

# Upper Bound Analysis for Rectangular Shaped Sintered Metal Powder Preform Considering Homogeneous Pattern of Deformation

Parveen Kumar, R. K. Ranjan, and Rajive Kumar

**Abstract**—The paper reports on an investigation into the various aspects of high speed open die forging of Cu metal powder preforms, which have been compacted and sintered from atomized Cu metal powder. An attempt has been made for the determination of the relative average die pressure developed for given geometries of the disc during the open die forging of sintered metal powder preform by using an upper bound approach. The deformation characteristics of metal powder preform has been demonstrated by applying an appropriate interfacial friction law and yield criteria. The results so obtained are discussed critically to illustrate the interaction of various process parameters involved and are presented graphically. The deformation pattern is influenced by many factors which interact with each other, the main factors are the interfacial friction, initial density of the preform and the geometry of the preform. Different frictional stresses are assumed on top and bottom interfaces of the die.

**Index Terms**—Interfacial friction, open die forging, sintered preform, upper bound method.

## I. INTRODUCTION

Sinter metal forming technology is a rapidly developing near net shape mass production technology. The final density of sinter formed products compares favorably with that of wrought products [1]-[2]. We can get the product of desired properties by using the sintered preform. Metal powder technology is creating interests in many parts of the world as the technology has extensive applications in the field of automobiles, aerospace, defense and other household products [3]-[5]. This paper concerns the pressure distribution at the workpiece interface and die load during the cold forging of the disc at a particular speed, during the analysis an appropriate interfacial friction law and yield criterion for porous metal is used, as characteristics of porous materials during compression have to be taken into consideration. The densification and compression of sintered preform takes place simultaneously therefore volumetric constancy is not possible, as preform's density changes due to closing of inter particle pores. Thus, suitable yield criterion is required, which is dependent on relative density of preform. The high interfacial pressure, which is applied for deformation, breaks the die-work piece interfacial lubricant film therefore, we

have to consider composite friction including both sliding and sticking friction [6]-[9]. To analyze the deformation behavior of the sintered preform which helps to decide various parameters of the process in practical work and for industrial propose. Upper bound method approach seems to be the best appropriate technique as Upper bound method is better than the equilibrium method [12]. The results so obtained are discussed critically to illustrate the interaction of various parameters involved and. Friction condition are of the greatest importance. The relative velocity between the workpiece material and the die surface together with high interfacial pressure and/or deformation modes will create the conditions essential for a adhesion in addition to sliding. For such a mechanism of composite friction, the shear equation

$$\tau = \mu \left[ p + \rho_o \phi_o \right] \quad (1)$$

where the first term on the right is the sliding friction and the second is the friction due to adhesion, which is due to change of the relative density of the preform. The pattern of metal flow during the compression of a metal powder preform is such that there exists two zones, an inner one where no relative movement between workpiece and die occurs (the sticking zone), and an outer zone where sliding occurs. Therefore, the appropriate interfacial friction laws for different conditions given by Rooks [10] are :

$$\tau = \mu \left[ P + \rho_o \phi_o \left\{ 1 - \left( \frac{r_m - r}{nr_o} \right) \right\} \right] \quad (2)$$

## II. UPPER BOUND APPROACH

### A. Velocity Field

$$U_y = \frac{(1-2\eta)}{2(1+\eta)} \left[ \frac{2U}{h} y - \frac{2U}{h} \alpha y \right] ;$$

$$U_x = \frac{2U}{h} \alpha x ; \quad U_z = -\frac{2U}{h} z$$

### B. Strain Rate

$$\epsilon_{xx} = \frac{2U}{h} ; \quad \epsilon_{zz} = -\frac{2U}{h} \alpha ; \quad \epsilon_{yy} = \frac{(1-2\eta)}{2(1+\eta)} \frac{2U}{h} - \frac{2U}{h} \alpha$$

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C. Compressibility Equations for Powder Components

$$\varepsilon_{xx} + \varepsilon_{yy} + \frac{1-2\eta}{2(1+\eta)} \varepsilon_{zz} = 0$$

