Tilted Bulk Disordered Distribution Cosmological Model

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***Abstract*—Bianchi type I bulk viscous fluid tilted cosmological model filled with disordered radiation and heat conduction is investigated. We assume that $\zeta\theta = K$ (constant), where ζ is the coefficient of bulk viscosity and θ is the expansion in the model. It has been assumed that the expansion in the model is only in two directions i.e. one of the component of Hubble parameter $\left(H_1 = \frac{A_4}{A}\right)$ is zero. The physical and geometrical aspects of the model in the presence and absence of bulk viscosity are also discussed.**

***Index Terms*—Tilted cosmological model, bulk viscosity, Bianchi type-I universe.**

I. Introduction

Homogeneous and anisotropic cosmological models have been widely studied in classical general relativity in the search for a relativistic picture of the universe in its early stages because they can be explained a number of observed phenomena quite satisfactorily. So that in recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic cosmological models in which matter do not move orthogonally to the hyper surface of homogeneity. These types of models are called tilted cosmological models. The general dynamics of tilted cosmological models have been studied by King and Ellis [1] and Ellis and King [2]. The cosmological models with heat flow have been also studied by Coley and Tupper ([3], [4]), Roy and Banerjee [5]. Ellis and Baldwin [6] have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it.

On the other hand, the matter distribution is satisfactorily described by perfect fluids due to the large scale distribution of galaxies in our universe. However, realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that when neutrino decoupling occurs, the matter behaves as a viscous fluid in an early stage of the universe. Viscous fluid cosmological models of early universe have been widely discussed in the literature. A realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. Misner ([7], [8]) has studied the effect of viscosity on the evolution of cosmological models. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in framework of general theory of relativity (Padmanabhan and Chitre [9]; Johri and Sudarshan [10]; Maartens [11]; Zimdahl [12]; Kalyani and Singh [13]; Singh, Beesham and Mbokazi [14].These motivate to study cosmological model with bulk viscosity.

Also, general relativity describes the state in which radiation concentrates around a star. Klein [15] worked on it and obtained an approximate solution to Einstein's field equation in spherical symmetry for a distribution of diffused radiation. Singh and Abdussattar [16] have obtained an exact static spherically symmetric solution of Einstein's field equation for disordered radiation. Bagora [17-18] obtained tilted Bianchi type I and III cosmological model for disordered radiation and stiff fluid distribution. Motivated by these studies, in this paper we investigated Bianchi type I bulk viscous fluid tilted cosmological model filled with disordered radiation and heat conduction. It has been assumed that the expansion in the model is only in two directions i.e. one of the component of Hubble parameter is zero. The physical and geometrical aspects of the model in the presence and absence of bulk viscosity are also discussed.

II. The Field Equations

We consider the Bianchi type-I metric in the form

$$ds^2 = -\,dt^2 + dx^2 + B^2\,dy^2 \tag{1}$$

where B and C are functions of 't' alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis [19] and for bulk viscosity given by Landau and Lifshitz [20] is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j - \zeta\theta(g_i^j + v_i v^j) \tag{2}$$

together with

$$g_{ij}v_i v^j = -1\,, \tag{3}$$

$$q_i q^j > 0\,, \tag{4}$$

$$q_i v^i = 0\,, \tag{5}$$

In the above, p is the isotropic pressure, ρ the matter density, q^i the heat conduction vector orthogonal to v^i. The fluid flow vector v^i has the components ($\sinh\lambda$, 0, 0, $\cosh\lambda$) satisfying (3), λ being the tilt angle.

The Einstein's field equation

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j\,. \qquad (c = G = 1)$$

The field equation for the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi\left[(\rho + p)\sinh^2\lambda + p + 2q_1\sinh\lambda - K\cosh^2\lambda\right] \tag{6}$$

$$\frac{C_{44}}{C} = -8\pi\,(p - K) \tag{7}$$

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$$\frac{B_{44}}{B} = -8\pi\,(p-K) \tag{8}$$

$$\frac{B_4C_4}{BC} = -8\,\pi\left[-(\rho+p)\cosh^2\lambda + p - 2q_1\sinh\lambda - \sinh^2\lambda\right] \tag{9}$$

$$(\rho+p)\sinh\lambda\cosh\lambda + q_1\cosh\lambda + q_1\frac{\sinh^2\lambda}{\cosh\lambda} - K\sinh\lambda\cosh\lambda = 0 \tag{10}$$

where the suffix '4' stands for ordinary differentiation with respect to the cosmic time '*t*' alone.

