Quantum Conductance of Three-Terminal Nanoring in the Presence of Rashba Interaction and an Impurity

F. Azadi Chegeni and E. Faizabadi

Abstract—**Quantum interference effects in quantum rings provide suitable means for controlling spin at mesoscopic scales. So an open one-dimensional quantum nano ring containing a potential impurity which is subjected to the Rashba spin-orbit interaction is considered. The strength of Rashba parameter can be controlled by a normal electric field. The quantum ring coupled with three leads (one input and two output leads) and patterned in two-dimensional electron gases (2EDG). One dimensional quantum wave guide theory, transfer matrix and Griffith's boundary conditions is used to calculated the transmission coefficients. Spin rotation and conductance of this arm-depended quantum ring is studied. We investigated that the position of impurity and the output leads can be effect on the conductance and spin rotation. We introduce the mesoscopic structure as a spin rotator of electrons.**

Index Terms—**Spintronic, Three-terminal quantum ring, Rashba spin-orbit interaction.**

I. Introduction

Recently quantum electronics from both experimental and theoretical physics communities has attracted a lot of interest [1]. These studies are focused on the role of Spin Orbit Interaction (SOI) in nanometric devices patterned in a two dimensional electron gas (2DEG) and on the interplay of SOI and quantum interference [2], [3]. Unlike electronic which is based on the charge of the electrons, the main concept of spintronic is combining of microelectronic and spin depended effects, that due to interaction between the spin of the electron, orbital degree of freedom and magnetic properties. Using spintronic in fabrication of device leads to higher production speed and lower power consumption. Due to the coupling of electron orbital motion and the spin degree of freedom at mesoscopic scale or nano scale, it is possible to manipulate and control the electron spin.

Thus great efforts have been devoted in the past few years to overcome the fundamental obstacles in the realization of spintronic devices, such as generation and the filtering of pure spin-polarized currents and their appropriate manipulation in a controllable environment. Due to spin-orbit interaction, Datta and Dass proposed a spin transistor more than ten years ago, that control the electron spin by means of a magnetic field and magnetic materials [4]-[12].

Most of these works for spin filters appeared based on intrinsic spin splitting properties of semiconductors associated with the Rashba SOI [13], [14]. The Rashba SOI is essentially the influence of an external field on a moving spin and can be seen as the interaction of the electron spin with the magnetic field, appearing in the rest frame of the electron.

Moreover for applications it is essential that the strength of Rashba effect and thus the spin splitting can be controlled by means of a gate electrode.

In this article we focus on the spin interference in coherent quantum nano ring conductors (QR) at low temperature (T<<1K) under the influence of electromagnetic potential [15]-[19], known as Aharonov-Bohm(AB)[20] and Aharonov-Casher(AC)[21] effects. In a coherent QR the dimensional of ring smaller than phase coherent length of electrons. In addition an extended literature is devoted to the study of persistent currents, from both of experimental and theoretical point of view, in close and open quantum rings by focusing on the spin polarization of these currents due to presence of Rashba interaction [22].

In 2003 Nitta et al. proposed a spin interference device allowing considerable modulation of the electric current. This device was a one-dimensional ring connected with two conductor leads [23]-[27].

In 2008 kalman et al. introduce the another one-dimensional spintronic device which is a semiconductor structure as a tree-terminal quantum ring[28].

In their device, electros entering in a totally unpolarized spin state become polarized at the outputs with different spin directions. This device can be deemed in a certain sense a spintronic analogue of the Stern-Gerlach apparatus [29].

In this paper we calculated the spin rotation and conductance of three-terminal quantum nano ring with an impurity in presence of Rashba spin-orbit interaction, which can be fabricated from e.g InAlAs/InGaAs based on heterostructure. We show for first time that the position of impurity and output leads can effect on the conductance and direction of outgoing spin. The rashba interaction coupling with impurity can be increases the spin rotation in special cases.

The paper is organized as follows; the Hamiltonian and model are introduced in next section. In sec 3, we present our results and discussed them. Finally, we end the paper by a brief summary.

II. Hamiltonian and Model

In the persence of Rashba spin-orbit interaction and a potential impurity, the Hamiltonian operator for a

Manuscript received October 10, 2011; revised November 20, 2011. This work was supported in part by the Department of physics, Iran university of Science and Technology, 16846 Tehran, Iran.

