Relations between Melody and Rhythm on Music Analysis: Representations and Algorithms for Symbolic Musical Data

Michele Della Ventura

Abstract—**What triggers the analysis of a musical piece is essentially a pragmatic impulse: that is working something out on its own terms rather than on the terms of something else.**

Musical analysis focuses on a certain musical structure (that may be represented by a phrase or by an entire composition), aiming at defining its constituents and explaining the way they operate. The basic procedure of musical analysis is the comparison: it is through collation that it defines the structural elements and reveals the functions.

This lecture presents a model of analysis that is capable of progressively exploring the symbolic level of the musical text, identifying its melodic and rhythmic cells on the basis of the information that every single one of them carries as well as the relationships existing among them.

Index Terms—**Entropy, melodic segment, musical surface, rhythmic segment, segmentation, symbolic musical data.**

I. Introduction

In a musical piece, one of the important elements is the theme which is the most identifiable and singable musical structure. It is the expression of emotions, ideas, moods and situations: it is like a line that drives our emotions, bringing them from one side of our soul to the other.

The theme represents the fundamental motif, often a recurrent one, of a composition, especially if it is a far-reaching composition; therefore it is a melodic fragment provided with individuality and recognizability, often to such an extent as to characterize the entire musical piece. The motifs are never explicitly indicated by the composer on the score, yet a fragment of notes extracted out of it does not necessarily constitute a motif. This lack of indication gives to the very motif a mysterious character as if it were a secret that listening and analysis have the assignment to reveal [1].

The musical theme is a melody composed of sounds and a rhythm: a pitch (sound), in fact, never shows up by itself, but the succession of pitches is always organized in time (rhythm). "*Melody in itself is weak and quiescent, but when it is joined together with rhythm it becomes alive and active*" [2].

Rhythm, therefore, bestows individuality and recognizability (see Fig. 1), yet, in the mean time, it characterizes the entire musical piece by means of rhythmic cells, that is groups of signs (or signs and rests) that create, within the musical discourse, recurring rhythmic schemata that could also be different at a melodic level (see Fig. 2) [3].

Fig. 1. Excerpt from the score of Vivaldi's "Autumn". The initial notes of the theme are represented on the first staff without any indication with respect to rhythm; the same notes are represented of the second staff together with the rhythm assigned by the composer.

Fig. 2. J. S. Bach's Partita in E minor BWV 830. The rhythmic cell of the 9th beat is represented on the first staff, while on the second and third staff there is the same cell that we can find respectively at the 34th beat and at the 13th beat.

The sounds and the rhythm, therefore, must never be considered separate entities, and for this reason, often a "rhythmic cell" existing in a musical piece, considered separately from the sounds, may identify at the same time a motif (or theme) [1].

The essence of the analysis is to define what is meant by melodic and rhythmic repetition; moreover which of the textual repetitions existing in the score may be associated to the notion of "motif" or "rhythmic cell" and, in case there are several different repetitions, what criterion to adopt in order to select only the part that may be considered interesting by an expert [4].

II. Information Theory

In the various algorithms realized for the rhythmic segmentation, the final choice of the segment is performed by comparing every single fragment found with a specially drawn up list of rules. The proposed model intends to present a new choosing methodology, based on the assessment of the *information* that every rhythmic fragment identified carries within.

The analysis based on the information theory sees music as a linear process supported by its own syntax [5]. However, it

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Michele Della Ventura is with the Conservatory of Music "A. Buzzolla" Viale Maddalena 2 ADRIA, Italy (e-mail: dellaventura.michele@tin.it).

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is not a syntax formulated on the basis of grammar rules, but rather based on the occurrence probability of every single element of the musical message compared to their preceding element. The fact that music is assimilated to a message transmitted by a sender to a receiver, or that information is mentioned, may give the impression of an analytical approach of the interpretive type, interested in the meanings of communication. In fact, here, the interest goes exclusively to the manner in which the expectations of the receiver are spurred, fulfilled or let down.

From the definition of "message" as a chain of discontinuous "units of meaning", follows that the musical "units of meaning" coincide with the minimal events of a composition: usually isolated notes, chords. Any event of such a chain demands a prediction as to the event that will follow it: there is information transmission when the prediction turns out to be disregarded, there is none when it turns out to be confirmed. In other words, the greater the unpredictability degree of the content of a certain message, the larger the amount of information contained in the message. Moreover, the fact that the events of a composition are modularly organized indicates the possibility to calculate, using a formula, or to express, using an index, the total *"information"* transmitted by a certain musical module.

