Steady-state Analysis of Bernoulli Feedback on $Geom^X/G/1$ Queue with Multiple Vacation and Set-up Times

Songfang Jia, Yanheng Chen, and Jinkui Liu

Abstract—**The problem $Geom^X/G/1$, a queuing model of discrete time with Bernoulli feedback, multiple vacation and Set-up times is considered. The generating function of the steady state queue length, waiting time and their stochastic decomposition are derived. And we also get the generating function of busy period, give some special cases, and present some numerical examples to study the effect of the varying parameters on the main Perf-ormance characteristics of our system.**

Index Terms—**Bernoulli feedback; multiple vacation; bulk arrive; set-up times; stochastic decomposition; discrete time queue**

I. Introduction

In recent years, the discrete time vacation queue is used so widely in the telecommunications systems and computer communication networks that the discrete-time queuing system m-odel and analysis is very important and very meaningful. The- refore, since Meisling[1] first proposed and studied this type of queuing system in 1958, the discrete time queuing model with various kinds of vacation rules has been extensively studied (see[2-7]).

On one hand, for Geom/G/1 vacation queue, an excellent and complete study on discrete-time queuing systems with vacations has been presented by Takagi[8] who gave the an-alysis of finite and infinite buffer $Geom/G/1$ type queues (in-cluding batch arrivals) with different vacation policies. And Fiems and Bruneel[9] analyzed the discrete-time $Geom^X/G/1$ queue under multiple vacations governed by a geometrically distributed timer in 2002. Most recently, Chang and Choi[10] have analyzed a single server batch arrival bulk-service queue where customers are served in batches of random size and the server takes multiple vacations whenever queue is empty. And Chuan ect[11] gave the analysis of a Multi-Vacation $Geo^{\lambda_1,\lambda_2}/G/1$ Queue with variable arrival rate. Differ from the single or multiple vacation policy, however, Zhang and Tian[12] adopted the policy of multiple adaptive vacations. This type of limited number of vacations policy reflects the trade-off. Between the benefit of working on the queue and the benefit of doing other jobs represented by the vacations. Based on this vacation policy, they got a fully new model and generalized many former results. Subsequently Sun etc [13] gave a steady state analysis of the batch-arrival $Geom/G/1$ queue with multiple adaptive vacations. In all these references, it is assumed that the server cannot take service during the vacations in the discrete-time queue system, based on this; Li ect [14] considered a steady-state analysis of a discrete time batcharrival queue with working vacations. On the other hand, feedback model often arise in life. For example, a remote multi-channel communication system will transmit again when the data transmission error occurs, which can be regarded as the queuing model with feedback. And the discrete time queuing model with feedback are more appro-priate than their continuous-time counterparts for modeling computer and telecommunication systems, since the basic units in these systems are digital such as a machine cycle time, bits and packeds etc. So, since Takacs[15] proposed feedback me-chanism in 1963, the discrete time queuing model with feedb-ack got a small amount of research, Disney ect [16] analyzed the stationary queue-length and waiting-time distributions in single-server feedback queues, then Atencia and Moreno [3] gave the discrete time $Geom^X/GH/1$ retrial queue with Berno-ulli feedback. Recently, the work of the [7] analyzed the $Geom/G/1$ queuing system with multiple adaptive vacations and Bernoulli feedback by full probability decomposition method. In this paper, we will apply the embedded Markov chain meth-od to discuss the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times, and obtain the probability generating function of the steady-state queue length, the probability generating function of the stationary waiting times, and the property of stochastic decomposition results of them. And we also analyze the busy period of the system model and get the generating function of busy period. Finally, we give several special cases of our model, and some numerical exam-ples show the influence of the parameters on some crucial perf-ormance characteristics of the system.