Quantum conductance of three-terminal nanoring in the presence of Rahba interaction and an impurity.

Fatemeh Azadi Chegeni and Edris Faizabadi are with Department of physics, Iran university of Science and Technology,16846 Tehran, Iran. (E-mail:azadif2@gmail.com; edris@iust.ac.ir).

one-dimensional ring structure in the Gausin coordinate is given by:

$$\mathrm{H} = \{\hbar\omega\left[\left(-i\frac{\partial}{\partial\varphi} + \frac{\omega}{2\Omega}(\sigma_x\cos\varphi + \sigma_y\sin\varphi)\right)^2 - \frac{\omega^2}{4\Omega^2}\right] + \mathrm{V}\delta(\varphi - \varphi_0)\} \quad (1)$$

where σ_x and σ_y are the pauli matrices, $\omega = e\hbar E_z/\left(\sqrt{2}m^*ac\right)^2$ is the frequency associated to the spin-orbit interaction, $\hbar\Omega = \hbar^2/2m^*a^2$ is the kinetic energy, m^* is the effective mass of the electron and $\mathrm{V}\delta(\varphi - \varphi_0)$ is the impurity term. Rashba result from the asymmetric confinement along the direction (z) perpendicular to the plane of the ring (x-y), and represents the average electric field along the z direction [30], [31]. We can solve the eigenvalue problem in a straight forward manner.

$$E_\mu = \hbar\Omega\left[k^2 + (-1)^\mu kw + \frac{1}{4}\right] = \hbar\Omega\left[\left(k - \frac{1}{2} - \frac{\phi^\mu}{2\pi}\right)^2 - \omega 24\Omega 2 \right. \quad (2)$$

where $\mu = 1,2 \ and \ \phi^\mu$ =- $\pi[1 + (-1)^\mu w]$,is the aharonov-casher phase. The energy of the incoming electrons $E = \hbar^2k^2/2m^*$ has to be conserved, therefore the condition $E/\hbar\Omega = k^2\rho^2 = E_\mu$, determines the possible value of k;

where

$$k_j^\mu = (-1)^{\mu+1}\left(\frac{w}{2} + (-1)^\nu q\right) \quad (3)$$

$\mu = 1,2$, $w = \sqrt{1 + (\omega/\Omega)^2}$, $q = \sqrt{(\omega/\Omega)^2 + (ka)^2}$ and ϕ^μ=-$\pi[1 + (-1)^\mu w]$ referred to Aharanov-Casher phase. to the four different k_j^μ the following four eigenstates are found to be

$$\chi_j^\mu\left(k_j^\mu\varphi\right) = e^{ik_j^\mu\varphi}\ \chi_j^\mu(\varphi) \qquad \mu, j = 1,2; \quad (4)$$

can be described of the pauli matrix σ_z as :

$$\chi_j^1(\varphi) = \begin{pmatrix} e^{\frac{-i\varphi}{2}}\cos\frac{\theta}{2} \\ e^{\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix},$$

$$\chi_j^2(\varphi) = \begin{pmatrix} e^{\frac{-i\varphi}{2}}\sin\frac{\theta}{2} \\ -e^{\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}, \quad \mu, j = 1,2; \quad (5)$$

In which angle θ given by, $tg(\theta) = \omega/\Omega$. The wave function for a given energy E in the different sections of the ring is the linear combination of these states.

$$\Psi_i(\varphi) = \sum_{\mu,j=1,2} a_{ij}^\mu\ \psi(k,\varphi) \ , i = I, II, III, IIII \quad (6)$$

And i denotes the section of the ring in the counterclockwise direction, starting from the position of the incoming lead.

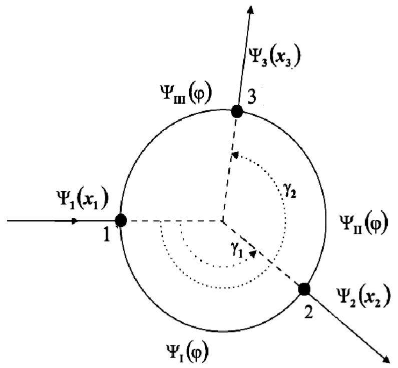


Fig. 1. The geometry of the device and the relevant wave function in the different domains. The parameter φ is measured from junction 1 in counterclockwise direction.