According to W. Weaver and C. Shannon, the concept of information [6] includes:

The symbolic apparatus through which it manifests. Information is an abstract concept that materializes through a set of symbols, while the meaning assigned to the symbols that represent it allows the information to be interpreted.

The semantic content conveyed by it. The set of symbols *that* convey a certain piece of information contains a precise meaning that characterizes its value; and it is this very meaning the one that constitutes the semantic part of the piece of information. In this regard, the manner in which information is sensitive to the state of the receiver: information loses value if it is already known to the receiver.

The pragmatic effect produced by it. Every single piece of *information* carries meanings with the intent to produce a certain effect.

In a communication which takes place by way of a given alphabet of symbols, information is associated to every single transmitted symbol [7]. Therefore, information can be defined as *the reduction of uncertainty that would have been obtained a priori on the transmitted symbol.* The wider the range of messages that the source can transmit (and the greater the uncertainty of the receiver as to the possible message) the larger the amount of information transmitted and along with it, its extent: the *entropy*.

In the information theory, the entropy is a positive value and not a negative one as it is originally in physics. From a mathematical perspective, the extent of the content of a certain piece of information *(I)* is obtained using Shannon's formula:

$$I = \log_2 \frac{p'}{p}$$

where p is the probability for the message to be transmitted, p' is connected to the use that the observer makes of the message and it coincides with the probability to materialize of the content of the information expected by the observer after the message has been transmitted.

III. Entropy

In information theory [6] (in connection with the signal theory [8]) *entropy* measures the amount of uncertainty or of information existing in a random signal. For every single symbol (of a message) that we transmit, we have a certain amount of information associated to that specific symbol. In most practical applications of information theory a choice among the messages of a set must be made and every message has its own probability of being transmitted. Shannon provided the definition of entropy of such a set, identifying it as the information content that The choice of one of the messages will transmit. If every single message has a probability p' of being transmitted, the entropy is obtained by summing all the set of the functions $p' \log 2\, p'$, every one relating to a message, that is:

$$H(X) = E[I(x_i)] = \sum_{i=1}^{n} I(x_i) \bullet P(x_i) = \sum_{i=1}^{n} P(x_i) \bullet \log_2 \frac{1}{P(x_i)}$$

The term entropy, borrowed from thermodynamics, designates, therefore, the average information content of a message.

In light of everything said so far, the musical message can be defined as a sequence of signals organized according to a code.

IV. Musical Message and Information

To compare various segments among them, in order to determine which is more important, each entropy is calculated: the less the entropy value, the greater the information carried by the segment [1].

In order to calculate the entropy one must refer to a specific alphabet: the alphabet is language – specific [9] and, as it may be immediately inferred from the formula (based on the probability of certain symbols rather than other symbols to be transmitted) it proves to be associated to language. In the case of musical language a distinction was made between the alphabet used for the melodic analysis and the one used for the rhythmic analysis:

1) For the **melodic analysis** the various melodic intervals were classified as symbols of the alphabet [1]. The classification of an interval consists in the *denomination* (generic indication) and in the *qualification* (specific indication). The *denomination* corresponds to the number of degrees that the interval includes, calculated from the lowest one to the highest one; it may be of a 2^{nd}, a 3^{rd}, 4^{th}, 5^{th}, and so on.; the *qualification* is deduced from the number of tones and semi-tones that the interval contains; it may be: *perfect, major, minor, augmented, diminished, more than augmented, more than diminished, exceeding, deficient.*
2) For the **rhythmic analysis** the duration of the different sounds was taken into consideration: *duration intended as the time interval in which sound becomes perceptible,*

regardless of whether it is due to a single sign or to several signs joined together by a value connection [10]. This principle allows us to go past musical concepts such as "accent", "meter" or "bar" (see Fig. 3).

Fig. 3. J. S. Bach's two voice invention in E major BWV 777 (the first three beats of the first staff).

If we were to analyze a score, the sound duration will not be expressed in seconds but calculated (automatically by the algorithm) on the basis of the musical sign (be it sound or rest) with the smallest duration existing in the musical piece. The duration of every single sign will therefore be a (integer) number directly proportional to the smallest duration. In order to distinguish between sound and rest, the latter is associated to a duration value directly proportional to the smallest duration, but negative [11].

Fig. 4. J. S. Bach's two voice invention in E major BWV 777 (the first three beats if the first staff). Graphic representation of the sounds based on duration.