D. Deformation Energy

$$W_d = \frac{2\rho^k \sigma_0}{\sqrt{3}(1-2\eta)} \int_v \left( \frac{1}{2} (\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2) \right)^{\frac{1}{2}} dv$$

$$W_d = \frac{4\rho^k \sigma_0 U}{\sqrt{3}(1-2\eta)h} \int_v \left( \alpha^2 + \frac{(1-2\eta)^2}{8(1+\eta)^2} - \frac{(1-2\eta)}{2(1+\eta)} \alpha + \frac{1}{2} \right)^{\frac{1}{2}} dv$$

$$W_d = \frac{wl\rho^k \sigma_0 U}{2\sqrt{3}(1-2\eta)} \left( \alpha^2 + \frac{1-2\eta}{8(1+\eta)^2} - \frac{1-2\eta}{2(1+\eta)} \alpha + \frac{1}{2} \right)^{\frac{1}{2}}$$

Friction Power

$$W_f = \int_v \tau |\Delta v| ds \quad ; \quad \Delta v = (U_x^2 + U_y^2)^{\frac{1}{2}}$$

$$\Delta v = \frac{2U}{h} \left[ \alpha^2 x^2 + \frac{(1-2\eta)^2}{4(1+\eta)^2} (1-\alpha)^2 y^2 \right]^{\frac{1}{2}}$$

$$W_f = \frac{2U}{h} \tau \iint_s \left[ \alpha^2 x^2 + \frac{(1-2\eta)^2}{4(1+\eta)^2} (1-\alpha)^2 y^2 \right]^{\frac{1}{2}} dx dy$$

E. Energy Dissipation Due To Inertia Force

$$W_a = \frac{\rho_p}{g} \int_v a_i U_i dv$$

$$= \frac{2N\rho_p}{g} \int_0^h \int_0^w \int_0^l (a_x U_x + a_y U_y + a_z U_z) dx dy dz \quad (3)$$

$$a_x = U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} + U_z \frac{\partial U_x}{\partial z} + \frac{\partial U_x}{\partial t}$$

$$a_y = U_x \frac{\partial U_y}{\partial x} + U_y \frac{\partial U_y}{\partial y} + U_z \frac{\partial U_y}{\partial z} + \frac{\partial U_y}{\partial t}$$

$$a_z = U_x \frac{\partial U_z}{\partial x} + U_y \frac{\partial U_z}{\partial y} + U_z \frac{\partial U_z}{\partial z} + \frac{\partial U_z}{\partial t} \quad (4)$$

$$\frac{\partial U_x}{\partial x} = \frac{2U}{h} \alpha \frac{\partial U_x}{\partial y} = 0;$$

$$\frac{\partial U_x}{\partial z} = 0; \quad \frac{\partial U_x}{\partial t} = \frac{2\dot{U}}{h} \alpha x$$

$$\frac{\partial U_y}{\partial x} = 0; \quad \frac{\partial U_y}{\partial y} = \frac{(1-2\eta)2U}{2(1+\eta)h} - \frac{2U}{h} \alpha;$$

$$\frac{\partial U_y}{\partial z} = 0; \quad \frac{\partial U_y}{\partial t} = \frac{(1-2\eta)2\dot{U}}{2(1+\eta)h} - \frac{2\dot{U}}{h} \alpha y$$

$$\frac{\partial U_z}{\partial x} = 0; \quad \frac{\partial U_z}{\partial y} = 0; \quad \frac{\partial U_z}{\partial z} = -\frac{2U}{h}; \quad (5)$$