By using the Griffith's boundary conditions at each junction, one-dimensional scattering problem is solvable. The Griffith's boundary conditions are: (i) the wave function must be continuous and (ii) the current density must be conserved. The wave function in the each lead can be expanded by:

$$\psi_1(x_1) = \binom{(f1)\uparrow}{(f1)\downarrow}e^{ikx_1} + \binom{(r1)\uparrow}{(r1)\downarrow}e^{-ikx_1} \quad (7)$$

$$\psi_2(x_2) = \binom{(t2)\uparrow}{(t2)\downarrow}e^{ikx_2} \quad (8)$$

$$\psi_3(x_3) = \binom{(t3)\uparrow}{(t3)\downarrow}e^{ikx_3} \quad (9)$$

Fig .1. Shows the geometry of the device and the relevant wave function in the different domains. The parameter φ is measured from junction 1 in counterclockwise direction.

Considering the input junction, the associated Griffith's boundary conditions are:

$$\psi_1(0) = \psi_I(0) = \psi_{III}(2\pi) \quad (10)$$

and

$$J_1(0) - J_I(0) + J_{III}(2\pi) = 0. \quad (11)$$

The appropriately normalized spin current densities in the leads are given by

$$J_l(\mathrm{x}_l) = 2aRe\left(\Psi_l^\dagger(x_l)\left(-i\frac{\partial}{\partial x_l}\right)\Psi_l(x_l)\right) \quad (12)$$

and in the ring by

$$J_i(\varphi) = 2Re\left(\Psi_i^\dagger(\varphi)\left(\frac{\omega}{2\Omega}\sigma_r(\varphi) - i\frac{\partial}{\partial x_l}\right)\Psi_l(x_l)\right) \quad (13)$$

where $\sigma_r = \sigma_x\cos\varphi + \sigma_y\sin\varphi$,i= $I, II, III, IIII$ and l=1,2,3,4. For the sake of definiteness, we present the details for the incoming junction (1). The results for the other junctions can be obtained in a similar manner. We simplify equation (11) by using (12)

Analogous equations can be written for the other two junctions. Besides, the same boundary conditions can be applied to δ-function potential in the arm. By applying the aforementioned conditions to spin up and down, we can find 22 equations which can be solved by numerical methods.

$$a\frac{\partial\Psi_1}{\partial x_1}\Big|_{x_1} - \frac{\partial\Psi_I}{\partial\varphi}\Big|_{\varphi=0} + \frac{\partial\Psi_{III}}{\partial\varphi}\Big|_{\varphi=2\pi} \tag{14}$$

Now we study the conductance of ring in presence of Rashba spin-orbit interaction and a potential impurity. In the landauer formalism the conductance is given by

$$G_\uparrow = \frac{e^2}{h}\left(\left|(t_{II})_\uparrow\right|^2 + \left|(t_{III}^{(\ )})_\uparrow\right|^2\right), \tag{15}$$

$$G_\downarrow = \frac{e^2}{h}\left(\left|(t_{II})_\downarrow\right|^2 + \left|(t_{III})_\downarrow\right|^2\right) \tag{16}$$

where $G_\uparrow$ and $G_\downarrow$ are the total conductance for spin up and spin down, $(t_{II})_\uparrow$, $(t_{II})_\downarrow$ denotes the quantum probability of transmission for output junction II for up and down spin and $(t_{III}^{(\ )})_\uparrow$, $(t_{III})_\downarrow$ are the quantum probability of transmission for output junction III for up and down spin [32].