The characteristic of the language on which it was developed is extracted from every single musical piece, analyzing a posteriori, respectively, the distribution of the intervals for the melodic analysis (considering, for every single interval, its evolution whether it is ascendent of descendent) and, the durations of the sounds for the rhythmic analysis (distinguishing sounds from rests).

For every single musical piece a table, which represents its own alphabet, is filled in: in the melodic analysis every interval has been considered for its ascending or descending trend (Fig. 6 shows an example of the invention with two voices of BACH n1 in D Major BWV 772 relative to the first staff only) and in the rhythmic analysis sounds and rests have been distinguished (Fig. 7 shows an example drawn out of J. S. Bach's Two Voice Invention in E Major BWV 777).

Determining the alphabet thus identified is not enough in order to calculate the entropy of a musical segment: it is necessary to consider the manner in which the durations succeed one another inside the musical piece.

In order to do this, the *Markov process* (or Markov's stochastic process) is used: we chose to deduce the transition probability that determines the passage from a state of the system to the next uniquely from the immediately preceding state [1].

Therefore the *transition matrix* is created. It is made up of the transition probabilities between the states of the system (*conditional probability*). In our case, the matrix represents the probabilities for a kind of interval or duration to resolve to another kind of interval or duration.

In order to better clarify these concepts, let us consider the following example relating to the melodic analysis.

Fig. 5. J. S. Bach's two voice invention in E major BWV 777 (the first three beats of the first staff).

In the score we consider the first interval mi-re#; this corresponds to a minor interval (2m), descending (d) that precedes re#-re interval, that is again a 2[nd] minor interval.

Transition matrix will be updated, incrementing by one unit, the corresponding box at the crossing of the two intervals (see Table I).

TABLE I: EXAMPLE OF TRANSITION MATRIX

Interval			2m		2M		3m		3M		4G		…
			1		2		3		4		5		
			a	d	a	d	a	d	a	d	a	d	
2m	1	a											
		d		1									
2M	2	a											
		d											
3m	3	a											
		d											
3M	4	a											
		d											
…													

TABLE II: EXAMPLE OF A PARTIAL MATRIX OF THE TRANSITIONS DRAWN OUT OF J.S. BACH'S TWO VOICE INVENTION IN E MAJOR BWV 777, RELATING TO THE MELODIC ANALYSIS

Interval			2m		2M		3m		3M		4G		…
			1		2		3		4		5		
			a	d	a	d	a	d	a	d	a	d	
2m	1	a											
		d		2									
2M	2	a											
		d											
3m	3	a											
		d											
3M	4	a											
		d											
…													

Interval			2m		2M		3m		3M		4G		…
			1		2		3		4		5		
			a	d	a	d	a	d	a	d	a	d	
2m	1	a	1	3	2			1					
		d	3	4		4	1					1	
2M	2	a	3		3			1		2			
		d		3	5	4					1		
3m	3	a				1							
		d				1	1	2		3			
3M	4	a				1							
		d		1				2	1				
…													

TABLE III: EXAMPLE OF A MATRIX OF THE TRANSITIONS DRAWN OUT OF J.S. BACH'S TWO VOICE INVENTION IN E MAJOR BWV 777, RELATING TO THE RHYTHMIC ANALYSIS

Duration		1		2		4		8	
		𝅘𝅥𝅰	𝅀	𝅘𝅥𝅯	𝄿	𝅘𝅥𝅮	𝄾	𝅘𝅥	𝄽
1	𝅘𝅥𝅰	19		16		2		1	
	𝅀								
2	𝅘𝅥𝅯	15		29		7			
	𝄿			2		3			
4	𝅘𝅥𝅮	4		5	3	63	2		
	𝄾				2	1			
8	𝅘𝅥					1			
	𝄽								

Now, considering this last interval, we can notice that following re-do# is a 2nd minor interval (2m), descending (d), so we update matrix incrementing one unit the corresponding box at the crossing between 2nd major ascending intervals with 2nd minor ascending intervals.

This procedure repeats itself up to the end of the score (Table II).

The same procedure above described has been used to find the transition matrix of the rhythmic analysis (Table III).

V. The Results Obtained

The proposed model of analysis was verified through the realization of an algorithm which, on the basis of the above-mentioned considerations, allows:

1) The automatic dimensioning of the table which represents the alphabet and the transition matrix, based on the characteristics of the musical piece being examined;
2) Indicating the analysis results by means of numeric values which are representative for the entropy value of the single identified segments (be them melodic or rhythmic). From the definition of entropy it can be inferred that there will not be values above zero, hence it is important to evaluate the number of decimals that may influence the identification of the most important segments: bigger the number of decimals, higher the accuracy of the analysis;
3) Presenting the results in order, in tables to be, if necessary, converted into diagrams so as to increase their degree of comprehensibility.

