III Type Bianchi Cosmological Model in Tilted Universe

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***Abstract*—Bianchi type-III tilted cosmological model for stiff fluid is investigated. It has been assumed that the expansion in the model is only in two directions i.e. one of the Hubble parameter ($\frac{A_4}{A}$) is zero. The physical and geometrical consequences of the model are discussed.**

***Index Terms*—Bianchi type-III universe, cosmology, stiff fluid, tilted model.**

I. Introduction

Homogeneous and anisotropic cosmological models have been studied widely in the framework of general relativity. These models are more restricted than the inhomogeneous models. But in spite of this, they explain a number of observed phenomena quite satisfactorily. In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic cosmological models in which matter does not move orthogonal to the hypersurface of homogeneity. Such types of models are called tilted cosmological models.The general dynamics of these models have been studied in detail by King and Ellis [1], Ellis and King [2], Collins and Ellis [3]. Ellis and Baldwin [4] have investigated that we are likely to be living in a tilted universe and they have indicated that how we may detect it.

Bianchi type III cosmological models are interesting because these models allow not only expansion but also rotation and shear.In general, these are anisotropic. The type III model also has an interesting geometric interpretation. The type III Lie group can be considered as the Thurston geometry H^2 x R which plays an important role in 3-dimensional geometry [5, 6, 7]. The tilted Bianchi type III model is the last of the ever expanding Bianchi models left to study in terms of its late-time behavior. Letelier and Tabensky [8] have investigated cylindrical self-gravitating fluid for stiff matter. Tabensky and Taub [9] have studied plane symmetric self gravitating fluid with pressure equal to energy density. Wesson [10] has investigated an exact solution to Einstein field equations with stiff equation of state. Wainwright et al. [11] have derived some exact solution which generalized Bianchi type III, V and VI_h models for vacuum and for stiff perfect fluid. Roy and Prasad [12] have retained the perfect fluid to be stiff but have relaxed the comoving restriction over its flow. Hurwitz and Spero [13] studied a Bianchi type III cosmology and discussed the matter contents effect on isotropic expansion. Wang [14] obtained the Bianchi type III cosmological model for a cloud string in the presence of bulk viscosity and magnetic field. Wang assumed that there is an equation of state $\rho = k\lambda$ and the scalar of expansion is proportional to the shear $\theta \propto \sigma$, which leads to a relation between metric potentials $B = mC^n$, to obtain this model. Coley et al. [15] studied on Bianchi model with vorticity specially type III bifurcation. They have shown that for $1 \le \gamma \le 2$ the late-time asymptote is the self similar vacuum spacetime given by $ds^2 = -dt^2 + t^2 (dx^2 + e^{-2x} dy^2) + dz^2$ .They also discussed that the tilt depends on γ i.e. if $\gamma = 1$, the tilt tends to zero while for $1 \le \gamma \le 2$, the tilt is asymptotically extreme. Bagora [16] has investigated Bianchi type III cosmological model for stiff fluid. Now we have investigated Bianchi type III stiff fluid cosmological models for perfect fluid distribution in general relativity. The physical and geometrical features of the model are also discussed.

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II. The Metric and Field Equations

We consider the Bianchi type III metric into the form

$$ds^2 = -dt^2 + dx^2 + B^2 e^{2x} dy^2 + C^2 dz^2 \qquad (1)$$

where B and C are functions of ‘t’ only.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis [17] is taken into the form.

$$T_i^j = (\in +p)v_i v^j + p g_i^j + q_i v^j + v_i q^j \qquad (2)$$

Together with

$$g_{ij} v^i v^j = -1 \qquad (3)$$

$$q_i q^j > 0 \qquad (4)$$

$$q_i v^i = 0 \qquad (5)$$

where p is the pressure, ∈ the density and q_i the heat conduction vector orthogonal to v^i. The fluid flow vector has the components $(\sinh\lambda, 0, 0, \cosh\lambda)$ satisfying (3), λ is tilt angle.

Using units in which c = G = 1 the Einstein’s field equation.

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j \qquad (6)$$

For the line element (1) leads to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-8\pi\left[(\in+p)\sinh^2\lambda+p+2q_1\sinh\lambda\right] \tag{7}$$

$$\frac{C_{44}}{C}=-8\pi p \tag{8}$$

$$\frac{B_{44}}{B}-1=-8\pi p \tag{9}$$

$$\frac{B_4C_4}{BC}-1=-8\pi\left[-(\in+p)\cosh^2\lambda+p-2q_1\sinh\lambda\right] \tag{10}$$

$$(\in+p)\sinh\lambda\cosh\lambda+q_1\cosh\lambda+q_1\frac{\sinh^2\lambda}{\cosh\lambda}=0 \tag{11}$$

where the suffix '4' stands for ordinary differentiation with respect to the cosmic time 't' Alone.