III. Result and Discussion

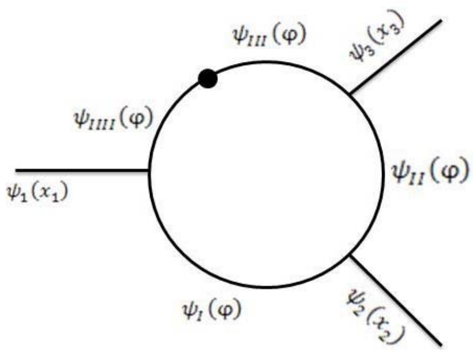
By using the method in the previous section, we have investigated the quantum conductance as a function of incident electrons energy. In a real ring the coupling between the leads and ring can be complicated. This asymmetry can be a consequence of fabrication defects and be induced by Lorentz force [33]. Calculations are performed for Rashba coefficient (ω / Ω) α = 5, v = 0.05 mev and effective mass m^*= 0.023m□, which v is the strength of potential impurity, and m□ is the rest mass of electrons.

We approach the scattering problem by using the quantum waveguide theory [34], [35]. For the strictly one-dimensional ring the wave function in different regions for each value of the spin are different. Actually, by applying the electric field perpendicular to the plane of ring, which leads to Rashba spin-orbit effect, the symmetry is broken; as described before, the electrons which move along the upper arm have a different phase compared to the electrons moving along the lower arm, this asymmetry leads to spin rotation[36], [37].

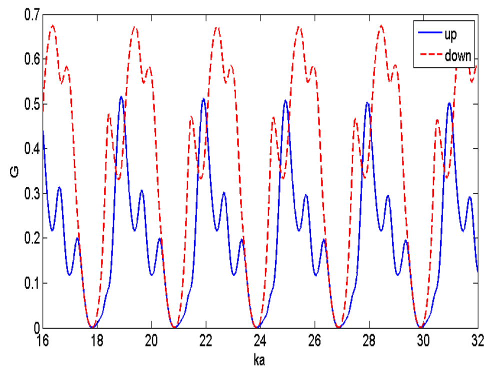
Wave function in different regions correlated together and transmitted into the output leads. For these values of the parameters the dependence of the transmission probability on incident electron energy is calculated. When we calculate the transmission probability in two output junctions by using the Landauer-Buttiker conductance formula which describes the transport properties of a system subject to a constant, low bias voltage[38], [39], the conductance in nano or mesoscopic structure cab be derived.

In this section we show the conductance for different cases of asymmetry.

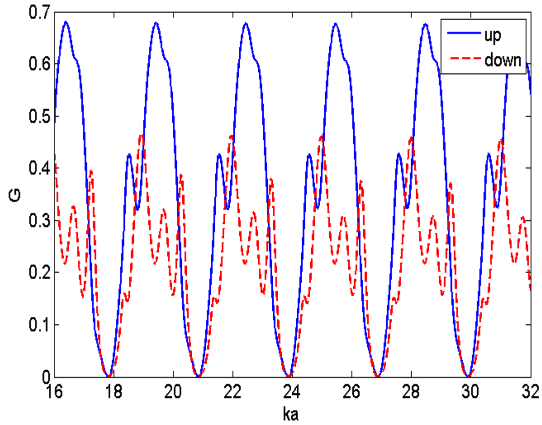
Fig. 2. Shows the quantum conductance of a three-terminal quantum nanoring. For these values of the parameters the dependence of the quantum conductance on incident electron energy is illustrated. The electrons which move thought the ring feel the δ-function potential. The strength of δ-function potential impurity, position of impurity and output leads can be influence on the conductance. In Fig. 1(b) the incoming spin state chosen to be up, Fig. 1(c) related to the electrons with down spin- the spin eigenstate of σ_z- and in Fig. 1(d) the spin current is non-polarized. When $\gamma_1 = 2\pi/3$, $\gamma_2 = 4\pi/3$ and $\gamma_3 = 5\pi/3$, which γ_1, γ_2 is the position of output leads and the impurity is located at an angle γ_3. Since as mentioned in the pervious section, in zero temperature the conductance is given by Landauer-Buttiker formalism. The conductance is expressed in unit of conductance ($G_0 = e^2/h$); which e is the electrons charge and h denotes the plank constant. The periodic behavior can be seen in the curves, these oscillations referred to constructive and destructive interference. In Fig. 1(b),(c),(d), it can be seen the conductance for some value of ka such as 17.9, 20.9 and 23.9, is zero. In this value of incident electron energy the interference between wave functions in different regions of ring are totally destructive so quantum ring acts as a nonconductor device and incident electrons totally reflected in input terminal. Comparing the curves, it can be seen when the incoming spin state chosen to be up the conductance for spin down electrons is higher. In Fig. 1(c) the opposite result is shown and Fig. 1(d) express that when the incoming spin current is non-polarized and the percent of spin up in higher the conductance for spin down electrons is more. Therefore, under these situations the coupling between Rashba spin-orbit interaction and a potential impurity can be rotated the electrons spin. Blue (solid line) and red (dashed line) curves related to conductance for spin up and down. Our results depend on the geometry of device. The position of output leads and impurity changes the periodic behaviors. We studied the spin-dependent interference in the ring in different cases.