The most interesting aspect emerged out of this kind of analysis was the focus on the importance of rhythm wit respect to sounds. As already mentioned in the first paragraph, a melody without rhythm (therefore made up only of sounds) is weak and inert, yet a rhythm without sounds has meaning anyway. The algorithm allowed the calculation of entropy separately for every single melodic and rhythmic segment and by comparing them it was possible to find that:

1) In the case of melodic analysis, the single "themes" were identified;
2) In the case of rhythmic analysis, the "rhythmic cells" were identified: sometimes existing singly in the musical piece and sometimes identifying a certain "theme";
3) For every single melodic segment identified, the entropy value of the represented rhythmic segment was calculated and this value turned out to be consistent with the ones emerged from the melodic analysis: the melodic segment with the lowest value of entropy had the lowest value at a rhythmic level.

The results of the analysis of two musical pieces are shown below:

J. S. Bach's Two Voice Invention in E Major BWV 777,

The first movement of L. Van Beethoven's "Pastoral" Symphony No. 6, op. 68.

A. J. S. Bach's Two Voice Invention in E Major BWV 777

The Two Voice Invention BWV 777, is part of a collection of fifteen pieces of which every single piece contains two voices, that is two melodies that move independently, which interweave, communicate in a jocular manner with each other, conferring unity and consistency to the musical piece (Fig. 6).

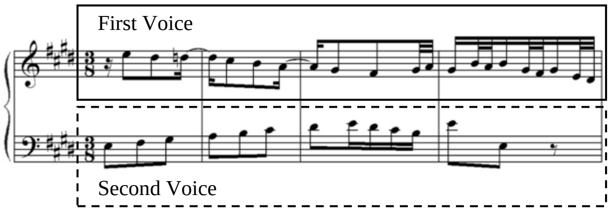

Fig. 6. The first four beats of Bach's two voice invention BWV 777.

An excerpt of the final Table (Table IV) restored by the algorithm is shown below, where in the first two columns there are the initial position of the segment so that it can be identified on the score and its length (how many notes it is made up of) whereas the entropy value of that particular segment is represented in the third and in the fourth column.

TABLE IV: Excerpt of the Final Table Restored by Algorithm

		Melodic analysis	Rhythmic analysis
Position beginning segment	Length segment	Entropy	Entropy
93	13	0,000000000044349	0,00000000000005
1	19	0,000003715139754	0,000000000000329
118	36	0,000003977415488	0,000000000007129
157	30	0,000013327743474	0,000000000013643
….	….	….	….

The first segment, identified by the algorithm with the melodic analysis, corresponds to the exposition of the second voice and the second segment corresponds to the first voice (Fig. 6). The same segments have the lowest value of entropy at a rhythmic level. This result is the most important, because it reinforces the theory that melody and rhythm are two fundamental inseparable components as far as musical structuring is concerned: a melody evolves along the rhythm in the absence of which it does not exist [10].

For a better data evaluation, and to appreciate them in their real meaning, we have projected the entropy values on a graph (Fig. 7), using as a reference not "uncertainty" represented by it, but instead the information (less uncertainty means more information).

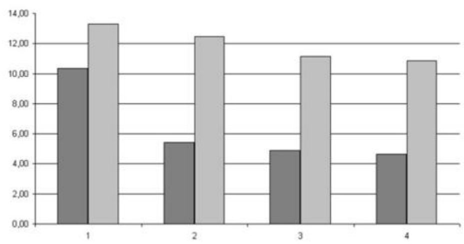

Fig. 7. Graphic representation of information carried by various segments found by the segmentation process. The scale used is the logarithmic scale that shows very low values as well as very high ones.

B. The First Movement of L. Van Beethoven's "Pastoral" Symphony No. 6, op. 68

The choice of this piece was suggested to evaluate the strength of algorithm, through a polyphonic piece. In this musical piece, the entire first movement is nothing less than a constant metaphorical transformation of the material contained in the first four opening beats (Fig. 8).

Fig. 8. The first four beats of Beethoven's "Pastoral" Symphony No. 6.

An excerpt of the final table restored by the algorithm is shown below (Table V).