II. Model Description and Embedded Markov Chain

The Bernoulli feedback on $Geom^X/G/1$ queue with multi-ple vacation and set-up times studied here is as follows:

(1) We consider the epoch n to clarify the state of the syst-em, and suppose that the arrivals of the customers occurs onl-

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Songfang Jia is with the College of Mathematics and Stastics, Chongqing, Three Gorges University, Wanzhou, Chongqing, China (corresponding author to provide phone: 023-5810-2361; fax:023-5810-2361; e-mail: jiasongfang@163.com).

Yanheng Chen is with the College of Mathematics and Stastics,Chongq-Ing Three Gorges University, Wanzhou, Chongqing, China (e-mail: math_yan@126.com).

Jinkui Liu is with the College of Mathematics and Stastics, Chongq-Ing Three Gorges University, Wanzhou, Chongqing, China (e-mail: liujinkui2006@126.com).

y at $t=n^-$ time (that moment on the eve of $t=n$) $n=0,1,\cdots$ the start and end of service occur at $t=n^+$ time (that moment on the end of $t=n$) $n=0,1,2,\cdots$ .This model is known as the late model system.

(2) Customers arrive in batches, the probability, PGF, mean and the ith moment of batch reach customer are respectively λ_k, $k=0,1,\cdots$ $\Lambda(z)$, λ, $\lambda^{(i)}$ .The intervals of batch reach customer which is independent and obey the parameter p for the same geometrical distribution.

(3) There is only a server in the system, service rule is first come first serve. Customers need service time s, which is independent and obey the same general discrete distribution. Its probability, PGF, mean and the ith moment respectively are $g_k, k=0,1,2,\cdots$, $G(z)$, g, $g^{(i)}$.

(4)When a customer has served, one will be immediately discharged to wait for the next service at the end of the queue with the probability $1-\alpha$, or leave the system forever with probability $\alpha(0<\alpha<1)$ .That means, the total service number of a customer ξ obeys the geometric distribution of the parameter α

$$P(\xi=k)=\alpha(1-\alpha)^{k-1}, k=1,2,\cdots$$

(5) One cycle begins as soon as the system becomes empty.

Vacation strategy is as follows: when a length V for vacation is end, if there is any customer waiting and the server will no longer take another vacation and serve the customers after a length U for setup time; The server will take another vacat-ion if there is no customer waiting in the system and serve the customers after a setup time.

The probability and probability generating function of vacation time, setup time respectively are

$$v_k, k=0,1,2,\cdots \qquad V(z)=\sum_{k=1}^{\infty} v_k z^k$$

$$u_k, k=0,1,2,\cdots \qquad U(z)=\sum_{k=1}^{\infty} u_k z^k$$

The probability of reaching $j(j=0,1,2,\cdots)$ customers in vacation time and setup time respectively are

$$C_j^{(V)}=\sum_{l=1}^{\infty} v_l \sum_{k=0}^{l} C_l^k p^k \bar{p}^{l-k} p\left(\sum_{i=1}^{k}\Lambda_i=j\right)$$

$$C_j^{(u)}=\sum_{l=1}^{\infty} u_l \sum_{k=0}^{l} C_l^k p^k \bar{p}^{l-k} p\left(\sum_{i=1}^{k}\Lambda_i=j\right)$$

(6) Above all random variables are independent.

Let $\tilde{s}$ represent each customer's total service time, we have by the complete probability formula

$$\tilde{g}_j=p(\tilde{s}=j)=\sum_{k=1}^{j} p(\xi=k)p(\tilde{s}=j|\xi=k)$$

$$=\sum_{k=1}^{j}\alpha(1-\alpha)^{k-1}p\left(\sum_{i=1}^{k}s_i=j\right), \qquad j=1,2,\cdots.$$

where s_i stands for the ith service time of the customer. and the PGFs and the average of the total service time are respectively listed as

$$\tilde{G}(z)=\sum_{j=1}^{\infty}\tilde{g}_j z^j=\frac{\alpha G(z)}{1-(1-\alpha)G(z)}, |z|<1,$$

$$E(\tilde{s})=\left[\frac{d\tilde{G}(z)}{dz}\right]_{z=1}=\frac{g}{\alpha}.$$

During busy period, define L_n be the number of customers

When the nth customer leaves the system and $\{L_n|n\ge1\}$ be the Markov chain of the discrete-time queue length process.