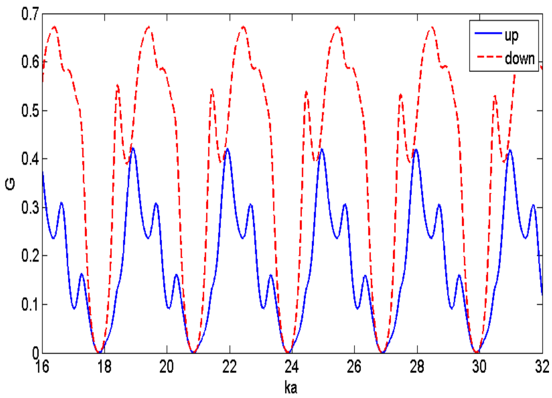
(a)



(b)

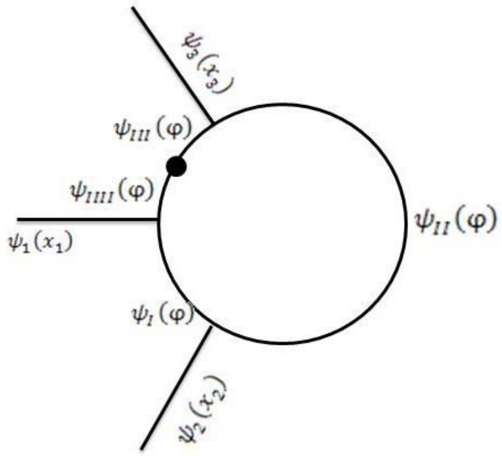


(c)

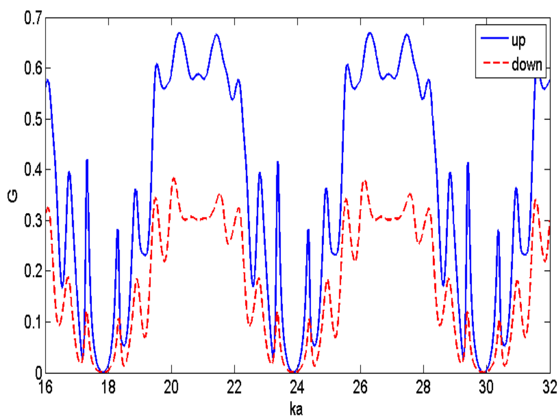


(d)

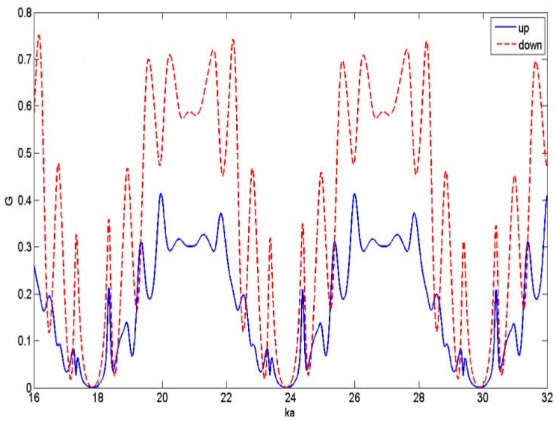
Fig. 2. (a) plot of our system, the conductance $G/G_0(G_0 = e^2/h)$ of ring , (b) input electrons with spin up=100%, (c) down=100% and (d) up=60% and down=40%.