TABLE V: EXCERPT OF THE FINAL TABLE RESTORED BY ALGORITHM

		Melodic analysis	Rhythmic analysis
Position beginning segment	Length segment	Entropy	Entropy
48	40	0,000000000000098	0,000000000000049
128	42	0,000000000146605	0,000000000000051
312	48	0,000000025636030	0,000000000000052
321	45	0,000000026315300	0,0000000000000524
….	….	….	….

Once more, the first segment, identified by the algorithm corresponds to the main theme, that has the lowest value of entropy both for the melodic analysis and for the rhythmic analysis (see Table V and Fig. 9).

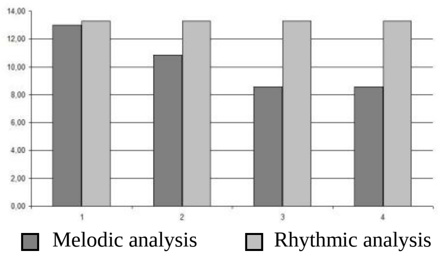

Fig. 9. Graphic representation of information carried by various segments found by the segmentation process.

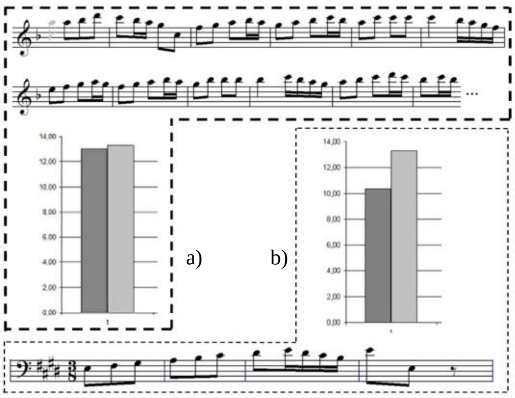

Fig. 10. Comparison between entropy values between main themes: the first theme, from Beethoven's "Pastoral" Symphony No. 6, and the second one from Bach's two voice invention BWV 777.

Analyzing at the main time melody and rhythm, it came out a very important result as you can see in the graphics of my studies: if a theme has many notes (see Fig. 10 (a)), the gap of entropy value between melodic and rhythmic level is minor than in the other one with few notes (see Fig. 10 (b)).

VI. CONCLUSION

Computational musical analysis is a continuously changing discipline, either because of the relapses originating from technological research, or because of the continuous influences deriving from the studies on cognitive sciences that, focusing on the comprehension of the human mental processes involved in the musical activities, may assume a decisive role in this kind of analysis.

The analysis model proposed follows an innovative point of view that allows for the discovery and exploitation of the information which is consistent with our perception: first of all there was an attempt to find a unique analysis system, valid for any *musical form*; then there was an attempt to look at the results from a strictly scientific perspective, using statistics.

The high degree of complexity of musical phenomena imposes certain forms of achievement that must be adequate and that, for completeness' sake, must cope with the problems under a sufficiently large number of angles.

Thus, even from a theoretical - musical point of view, the possibility to integrate different approaches appears as a precursory way of interesting developments. And it is really thanks to the new techniques of artificial intelligence that such forms of integration and verification of the results become achievable.

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Michele Della Ventura defined his professional training, since high school, within the framework of two distinct areas of study: music and mathematics.

He embarked on a course of study of the Pianoforte under the guidance of Donato Cuzzato, continuing then with Francesco Bencivenga under the guidance of whom he brilliantly graduated from the "A. Steffani" Conservatory of Music of Castelfranco Veneto in 1993.

His dual formation, in Information Technology and music, drives him into carrying out research activities on the relation between Music and Mathematics, attending several seminars on the "Algebraic Formalization of Musical Structures" held by Moreno Andreatta (researcher at IRCAM in Paris) during which he achieves the algebraic formalization of the "Sechs Kleine Klavierstücke op. 19" by Arnold Schönberg.

The development of education-related technologies draw him to focus his attention on the innovations of information technology associated to musical programming languages and to attend a Post-Graduate Master's Degree on E-Learning (E-Learning: methods, techniques and applications) at the University Tor Vergata of Rome, graduating from it with the highest marks with the thesis "Learning and new technologies".

His research activity continues within the framework of computer-aided musical analysis, publishing articles and holding national and international conferences and seminars on "Music and new technologies".

He combines his research and IT consultant activity with the position of Music informatics Professor at Music Academies and Conservatories and the position of Musical Technologies Teacher in High Schools specializing in Music.