Put these values in equation (4), we have,

$$a_x = \frac{4U^2 \cdot \alpha^2 x}{h^2} + \frac{2U \dot{\alpha} x}{h};$$

$$a_x U_x = \frac{8U^3 \cdot \alpha^3 \cdot x^2}{h^3} + \frac{4U \dot{U} \cdot \alpha^2 \cdot x^2}{h^2}$$

$$a_z = \frac{4U^2 \cdot z}{h^2} - \frac{2\dot{U}}{h} z$$

$$a_z U_z = -\frac{8U^3 z^2}{h^3} + \frac{4U \dot{U} z^2}{h^2}$$

$$a_y = \frac{(1-2\eta)^2 4U^2 \cdot y}{4(1+\eta)^2 h^2} - 2 \frac{(1-2\eta)4U^2 \alpha \cdot y}{2(1+\eta)h^2}$$

$$+ \frac{4U^2 \alpha^2 \cdot y}{h^2} + \frac{(1-2\eta)2\dot{U} \cdot y}{2(1+\eta)h} - \frac{2\dot{U}}{h} \alpha \cdot y$$

$$a_y U_y = \left[ \frac{(1-2\eta)^3 U^3 \cdot y^2}{(1+\eta)^3 h^3} + \frac{4(1-2\eta)^2 U^3 \cdot \alpha y^2}{(1+\eta)^2 h^3} \right.$$

$$+ \frac{4(1-2\eta)U^3 \cdot \alpha^2 y^2}{(1+\eta)h^3} + \frac{(1-2\eta)U \dot{U} \cdot y^2}{(1+\eta)^2 h^2} -$$

$$\left. \frac{2(1-2\eta)U \dot{U} \alpha \cdot y^2}{(1+\eta)h^2} \right] (1-\alpha)$$

From equation (3), after integration we get-

$$W_a = \frac{2N\rho_p}{g} \left[ \left\langle \frac{8U^3 \alpha}{h} + 4U \dot{U} \right\rangle \frac{wl^3 \alpha^2}{3h} + \left\langle (1+\alpha) \left( (A^3 - 4A^2 \alpha + 4A \alpha^2) \frac{U^3}{h} + \right) \frac{w^3}{3h} + \left\langle (4U \dot{U} h - 8U^3) \right\rangle \frac{lw}{3} \right] \quad (6)$$

$$A = \frac{(1-2\eta)}{(1+\eta)}$$

F. Die Load

For plastic deformation, external power  $\dot{J}$  is

$$J = W_i + W_f + W_a + W_t \quad (7)$$

( $W_i$ ) denotes therate of internal energy dissipation  
 ( $W_f$ ) denotes the frictional shear energy losses  
 ( $W_a$ )denotes energy dissipation by inertia forces  
 ( $W_t$ ) covers the power supplied by body tractions.

In this case no external surface traction is stipulated. Therefore,

$$W_t = 0. \quad J = \int F_i U_i ds = PU \quad (8)$$

Optimization of total power

$$J = W_d + W_f + W_a \text{ Now, } p = \frac{P}{\left(\frac{wl}{4}\right)}; \text{ for}$$

$$\rho_0 \phi_o = x.p = x \frac{P}{\left(\frac{wl}{4}\right)} \quad x = 0.1, 0.2, 0.3,$$

$$J = p \frac{wl}{4} U \quad (9)$$

### III. EXPERIMENTAL PROCEDURE

#### A. Preparation of Specimens

##### 1) Compaction

Metal powder was compacted in a closed rectangular die (bore 15 mm X 10 mm) using a 150 Ton hydraulic press at various recorded pressures. The compaction arrangement is shown in Fig. 1

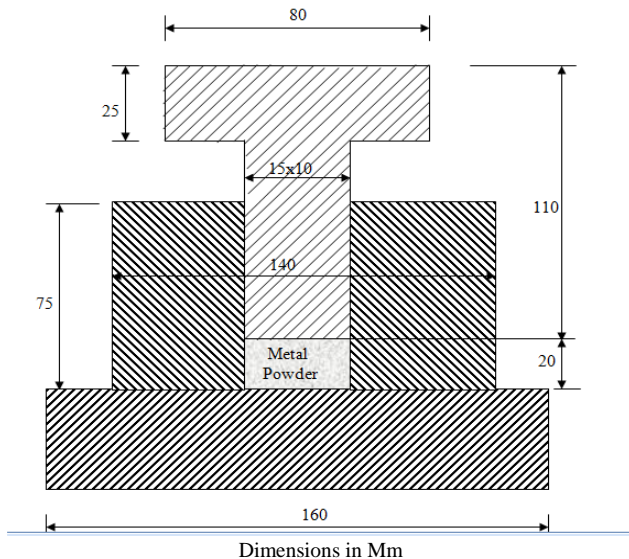


Fig. 1. Compaction of metal powder perform.