III. Solution of Field Equations

Equations from (6) to (10) are five equations in six unknown B, C, ρ, p, q_1 and λ. For the complete determination of these quantities, we assume that the model is filled with disordered radiation which leads to

$$\rho = 3p \tag{11}$$

Also, we assume that

$$\zeta\theta = K \tag{12}$$

The condition $\zeta\theta = K$ is due to the peculiar characteristic of the bulk viscosity. It acts like a negative energy field in an expanding universe (Johri and Sudharshan[21]) i.e $\zeta\theta = K$. According to that expansion is inversely proportional to bulk viscosity.

From equations (6) and (9) with (11), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} = 16\pi p + 8\pi K \tag{13}$$

Again using (8) in (13), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} = -2\left[\frac{B_{44}}{B} - 12\pi K\right] \tag{14}$$

Let us assume that BC = μ, $\frac{B}{C} = \nu$. (15)

Equations (7) and (8), with (15) lead to

$$\frac{\nu_4}{\nu} = \frac{a}{\mu} \tag{16}$$

where 'a' is constant of integration.

Using (15) and (16) in equation (14), we have

$$2\mu_{44} - \frac{\mu_4{}^2}{2\mu} = \frac{-a^2}{2\mu} + 24\pi K\mu \tag{17}$$

Equation (17) gives

$$\mu_4{}^2 = a^2 + 16\pi K\mu^2 + b\mu^{1/2} \tag{18}$$

where 'b' is constant of integration and $\mu_4 = f(\mu)$.

Equation (16) leads to

$$\log\nu = \int \frac{a\,d\mu}{\mu\sqrt{a^2 + 16\pi K\mu^2 + b\mu^{1/2}}} \tag{19}$$

Hence the metric (1) reduces to the form

$$ds^2 = -\frac{d\mu^2}{f^2} + dx^2 + \mu\nu dy^2 + \frac{\mu}{\nu}dz^2 \tag{20}$$

By introducing the following transformations

$\mu = T, x = X, y = Y, z = Z.$

The metric (20) reduces to the form

$$ds^2 = \frac{-dT^2}{a^2 + 16\pi KT^2 + bT^{1/2}} + dX^2 + T\nu dY^2 + \frac{T}{\nu}dZ^2 \tag{21}$$

where ν is determined by (19) when $\mu = T$.

IV. Some Physical and Geometrical Features

The isotropic pressure p and matter density ρ for the model (21) are given by

$$8\pi p = \frac{1}{8T^{3/2}}\left[b + 32\pi KT^{3/2}\right] \tag{22}$$

$$8\pi\rho = \frac{3}{8T^{3/2}}\left[b + 32\pi KT^{3/2}\right] \tag{23}$$

The tilt angle λ is given by

$$\cosh\lambda = \frac{3b}{2(b - 32\pi KT^{3/2})} \tag{24}$$

$$\sinh\lambda = \sqrt{\frac{b + 64\pi KT^{3/2}}{2(b - 32\pi KT^{3/2})}} \tag{25}$$

The scalar of expansion θ calculated for the flow vector v^i for the model (21) is given by

$$\theta = \frac{(b - 8\pi KT^{3/2})}{T}\sqrt{\frac{3b(a^2 + 16\pi KT^2 + bT^{1/2})}{2(b - 32\pi KT^{3/2})^3}} \tag{26}$$

The flow vector v^i and heat conduction vector q^i for the model (21) are given by

$$v^1 = \sqrt{\frac{b + 64\pi KT^{3/2}}{2(b - 32\pi KT^{3/2})}} \tag{27}$$

$$v^4 = \sqrt{\frac{3b}{2(b - 32\pi KT^{3/2})}} \tag{28}$$

$$\frac{(b + 64\pi KT^{3/2})}{64\pi T^{3/2}}\sqrt{\frac{3b}{2(b - 32\pi KT^{3/2})}} \tag{29}$$

$$q_1 = \frac{-3b}{64\pi T^{3/2}} \sqrt{\frac{b+64\pi K T^{3/2}}{2(b-32\pi K T^{3/2})}} \tag{30}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are given by

$$\sigma_{11} = \frac{b(80\pi K T^{3/2} - b)}{2T} \sqrt{\frac{3b(a^2 + 16\pi K T^2 + bT^{1/2})}{2(b-32\pi K T^{3/2})^5}} \tag{31}$$

$$\sigma_{14} = \frac{-b(80\pi K T^{3/2} - b)}{2T} \sqrt{\frac{(a^2 + 16\pi K T^2 + bT^{1/2})(b+64\pi K T^{3/2})}{2(b-32\pi K T^{3/2})^5}} \tag{32}$$

$$\omega_{14} = 72\pi K b T^{3/2}(b+32\pi K T^{3/2}) \sqrt{\frac{(a^2 + 16\pi K T^2 + bT^{1/2})}{2(b+64\pi K T^{3/2})(b-32\pi K T^{3/2})^5}}, \tag{33}$$