Then we have

$$L_{n+1}=\begin{cases}L_n-1+A_{n+1}, L_n\ge1\\ Q_b+A_{n+1}-1, L_n=0\end{cases}.$$

where A_{n+1} represents the number of customers who arrive during the internal of the $(n+1)th$ customer's total service times, Q_b stands for the number of customers when a busy period begins. Then the probability distributions and PGFs of them are listed as

$$k_l=p(A=l)$$

$$=\sum_{j=1}^{\infty}p(\tilde{s}=j)\sum_{k=0}^{j}C_j^k p^k \bar{p}^{j-k} p\left(\sum_{i=1}^{k}\Lambda_i=l\right), l=0,1,2,\cdots$$

$$A(z)=\sum_{l=0}^{\infty}k_l z^l=\tilde{G}(\bar{p}+p\Lambda(z))$$

$$b_j=p(\alpha+A-1=j)$$

$$=\frac{1}{1-v(\bar{p})}\sum_{i=1}^{j}C_i^{(v)}\sum_{r=0}^{j+1-i}C_r^{(u)}k_{j+1-i-r}, j=0,1,2,\cdots$$

$$R(z)=\sum_{j=0}^{\infty}z^j b_j$$

$$=\frac{1}{z}A(z)\frac{1}{1-v(\bar{p})}U(\bar{p}+p\Lambda(z))[V(\bar{p}+p\Lambda(z))-v(\bar{p})]$$

Based on Foster rule, we can confirm that the Markov chain $\{L_n, n\ge1\}$ is positive recurrent if and only if

$$\rho=\frac{p\lambda g}{\alpha}<1 \qquad \text{(see[17])}.$$

III. Stochastic Decomposition Results

The system is positive recurrent If $\rho<1$, then we are Listed as

$$\beta=pE(U)+\frac{p}{1-v(\bar{p})}E(V).$$

Theorem 1. The steady-state queue length of the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times, immediately after the service completions, denoted by Π. The probability generating function of whch is listed as

$$\Pi(z)=\frac{(1-\rho)(1-\Lambda(z))\tilde{G}(\bar{p}+p\Lambda(z))}{\tilde{G}(\bar{p}+p\Lambda(z))-z}\cdot\frac{1-\frac{1}{1-v(\bar{p})}U(\bar{p}+p\Lambda(z))[V(\bar{p}+p\Lambda(z))-v(\bar{p})]}{\beta(1-\Lambda(z))} \tag{1}$$

where $\tilde{G}(\bar{p}+p\Lambda(z))=\dfrac{\alpha G(\bar{p}+p\Lambda(z))}{1-(1-\alpha)G(\bar{p}+p\Lambda(z))}$.

Proof Denote $\{\pi_k^+, k\ge0\}$ be the steady-state distribution of the queue length, which satisfies $\Pi^+ \tilde{P} = \Pi^+$, that means

$$\pi_j^+ = \pi_0^+ b_j + \sum_{i=1}^{j+1} \pi_i^+ k_{j+1-i}, \quad j = 0,1,2,\cdots.$$

Multiplying z^j on both sides of the distribution function and summing over $j = 0,1,2,\cdots$, we obtain

$$\begin{aligned}\Pi^+(z) &= \pi_0^+ R(z) + \sum_{j=0}^{\infty}\sum_{i=1}^{j+1} \pi_i^+ k_{j+1-i} z \\ &= \pi_0 R(z) + \frac{1}{z} A(z)[\Pi^+(z) - \pi_0^+]\end{aligned} \tag{2}$$