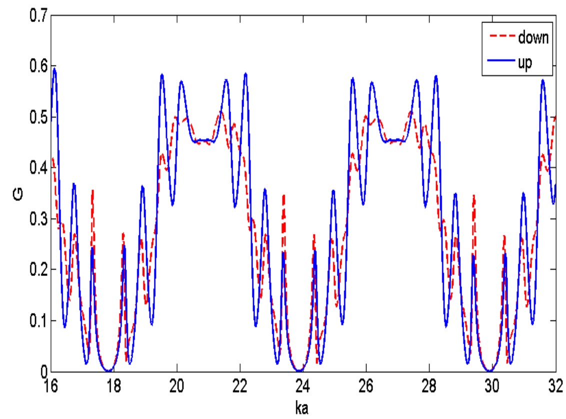
(a)



(b)



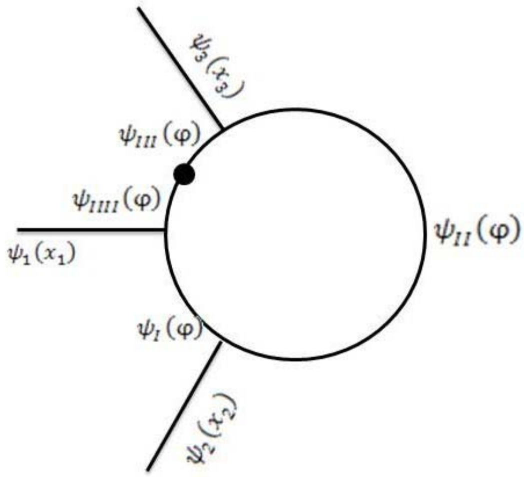
(c)



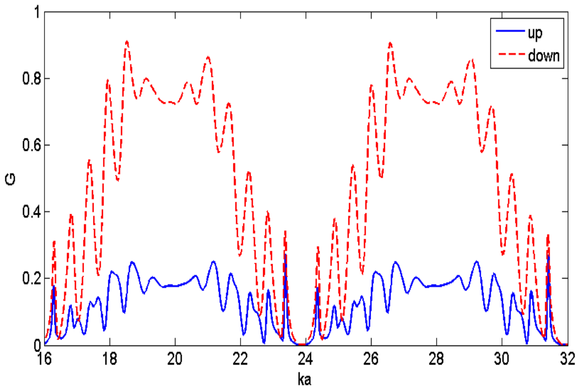
(d)

Fig. 3. (a) plot of our system, the conductance $G/G_0(G_0 = e^2/h)$ of ring , (b) input electrons with spin up=100%, (c) down=100% and (d) up=60% and down=40%.

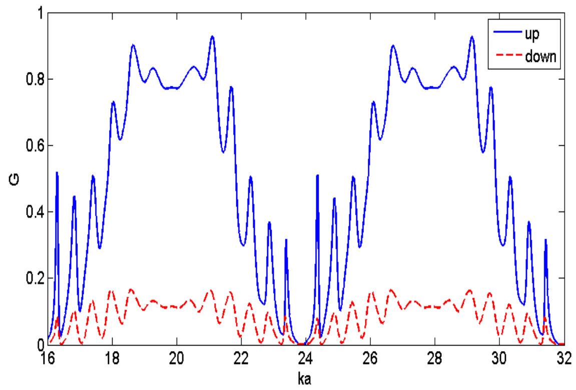
Fig. 3 shows the effect of asymmetry on the conductance. In this case there is not a perfect symmetry between the arms. The outgoing leads are located at $\gamma_1 = \pi/3$, $\gamma_2 = 5\pi/3$ and impurity located at $\gamma_3 = 11\pi/3$. It is obvious that the value of incoming electrons energy that our device is nonconductor is decreased. It is very good result for us and the correlation between elecrons and impurity in persence of SOI decrease the spin rotation. In this case the quantum conductance for incoming spin states is more. Namely the input electrons are rotated the same direction at the output leads of the ring.