III. Solution of The Field Equations

Equations from (7) to (11) are five equations in six unknown B, C, $\in$, p, λ and q_1. For the complete determination of these quantities, we assumed that the model is filled with stiff fluid, which leads to

$$\in=p. \tag{12}$$

From equations (7), (10) and (12), we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}-1=0. \tag{13}$$

From equations (8) and (9), we have

$$\frac{C_{44}}{C}-\frac{B_{44}}{B}+1=0. \tag{14}$$

Equations (13) and (14) lead to

$$\frac{2C_{44}}{C}+\frac{2B_4C_4}{BC}=0. \tag{15}$$

This gives

$$\frac{\nu_4}{\nu}=\frac{\mu_4}{\mu}-\frac{2a}{\mu}, \tag{16}$$

where $BC=\mu$, $\frac{B}{C}=\nu$ and 'a' is constant of integration.

Equation (14) leads to

$$\frac{\mu_{44}}{\mu}=1 \tag{17}$$

From equation (17), we have

$$\mu_4=\sqrt{\mu^2+b} \tag{18}$$

where $\mu_4=f(\mu)$ and 'b' is constant of integration.

Again From (16), we have

$$\log\nu=\log\mu-2a\int\frac{d\mu}{\mu\sqrt{\mu^2+b}} \tag{19}$$

Hence the metric (1) reduces to the form

$$ds^2=-\frac{d\mu^2}{f^2}+dx^2+\mu\nu e^{2x}dy^2+\frac{\mu}{\nu}dz^2 \tag{20}$$

where ν is determined by (19).

By introducing the following transformations

$$\mu=T,\ x=X,\ y=Y,\ z=Z.$$

The metric (20) becomes

$$ds^2=\left[\frac{1}{T^2+b}\right]dT^2+dX^2+T\nu e^{2x}dY^2+\frac{T}{\nu}dZ^2 \tag{21}$$

where

$$\nu=N\exp\left[\log T-2a\int\frac{dT}{T\sqrt{T^2-b}}\right] \tag{22}$$

where N is constant of integration.

IV. Some Physical and Geometrical Features

The pressure and density for the model (21) are given by

$$8\pi\in=8\pi p=\frac{a}{T^2}[\psi_1-a]. \tag{23}$$

The tilt angle λ is given by

$$\cosh\lambda=\sqrt{\frac{2a^2+T^2-2a\psi_1}{2(T^2+b^2-a\psi_1)}} \tag{24}$$

$$\sinh\lambda=\frac{T}{\sqrt{2(a\psi_1-T^2-a^2)}} \tag{25}$$

The scalar of expansion θ calculated for the flow vector v^i is given by

$$\theta=[(2-5a)T^4+(6a^2-4ab-8a^2)T^2-8a^3b+4a^2b+(4a^2T^2+4a^4+2T^4-2aT^2)\psi_1]\times\frac{1}{2T(T^2+a^2-a\psi_1)^{3/2}\sqrt{2(2a^2+T^2-2a\psi_1)}}. \tag{26}$$

The components of fluid flow vector v^i and heat conduction vector q^i for the model (21) are given by

$$q^1=\frac{2a^2+T^2-2a\psi_1}{8\pi T\sqrt{(a\psi_1-T^2-a^2)}} \tag{27}$$

$$q^4=\frac{1}{8\pi}\sqrt{\frac{2a^2+T^2-2a\psi_1}{2(T^2+a^2-a\psi_1)}} \tag{28}$$

$$\nu^1 = \frac{T}{\sqrt{2(a\psi_1 - T^2 - a^2)}} \tag{29}$$

$$\nu^4 = \sqrt{\frac{2a^2 + T^2 - 2a\psi_1}{2(T^2 + a^2 - a\psi_1)}} \tag{30}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are given by

$$\sigma_{11} = \frac{1}{6T}\sqrt{\frac{2a^2 + T^2 - 2a\psi_1}{2(T^2 + a^2 - a\psi_1)^3}}\Big[(T^2 + a^2 - a\psi_1)$$
$$(\psi_2 - \psi_3\psi_1) - T^2\sqrt{(2a^2 + T^2 - 2a\psi_1)}\Big], \tag{31}$$

$$\sigma_{22} = \frac{-\nu e^{2x}}{3\sqrt{2(T^2 + a^2 - a\psi_1)^2(2a^2 + T^2 - 2a\psi_1)}}$$

$$\Big[\big[3a(2a^2 + T^2 - 2a\psi_1) - \psi_3\psi_1 + \psi_2 - 2T^2\sqrt{(2a^2 + T^2 - 2a\psi_1)}\Big], \tag{32}$$

$$\sigma_{33} = \frac{1}{3\nu\sqrt{2(T^2 + a^2 - 2a\psi_1)^3(2a^2 + T^2 - 2a\psi_1)}}$$
$$\big[3a(2a^2 + T^2 - 2a\psi_1) - \psi_4\psi_1 - \psi_5 - T^2\sqrt{(2a^2 + T^2 - 2a\psi_1)}\big], \tag{33}$$

$$\sigma_{44} = \frac{T}{6\sqrt{2(T^2 + a^2 - a\psi_1)^5(2a^2 + T^2 - 2a\psi_1)}}$$

$$[\psi_2 - \psi_3\psi_1 - T^2(T^2 + a^2 + a\psi_1)\sqrt{(2a^2 + T^2 - 2a\psi_1)}\Big], \tag{34}$$

$$\omega_{14} = \frac{(aT^2 + 2ab - 2a^2\psi_1)}{2\sqrt{2(T^2 + a^2 - a\psi_1)^3}}. \tag{35}$$

