

Solitary Waves of Ultra Intense Laser Pulses in a Fully Relativistic Plasma Channel

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Abstract—We present a theoretical study of nonlinear interactions between a short, intense laser-beam and a fully relativistic plasma channel containing hot electrons and positrons. We have obtained the governing equation for the evolution of the spot size with relativistic corrections. Also, the acceptable solutions of relativistic solitary waves have been predicted.

Index Terms—Laser-plasma interactions, Plasma channel, Solitary waves.

I. INTRODUCTION

The development of ultra intense short laser pulses allows exploration of fundamentally new parameter regimes for nonlinear laser plasma interaction [1]-[3]. Now we have laser beams with intensities up to 10^{21} W/cm^2 in laboratories. At such a high intensities electrons quiver velocity becomes relativistic. In such a situation, consideration of the relativistic electron mass variation [4]-[8] as well as the relativistic ponderomotive force [9], [10] is very essential in the study of nonlinear laser plasma interactions.

When an ultra intense laser pulse propagates in plasma, several nonlinear phenomena are excited, as, for example, laser frequency variation, high-order harmonic generation, the appearance of coherent nonlinear structures (plasma channels, relativistic solitons, vortices), the generation of ultra intense quasistatic electric and magnetic fields, and electron and ion acceleration to relativistic energies.

The paper is organized as follows. In section 2, we present the relevant equations to formulate the proposed problem with appropriate assumptions and the relativistic versions of the equations describing the evolution of the spot size is obtained. In section 3, we discuss the solution to the governing equation. Section 4, summarizes the findings of this study. It is necessary to mention that, in the low velocity limit, all the expressions found in this study reduce exactly to their weak relativistic expressions [11].

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II. BASIC EQUATIONS

We first present the relevant equations describing the action of intense laser light on the plasma consisting hot electrons and positrons. We assume a circularly polarized laser pulse propagating along the z -direction in a hot and under dense plasma channel which has a parabolic density profile of the form $n(r) = n_0(1 + r^2/r_{ch}^2)$, where n_0 and r_{ch} are the initial axial electron density and the effective channel radius, respectively. By the ansatz,

$$\mathbf{a}(r, z, t) = \frac{1}{2} a(r, z, t) (\hat{x} + i\hat{y}) \exp[i(k_0 z - \omega_0 t)] \quad (1)$$

where $a(r, z, t)$ is the complex amplitude normalized by $m_0 c^2 / e$, m_0 and e are the rest mass and the charge of electron, c is the light velocity in vacuum and k_0 and ω_0 are the laser centre wave number and the frequency, respectively, we can obtain the wave equation as

$$\left(\nabla_{\perp}^2 + 2ik_0 \frac{\partial}{\partial z} - k_p^2 \eta \frac{r^2}{r_{ch}^2} + k_p^2 \frac{|a|^2}{2} - \eta^2 k_p^2 \nabla_{\perp}^2 \frac{|a|^2}{2} \right) a(r, z) = 0 \quad (2)$$

where $k_p = \omega_p / c$ is the plasma wave number and η is

defined as $\eta = \gamma_v \frac{1 - \beta_0 v}{\gamma_0 (\beta_0 - v)^2}$. Also, $\beta = \frac{u}{c}$, $v = \frac{V}{c}$

and $\gamma_v = 1 / (1 - V^2 / c^2)^{1/2}$ is the ‘effective relativistic factor’ associated with the velocity of the wave and should not be confused with the relativistic factor,

$\gamma = 1 / (1 - u^2 / c^2)^{1/2}$, related to the fluid velocity of the plasma. The subscript 0 represents quantities at infinity. In deriving equation (2), we have used the Coulomb gauge $\nabla \cdot \mathbf{a} = 0$ and the dispersion relation $\omega_0^2 = \omega_p^2 + k_0^2 c^2$.

Also, the paraxial approximation and slowly varying envelope approximation have been used. We note that in the limit $\beta_0 \rightarrow 0$ (i.e. $\gamma_0 \rightarrow 1$), equation (2) approaches its weak relativistic format in [11]. Using equation (2) and assuming that this equation has a solution as

$$a(r, z) = a_r(z) e^{-r^2/r_s^2(z)} e^{i[b(z)r^2 + \phi(z)]} \quad (3)$$

where $a_r(z)$, $r_s(z)$, $b(z)$ and $\phi(z)$ are the real amplitude, spot size, spatial chirp parameter and phase shift of the laser pulse respectively, after straightforward, but tedious derivations, equation (2) can be written as

$$\frac{1}{2} \left(\frac{\partial r_s}{\partial z} \right)^2 + \psi(r_s) = 0 \quad (4)$$

where

$$\psi(r_s) = \frac{1}{2}(1-p)[r_s(\beta_0 - v)]^{-2}(1 - \beta_0 v) + \frac{1}{2}v^2 N_c r_s^2 \times [(\beta_0 - v)]^{-2} - \frac{1}{8}\gamma_0(1 - \beta_0 v)a_0^2 r_s^{-4} - \psi_0 \gamma_0 \quad (5)$$