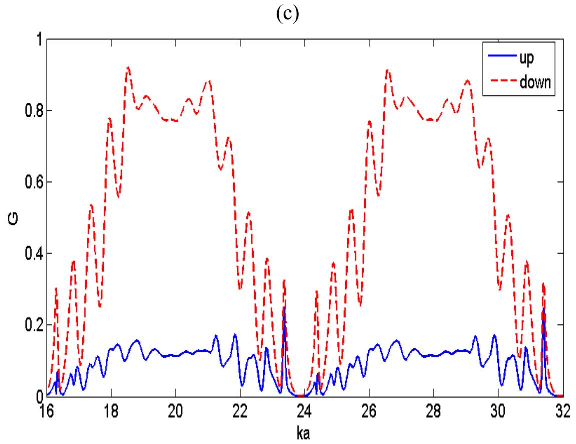
(a)



(b)



(c)



(d)

Fig. 4. (a) plot of our system, the conductance $G/G_0(G_0 = e^2/h)$ of ring , (b) input electrons with spin up=100%, (c) down=100% and (d) up=60% and down=40%.

Fig. 4 Shows another case of asymmetry. Graphs are plotted for $\gamma_1 = \pi/4$, $\gamma_2 = 7\pi/4$ and impurity located at $\gamma_3 = 15\pi/8$. In this case the conductance of ring also is a periodic function of ka. Our results show the spin direction of incoming electrons is rotated about 95% in the output leads and the conductance in some value of incident electrons energy, such as ka=18.5 and 26.5 is maximum (near 1). In fact the input spinor that changes its direction is due to the SOI and impurity, lead to spin rotation devices. By comparing the curves in Figs 2, 3 and 4 we see that in figure 4 the value of incoming electrons energy is nonconductor is decreased more than pervious figure (Fig 3).

IV. Conclusion

We have studied quantum conductance in a quantum nanoring by considering Rashba interaction and an impurity. By applying the Griffith's boundary conditions, the conductance dependence of position of impurity and output leads for different values of wave number is investigated. The spin rotation appears as a result of Rashba spin-orbit coupling that interaction between incident electrons and impurity plays an important role.

Acknowledgment

This work was supported in part by the Department of physics, Iran university of Science and Technology,16846 Tehran, Iran.

References

[1] T. Ando, Y. Arakawa, K. Furuya, S. Komiyama, and H.Nakashima, *Mesoscopic Physics and Electronics*, edited, Springer, Berlin, 1998.
[2] G. A. Prinz, *Science 282*, pp.16, 60, 1998.
[3] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Moln´ar, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, *Science 294*, pp. 14, 88, 2001.
[4] S. Datta and B. Das, *Appl. Phys. Lett. 56,* pp. 665, 1990.
[5] S. Murakami, N. Nagaosa, and S. C. Zhang, *"Science 301,"* pp. 13, 48, 2003.
[6] [Inspec] [MEDLINE] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, *"Science 306,"* pp. 19, 10, 2004.
[7] S. Bellucci, F. Corrente, P. Onorato, "Condens. Matter 19," *J. Phys.,* pp. 39, 50, 18, 2007.
[8] S. Bellucci, F. Corrente, and P. Onorato, "Condens. Matter 19," *J. Phys.* pp. 39, 50, 19, 2007.
[9] S. Bellucci and P. Onorato, Phys. Rev. B 77, 075303, 2008.
[10] S. Bellucci and P. Onorato, "Condens. Matter 19," *J. Phys.* pp. 39, 50, 20, 2007.
[11] F. Chi, J. Zheng, and L.-L. Sun, *Appl. Phys. Lett.* 92, 2008, pp.172, 104.
[12] F. Chin and J. Zheng, *Appl. Phys. Lett*. 92, pp. 062, 106，2008.
[13] E. I. Rashba and F. Tverd. Tela (Leningrad) 2, 1224 (1960) [*Sov. Phys. Solid State 2*, no. 1109, 1960].
[14] Y. A. Bychkov and E. I. Rashba, *J. Phys*. C 17, pp. 60, 39, 1984.
[15] D. Frustaglia, M. Hentschel, and K. Richter, *Phys. Rev. Lett.87*, pp. 256602-1, 2001.
[16] J. Nitta, T. Koga, and H. Takayanagi, *PhysicaE12*, pp.753, 2002.
[17] P. Földi, O. Kálmán, M. G. Benedict, and F. M. Peeters, *Phys. Rev. B73,* pp. 155, 325, 2006.
[18] R. Citro and F. Romeo, *Phys.Rev.B73*, pp. 233, 304, 2006.
[19] Z. Zhu, Y. Wang, K. Xia, X. C. Xie, and Z. Ma, *Phys. Rev.* vol. B 76, pp. 125, 311, 2007.
[20] Y. Aharonov and D. Bohm, *Phys.Rev.115*, pp. 485, 1959.
[21] Y. Aharonov and A.Casher, *Phys.Rev.Lett.53*, pp. 319, 1984.
[22] J. Splettstoesser, M. Governale, and U. Zulicke, *Phys. Rev. 2003,* vol.B68, pp. 165, 341, 2003.
[23] J. Nitta, F. E. Meijer, and H. Takayanagi, *Appl. Phys. Lett.* vol.75, pp. 695, 1999.
[24] J. C. Egues, G. Burkard, and D. Loss, *Appl. Phys. Lett.* vol. 82, pp. 26, 58, 2003.
[25] D. Frustaglia, M. Hentschel, and K. Richter, *Phys. Rev. Lett.* vol. 87, pp. 256, 602, 2001.
[26] R. Ionicioiu and I. D'Amico, *Phys. Rev.,* vol. B 67, pp. 041, 307, 2003.
[27] J. B. Yau, E. P. De Poortere, and M. Shayegan, *Phys. Rev. Lett.* vol. 88, pp. 146, 801, 2003.
[28] O. Kalman, P. Foldi, M. G. Benedict, and F. M. Peeters, *Phys. Rev.* vol. B 78, pp. 125, 306, 2008
[29] J. B. Yau, D. Portere, and S. hayegan, *Phys. Rev. Lett.* vol. 88 pp. 146, 801, 2003.
[30] F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, *Phys. Rev.* vol. B 66, pp. 033, 107, 2002.
[31] B. Molnar, F. M. Peeters, and P. Vasilopoulos, Phys. Rev. vol. B 69, pp. 155,335, 2004
[32] S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge Univ. Press, Cambridge, 1995.
[33] O. kalman, *Quantum Interference in Semiconductor Ring,* University of Szeged. Szeged, Hungary, 2009.
[34] J. B. Xia, *Phys. Rev.* vol.B 45, pp.35, 93, 1992.
[35] P. S. Deo and A. M. Jayannavar, *Phys. Rev.* vol. B 50, pp. 116, 29,1995