##### 2) Sintering

Sintering of copper compacts was carried out at 850°C respectively for two hours .All sintering operations were carried out in a muffle type silicon carbide furnace capable of providing sintering temperature unto 1300°C with an accuracy of ± 5°C. In order to minimize the non-uniformity of density distribution, the sintered compacts were re-pressed at

the same compaction pressure in the same die. The specimens were resintered at the same temperature and time.

##### 3) Machining

The specimens were made by machining the compacts to the desired dimensions. The surfaces of the specimens were polished with fine emery paper.

##### B. Measurements.

Experiments were conducted on a Universal Testing Machine using appropriate dies. The Cu metal powder preform of known relative density was placed inside the conical converging dies and was compressed at room temperature by applying the load. The compression was carried out in dry and lubricated conditions. Fine graphite powder was applied as lubricant. The following important measurements were made:

- 1) Increase in the relative density of the preform with increase in compressive load.
- 2) Variation of  $p/\sigma_0$  (Relative forging pressure)of the perform with percentage reduction in height
- 3) Variation of  $p/\sigma_0$  of the perform with change in density at 40 % reduction in height
- 4) Increase in the relative density of the preform with increase in  $p/\sigma_0$  (Relative forging pressure)

Electrolytic Copper powder of greater than 99% purity was used for preparation of test piece

TABLE I: PHYSICAL CHARACTERISTICS OF POWDER USED APPARENT DENSITY 2.60 G/CC TAP DENSITY 8.96G/C.

Screen Analysis (micron)	+100	-100 +150	-150 +200	-200 +240	-240 +350	-350
% Weight Retained	0	35	15	14.5	20	14.5

TABLE II: CHEMICAL ANALYSIS (WEIGHT % OF POWDER).MAXIMUM LIMITS OF IMPURITIES-

Copper	99.80%
Phosphorous	< 0.001%
Iron	< 0.006%
Silicon	< 0.002%

### IV. RESULT AND DISCUSSION

In Cu metal powder forming process compaction (densification) and deformation happen simultaneously. Material flows mainly in the direction of punch movement, with a little lateral flow. As the density increases, lateral flow increases. In the final stage of deformation. let us assume following set of data

$$n = 1, 2 ; x=0.1, 0.2,0.3; \\ \mu_1=0.3, \mu_2=0.3, 0.35, 0.4, 0.45; \\ \rho=0.70, 0.75, 0.80, 0.85, 0.90, 0.95$$

Percentage reduction in height of the perform = 10%, 20%, 30%, 40% .

We have considered different interfacial friction at the different surfaces.

Fig. 5 shows the variation of relative average forging pressure ( $p/\lambda$ ) with relative density of sintered preform by

Upper bound method. The curves express the theoretical results at 40% reduction of the preform for different values of the coefficient of friction at upper and bottom surfaces, ignoring the deformation due to inertia factor by upper bound and equilibrium method. As relative density of the sintered perform increases, the required amount of relative forging pressure increases, experimental values of the relative forging pressure is nearly in close agreement with the theoretical values obtained by upper bound method.

Fig. 6, Fig. 7, Fig. 8, Fig. 10 and Fig. 11 shows the variation in relative average forging pressure ( $p/\lambda$ ) with percentage reduction in height of the metal powder perform at different initial relative density by Upper bound method at  $n=1, 2$  with different values of the coefficient of friction at upper and bottom surface considering the inertia factor These figures show the variation of relative density with average relative forging pressure.

Fig. 9 shows the variation of relative average forging pressure ( $p/\lambda$ ) with relative density of sintered preform by Upper bound method at 40% reduction.