Here, $\psi_0 = \frac{1}{2}(1-p) \frac{1 - \beta_0 v}{(\beta_0 - v)^2} + \frac{1}{2}v^2 N_c - \frac{\gamma_0 a_0^2}{8}$,

$p = k_p^2 a_0^2 r_0^2 / 16$ is the normalized laser power by the critical power $P_c = 16\pi$ for relativistic self-focusing in plasma and $N_c = \gamma_0^2 k_p^2 r_0^4 / 4r_{ch}^2$ is a parameter related to the effect of reformed channel focusing. In equations (4) and (5) the dimensionless variables $z/Z_R \rightarrow z$ and $r_s/r_0 \rightarrow r_s$ where $Z_R = k_0 r_0^2 / 2$ is the Rayleigh length, are used. We also note that, a collimated incident laser pulse, i.e., $b_0 = \left(\frac{\partial r_s}{\partial z} \right)_{z=0} = 0$ and the initial condition $r_s = 1$ at $z = 0$ in integrating equation (4) is considered. Equations (4) and (5) are the relativistic versions of the equations describing the evolution of the spot size. It is interesting to note that for $\beta_0 = 0$ equations (4)-(5) obviously reduce to the expressions for the weak relativistic limits [11].

III. ACCEPTABLE SOLUTIONS OF RELATIVISTIC SOLITARY WAVES

Now we are going to turn our attention to investigate the existence of solitary wave solutions of equation (4). We follow the same technique used in [11]. The roots of $\psi(r_s) = 0$ can be easily found using the solutions of a cubic equation. The roots, in terms of the coefficient can be expressed as

$$r_{s1} = 1 \quad (6)$$

$$r_{s2} = \left[\left(N_p + \sqrt{N_p^2 - 16N_c a_0^2} \right) / 8N_c \right]^{1/2} \quad (7)$$

$$r_{s3} = \left[\left(N_p - \sqrt{N_p^2 - 16N_c a_0^2} \right) / 8N_c \right]^{1/2} \quad (8)$$

where

$$N_p = 4(1-p)\eta - \gamma_0^2 a_0^2 \quad (9)$$

Real and positive roots are acceptable. Different cases can be considered as,

(a) if $p > 1 - \gamma_0 a_0^2 \sqrt{N_c} - \gamma_0^2 a_0^2 / 4$, the equation has one real (i.e. $r_{s1} = 1$) and two non real complex conjugate roots.

(b) if $p < 1 - \gamma_0^2 (N_c - a_0^2 / 2)$, $\psi(r_s) = 0$ has three distinct real roots $r_{s3} < r_{s1} = 1 < r_{s2}$.

(c) if $1 - \gamma_0^2 (N_c - a_0^2 / 2) < p < 1 - \gamma_0 a_0^2 \sqrt{N_c} - \gamma_0^2 a_0^2 / 4$, three cases can be considered.

(c1) if $N_c = N_c^*$, where the critical channel parameter $N_c^* = \gamma_0^2 a_0^2 / 4$, $\psi(r_s) = 0$ has triple root, i.e., $r_{s1} = r_{s2} = r_{s3} = 1$.

(c2) if $N_c > N_c^*$, three kinds of cases are discussed as follows:

(c2.1) if $p = 1 - \gamma_0 a_0^2 \sqrt{N_c} - \gamma_0^2 a_0^2 / 4$, $\psi(r_s) = 0$ has three real roots: $r_{s1} = 1$ and twofold root $r_{s2} = r_{s3} = \sqrt{a_0 / 2\sqrt{N_c}} < 1$.

(c2.2) if $1 - \gamma_0^2 (N_c - a_0^2 / 2) < p < 1 - \gamma_0 a_0^2 \sqrt{N_c} - \gamma_0^2 a_0^2 / 4$, $\psi(r_s) = 0$ has three unequal real roots: $r_{s3} < r_{s2} < r_{s1} = 1$.

(c2.3) if $p = 1 - \gamma_0^2 (N_c - a_0^2 / 2)$, then $\psi(r_s) = 0$ has three real roots: twofold root $r_{s1} = r_{s2} = 1$ and $r_{s3} < 1$.

(c3) if $N_c < N_c^*$, if $N_c < N_c^*$, the following results are given:

(c3.1) if $p = 1 - \Lambda^2 a_0^2 \sqrt{N_c} - \Lambda a_0^2 / 4$, $V(r_s) = 0$ has three real roots: $r_{s1} = 1$ and $r_{s2} = r_{s3} = \sqrt{a_0 / 2\sqrt{N_c}} > 1$.

(c3.2) if $1 - \Lambda_0^2 (N_c - a_0^2 / 2) < p < 1 - \Lambda^2 a_0^2 \sqrt{N_c} - \Lambda a_0^2 / 4$, $V(r_s) = 0$ has three unequal real roots: $r_{s1} = 1 < r_{s3} < r_{s2}$.

(c3.3) if $p = 1 - \gamma_0^2 (N_c - a_0^2 / 2)$, $V(r_s) = 0$ has three real roots: twofold root $r_{s1} = r_{s3} = 1$ and $r_{s2} > 1$.

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