[36] A. M. Jayannavar, P. SinghaDeo, Phys.Rev. vol. B51, no.15, pp. 10, 175,1995.

[37] R. Citro and F. Romeo, *Phys.Rev.* vol. B75, pp. 073, 306, 2007 .

[38] R. Landauer, *Philos. Mag.* vol. 21, pp. 8, 63, 1970

[39] M. Buttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. vol. B 31, pp. 6, 207, 1985

Fatemeh Azadi Chegeni was born in Khorramabad, Lorestan, Iran, on September 21, 1984. She got the master degree in Solid state physicsc at Iran University of Science and Technology, Tehran, Iran, and bachelor degree applied physics at Lorestan University, Khorramabad, Iran. She is the Msc student and study solid state physics at Iran University of science and technology from fall 2008 until now. She works at Islamic azad university, Teaching basic physics from fall 2011 until now.

Seyyed Edris Faizabadi got the PHD degree in Shahid Beheshti University, Tehran, Iran, and master degree in Shahid Beheshti University, Tehran, Iran, and bachelor in Ferdousi University, Mashhad, Iran.

He is a Assistant Professor of physics at Iran University of Sceince and Technology. His Publications-Journal Paper as follows: Edris Faizabadi, Mahbobeh Omidi, “Edged topological disordered quantum ring in the presence of magnetic flux “, physics Letters A, Vol.374, pp. 1762-1768, 2010. Edris Faizabadi, Ali Bagheri.” Effects of valency percentage on the energy gap of zigzag single-wall carbon nanotube “, Physica E, Vol. 41, pp. 1828-1831, 2009. Mohammad Reza Khayatzadeh Mahani, Edris Faizabadi,” Efficient Spin Filtering in a Disordered Semiconductor Superlattice in the Persence of Dresselhaus Spin-orbit Coupling “, Physics Lettetrs A, Vol. 327, pp. 1926-1929, 2009.