Here

$$\psi_1 = \sqrt{T^2 + b},$$

$$\psi_2 = (4aT^4 + 5abT^2 + 4a^3T^2 + 4a^3b),$$

$$\psi_3 = (7a^2T^2 + 2a^3 + 2a^2b + T^4),$$

$$\psi_4 = (3a^2T^2 + T^4 + 2a^3 + 2a^2b),$$

$$\psi_5 = 2aT^4 + abT^2 + 4a^3T^2 + 4a^3b.$$

The rates of expansion H_i in the direction of x, y and z axes are given by

$$H_1 = 0, H_2 = \frac{1}{T}[\psi_1 - a], H_3 = \frac{a}{T}.$$

V. Conclusion

Generally, the model represents shearing, rotating and tilted type universe in which the flow vector is geodesic. So that it is anisotropic. The model starts with big bang singularity. Rate of expansion along x-axis vanishes, where as the rate of expansion along y and z is decreasing function of time. Also the expressions for pressure and density have singularity at T =0, i.e. at T = 0, $p \rightarrow \infty$ and T = ∞, $\in \rightarrow 0$.

References

[1] A. R. King and G. F. R. Ellis, "Tilted homogeneous cosmological models," Comm. Math. Phys, vol. 31, pp. 209-242, 1973.

[2] G. F. R. Ellis and A. R. King A R, "Was the big bang a whimper?," Comm. Math. Phys., vol. 38, no. 2, pp. 119-156, 1974.

[3] C. B. Collins and G. F. R. Ellis, "Singularities in Bianchi cosmologies," Phys. Rep, vol. 56, no. 2, pp. 67-105, 1979.

[4] G. F. R. Ellis and J. E. Baldwin, "On the expected anisotropy of radio source content," Mon. Not. Roy. Astro. Soc, vol. 206, pp. 377-381, 1984.

[5] W. P. Thurston, "Three-Dimensional Geometry and Topology," *Edited by Silvio Levy, U.S.A.: Princeton Univ. Press*, vol. 1, 1997.

[6] W. P. Thurston, "Three dimensional manifolds, Kleinian groups and hyperbolic geometry," *Bull. Amer. Math. Soc*, vol. 6, pp. 357-381, 1982.

[7] J. W. Morgan, "Recent progress on the poincar´e conjecture and the classification of 3-manifolds," Bull. Am. Math. Soc, vol. 42, no. 1, pp. 57-78, 2004.

[8] P. S. Letelier and R. R. Tabensky, "Cylindrical self-gravitating fluids with pressure equal to energy density," *Il Nuovo Cimento B*, vol. 28, no. 2, pp. 407-414, 1975.

[9] R. R. Tabensky and A. H. Taub, "Plane symmetric self-gravitating fluids with pressure equal to energy density," *Comm. Math. Phys*, vol. 29, no. 1, pp. 61-77, 1973.

[10] P. S. Wesson, "An exact solution to Einstein's equations with a stiff equation of state," *J. Math. Phys*, vol. 19, pp. 2283-2284, 1978.

[11] J. Wainwright, W. C. W. Ince and B. J. Marshman, "Spatially homogeneous and inhomogeneous cosmologies with equation of state p=μ," *Gen. Rel. Gravit*, vol. 10, no. 3, pp.259-271,1979.

[12] S. R. Roy and A. Prasad, "Inhomogeneous generalizations of Bianchi Type VIh universes with stiff perfect fluid and radiation," *Astrophys. Space Sci*, vol.225, no. 1, pp.151-156, 1995.

[13] H. A. Hurwitz and A. Spero, "A Bianchi type III cosmology - Matter content's effect on isotropic expansion," *Il Nuovo Cimento B*, vol. 34, pp.1-4, 1976.

[14] X. X. Wang, "Bianchi type-III string cosmological model with bulk viscosity and magnetic field," *Chin.Phys. Lett.* vol. 23, pp. 1702, 2006.

[15] A. A. Coley and S. Hervik, "Bianchi models with vorticity: The type III bifurcation," Class. Quant. Grav, vol. 25, pp. 198001-198006, 2007.

[16] A.Bagora, "Bianchi type III stiff fluid cosmological models for perfect fluid distribution," *Astrophys. Space Sci*, vol. 319, pp.155-159, 2008.

[17] G. F. R. Ellis, "General Relativity and Cosmology, R.K. Sachs [ed.]," New York: Academic Press, 1971, pp. 116.