Fig. 12, Fig. 13 shows variation in  $p/\lambda$  (Relative forging pressure) with % reduction in height of the metal powder perform by both the methods at  $\rho=0.8$   $\mu_1 = \mu_2$ , We take different values of  $\rho_0\phi_0$ .

These Figures shows the theoretical compressive relative pressure versus percentage reduction. The curves express the theoretical results for a particular initial relative density of the preform and for various values of the coefficient of friction at upper and bottom surfaces. The compressive relative pressure is found to increase with increase in percentage reduction in height and the coefficient of friction, compressive relative pressure is also found to increase with increase in the value of  $\rho_0\phi_0$ . Inertia factor is also playing a vital role in deforming the workpiece. it is expected that the result of this paper will help the academicians who are working in this field.

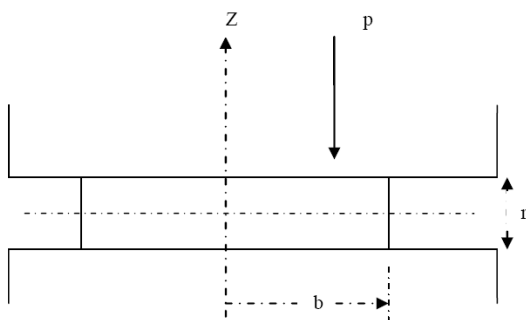


Fig. 2. Schematic diagram of disc (upper bound method).



Fig. 3. Sintering furnace.



Fig. 4. Hydraulic press.

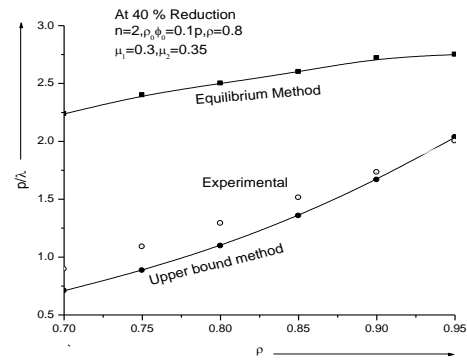


Fig. 5.

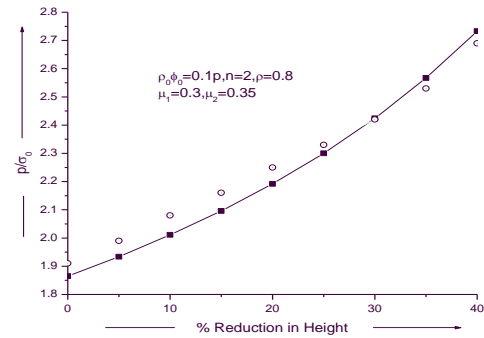


Fig. 6.

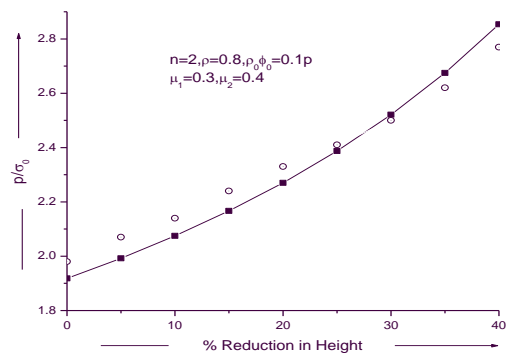


Fig. 7.

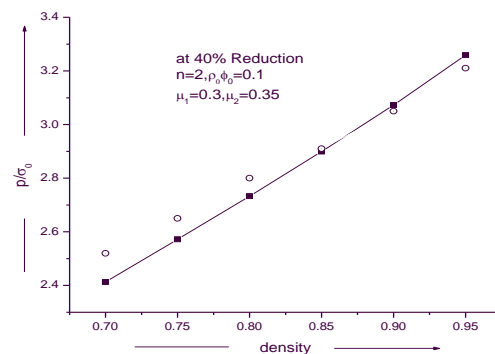


Fig. 8.