Substituting $R(z)$ and $A(z)$ into (2), which yields

$$\Pi^+(z) = \pi_0^+ \frac{\tilde{G}(\overline{p} + p\Lambda(z)) 1 - \frac{1}{1-v(\overline{p})} U(\overline{p} + p\Lambda(z))[V(\overline{p} + p\Lambda(z)) - v(\overline{p})]}{\tilde{G}(\overline{p} + p\Lambda(z)) - z}$$

Based on the normalization condition $\Pi^+(1) = 1$ and L'Hospital rule, we get

$$\pi_0^+ = (1-\rho)(\lambda\beta)^{-1}$$

then

$$\Pi^+(z) = \frac{(1-\rho)(1-\Lambda(z))\tilde{G}(\overline{p} + p\Lambda(z))}{\tilde{G}(\overline{p} + p\Lambda(z)) - z} \cdot \frac{1 - \frac{1}{1-v(\overline{p})} U(\overline{p} + p\Lambda(z))[V(\overline{p} + p\Lambda(z)) - v(\overline{p})]}{\beta(1-\Lambda(z))} \tag{3}$$

This is the probability generating function of the queue length of the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times when the customer will leave the sever. It should be pointed out that the queue length which remain when the customer leave the server is not usually of stationary distribution $\{\pi_k, k \geq 0\}$, that means

$$\pi_k = \lim_{t\to\infty} P(N(t) = k)$$

but is the distribution of the queue of the regeneration time queue. However, he PGF of the general Stationary distribution is as

$$\Pi(z) = \frac{\lambda(1-z)}{1-\Lambda(z)}\Pi^+(z) = \frac{(1-\rho)(1-z)\tilde{G}(\overline{p} + p\Lambda(z))}{\tilde{G}(\overline{p} + p\Lambda(z)) - z} \cdot \frac{1 - \frac{1}{1-v(\overline{p})} U(\overline{p} + p\Lambda(z))[V(\overline{p} + p\Lambda(z)) - v(\overline{p})]}{\beta(1-\Lambda(z))} \tag{4}$$

Corollary 1 When $\rho < 1$, Π can be decomposed into the sum of two independent random variables:

$$\Pi = \Pi_0 + \Pi_d$$

where Π_0 is queue length of the $Geom^X/G/1$ queue with Bernoulli feedback and no vacation, Π_d is the additional queue length, the PDFs of them are listed as

$$\Pi_0(z) = \frac{(1-\rho)(1-z)\tilde{G}(\overline{p} + p\Lambda(z))}{\tilde{G}(\overline{p} + p\Lambda(z)) - z}$$

$$\Pi_d(z) = \frac{1 - \frac{1}{1-v(\overline{p})} U(\overline{p} + p\Lambda(z))[V(\overline{p} + p\Lambda(z)) - v(\overline{p})]}{\beta(1-\Lambda(z))}$$

Proof We can have the stochastic decomposition by the equation（4）.

The mean of the queue length of the $Geom^X/G/1$ queue with Bernoulli feedback and no vacation is

$$E(\Pi_0) = (1-\rho)\left\{\frac{p\lambda\frac{g}{\alpha}}{1-p\lambda\frac{g}{\alpha}} + \frac{1}{2\left(p\lambda\frac{g}{\alpha}-1\right)^2}\left[p\lambda^{(2)}\frac{g}{\alpha} + \left(p^2\lambda^2 + 2p^2\lambda\lambda^{(2)}\right)\frac{\alpha g^{(2)} - 2(1-\alpha)g^2}{\alpha^2} + p^3\lambda^3\frac{\alpha^2 g^{(3)} + 6\alpha(1-\alpha)gg^{(2)} + 6(1-\alpha)^2 g^3}{\alpha^3}\right]\right\}$$

and mean of the additional queue length is

$$E(\Pi_d) = \frac{p^2\lambda}{1-v(\overline{p})} \cdot \frac{(1-v(\overline{p}))E[U(U-1)] + E[V(V-1)] + 2E[U]E[V]}{2\beta}$$

then $E(\Pi) = E(\Pi_0) + E(\Pi_d)$.