Thus

$$\sigma_{11}\nu^1 + \sigma_{14}\nu^4 = \frac{b(80\pi K T^{3/2} - b)}{4(b-32\pi K T^{3/2})^3 T} \sqrt{3b(a^2 + 16\pi K T^2 + bT^{1/2})(b+64\pi K T^{3/2})}\,(1-1) = 0 \quad .(34)$$

Similarly

$$\omega_{11}\nu^1 + \omega_{14}\nu^4 = 0 \tag{35}$$

In the absence of bulk viscosity, the above mentioned quantities lead to

$$8\,\pi\, p = \frac{b}{8T^{3/2}},\; 8\,\pi\,\rho = \frac{3b}{8T^{3/2}},\; \theta = \frac{1}{T}\sqrt{\frac{3(a^2 + bT^{1/2})}{2}},$$

$$\nu^1 = \sqrt{\frac{1}{2}},\; \nu^4 = \sqrt{\frac{3}{2}},\; q_1 = \frac{-3b}{64\pi T^{3/2}},$$

$$\sigma_{11} = \frac{-1}{2T}\sqrt{\frac{3(a^2 + bT^{1/2})}{2}},\; \sigma_{14} = \frac{1}{2T}\sqrt{\frac{(a^2 + bT^{1/2})}{2}},\; \omega_{14} = 0.$$

Again

$$\sigma_{11}\nu^1 + \sigma_{14}\nu^4 = \frac{-b}{2T}\sqrt{\frac{3(a^2 + bT^{1/2})}{2}} + \frac{b}{2T}\sqrt{\frac{3(a^2 + bT^{1/2})}{2}} = 0 \tag{36}$$

The physical significance of conditions (34), (35) and (36) are explained by Ellis [22]: The shear tensor (σ_{ij}) determines the distortion arising in the fluid flow, leaving the volume invariant. The direction of principal axis is unchanged by the distortion, but all other directions are changed. Thus we have
which leads to

$$\sigma_{ij}\nu^j = 0,$$

$$\sigma_{11}\nu^1 + \sigma_{14}\nu^4 = 0 \quad (\because \nu_1 \neq 0, \nu_4 \neq 0)$$

Shear (σ) is given by $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$.

Thus $\sigma^2 \geq 0$ and $\sigma = 0 \Leftrightarrow \sigma_{ij} = 0$.

The vorticity tensor (ω_{ij}) determines a rigid rotation of cluster of galaxies with respect to a local inertial rest frame. Thus, we have

$$\omega = \eta \;{}_{\ell}\omega^k \omega^\ell$$

where $\eta_{ijk\ell}$ is pseudo tensor and $\omega^i = \frac{1}{2}\eta_{ijk\ell}\nu_j\omega_{k\ell}$.

Thus $\omega_{ij}\nu^j = 0$.

This leads to

$$\omega_{11}\nu^1 + \omega_{14}\nu^4 = 0 \;\; (\because \nu_1 \neq 0, \nu_4 \neq 0)$$

The magnitude of ω_{ij} is ω and is defined as

$$\omega^2 = \frac{1}{2}\omega_{ij}\omega^{ij}.$$

Also $\omega = 0 \Leftrightarrow \omega_{ij} = 0$.

V. Conclusion

The model (21) represents a tilted model as $\rho \to \infty$ when $T \to 0$. The model starts with a big-bang at T=0 and the expansion in the model decreases as time increases. Also, $\sigma_{ij}\nu^j = 0$ and $\omega_{ij}\nu^j = 0$ are satisfied as $\sigma_{11}\nu^1 + \sigma_{14}\nu^4 = 0$ and $\omega_{11}\nu^1 + \omega_{14}\nu^4 = 0$. The model has point type singularity at T=0 (MacCallum, [23]).

In the absence of bulk viscosity, $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$, therefore ρ is the decreasing function of time. The reality conditions $\rho + p > 0$, $\rho + 3p > 0$ given by Ellis [24] are satisfied when $T^{3/2} > 0$. In the absence of bulk viscosity there is no rotation in the model. In absence of bulk viscosity the velocity component v^1 and v^4 are constant hence in absence of bulk viscosity flow is uniform. In general, the model is an expanding universe in which the lines of flow of matter are geodesic, shearing but non-rotating.

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