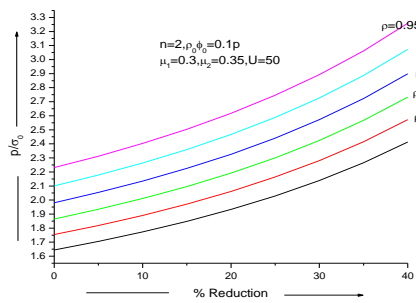


Fig. 9.

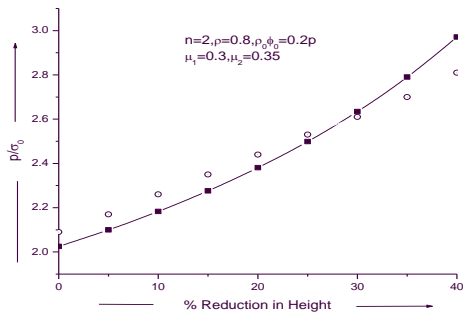


Fig. 10.

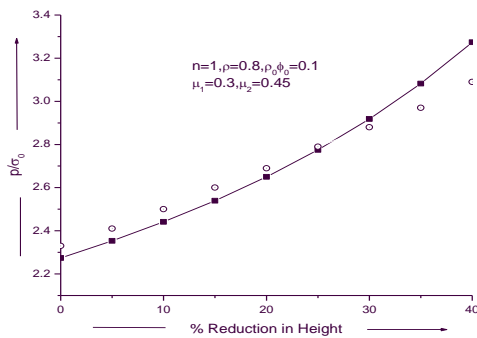


Fig. 11.

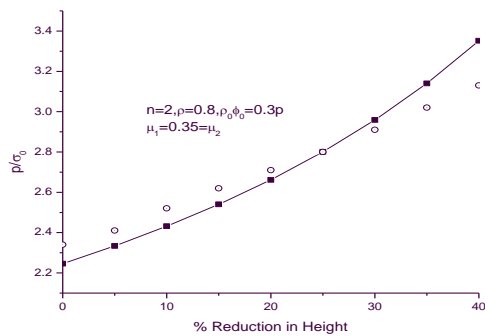


Fig. 12.

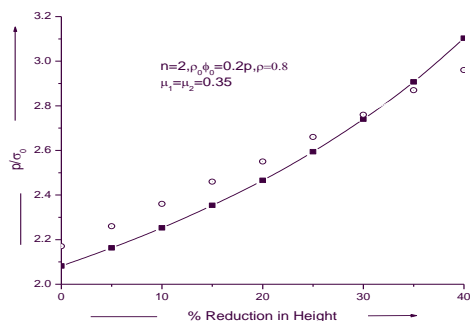


Fig. 13.

## V. CONCLUSION

In powder forging, mass constancy is to be assumed. During forging of metal powder preforms it is seen that compaction and compression both take place simultaneously. Initially the closing of pores dominates the compression process. During forging of powder preforms, the mode of deformation is quite different from wrought materials and it is function of both density and hydrostatic stress. The larger amount of applied load is utilized in densification and lesser amount is consumed for compression.

A composite interfacial friction law has been taken for studying the deformation characteristics of the sintered porous materials. The relative average forging pressure increases with increasing percentage reduction of height of the perform and coefficient of friction

## NOMENCLATURE

- $h$  = Instantaneous thickness of perform
- $\mu_1$  = Coefficient of friction of upper surface
- $\mu_2$  = Coefficient of friction of lower surface
- $P$  = Die load,  $\tau$  = Shear stress,  $p$  = ram pressure
- $\sigma_0$  = Yield stress of the non-work hardening matrix metal
- $J_2$  = Second invariant of deviatoric stress
- $x, y, z$  = Cartesian co-ordinates,
- $n$  = A constant quantity
- $\rho_0$  = A dimensional ratio ( $=\rho_r/\rho^*$ ),
- $\phi_0$  = specific cohesion of a contact surface
- $k$  = Constant equal to 2 in yield criterion
- $\rho^*, \rho_r$  = Densities of apparent and real contact
- $\eta$  = Constant and a function of  $\rho$  only
- $\rho$  = Relative density of the perform
- $\epsilon_r, \epsilon_\theta, \epsilon_z$  = Principal strain increment

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