation takes Beijing as an example. Results are satisfactory, but the scheme cannot work in the long run.

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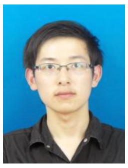

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