Theorem 2 The stationary batch waiting times of the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times, immediately after the service completions, denoted by W. The probability generating function of which is listed as

$$W(s) = \frac{(1-\rho)(1-s)}{p\Lambda[\tilde{G}(s)] - s - \overline{p}} \cdot \frac{p\left\{1 - \frac{1}{1-v(\overline{p})} U(s)[V(s) - v(\overline{p})]\right\}}{\beta(1-s)} \cdot \frac{1-\Lambda[\tilde{G}(s)]}{\lambda(1-\tilde{G}(s))} \tag{5}$$

where $\Lambda[\tilde{G}(s)] = \Lambda\left[\frac{\alpha G(s)}{1-(1-\alpha)G(s)}\right]$

Proof Because W is composed by two parts W_g and W_f, where W_g is the waiting times of the batch, W_f is the waiting times of the customer in the batch. Based the FCFS rule, the number of customers batch to reach when service is completed in the existing system is equivalent to the one within the inter-val time. So, $W_g(s)$ has the relationship

$$\begin{aligned}\Pi_g(z) &= \frac{(1-\rho)(1-z)\tilde{G}_g(\overline{p} + p\Lambda_g(z))}{\tilde{G}_g(\overline{p} + p\Lambda_g(z)) - z} \cdot \frac{1 - \frac{1}{1-v(\overline{p})} U(\overline{p} + p\Lambda_g(z))[V(\overline{p} + p\Lambda_g(z)) - v(\overline{p})]}{\beta(1-\Lambda_g(z))} \\ &= W_g(\overline{p} + p\Lambda_g(z))\tilde{G}_g(\overline{p} + p\Lambda_g(z))\end{aligned} \tag{6}$$

Also, because the service time of the $Geom^X/G/1$ queue with Bernoulli feedback and server setup/closedown times can be regarded as $\sum_{i=1}^{\Lambda}\tilde{G}_i$, which is the service time of the Bernou-lli feedback $Geom/G/1$ queue model with multiple adaptive vacations and server setup/closedown times. Thus

$$\tilde{G}_g(s)=\Lambda\left(\tilde{G}(s)\right) \quad (7)$$

making $s=\overline{p}+p\Lambda(z)$, and substituting(7)into (6), which yields

$$W_g(s)=\frac{(1-\rho)(1-s)}{p\Lambda\left(\tilde{G}(s)\right)-s+\overline{p}}\frac{p\left\{1-\frac{1}{1-v(\overline{p})}U(s)[V(s)-v(\overline{p})]\right\}}{\beta(1-s)}$$

We use a result of "discrete-time form" renewal theory to solve $W_f(s)$. In the batch queue for the length Λ, Let η stands for the customer number before the customer, which is nonnegative integer random variable, then the waiting times of the customer in the batch is $W(s)=\sum_{i=1}^{\eta}\tilde{G}_i$, the corresponding PGF is

$$W(s)=\frac{(1-\rho)(1-s)}{p\Lambda\left[\tilde{G}(s)\right]-s-\overline{p}} \frac{p\left\{1-\frac{1}{1-v(\overline{p})}U(s)[V(s)-v(\overline{p})]\right\}}{\beta(1-s)}\frac{1-\Lambda\left[\tilde{G}(s)\right]}{\lambda\left(1-\tilde{G}(s)\right)} \quad (8)$$

Corollary 2 When $\rho<1$, W can be decomposed into the sum of two independent random variables:

$$W=W_0+W_d$$

where W_0 is waiting time of the $Geom^X/G/1$ queue with Bernoulli feedback and no vacation, W_d is the additional waiting time, the PDFs of them are listed as

$$W_0(s)=\frac{(1-\rho)(1-s)}{p\Lambda\left[\tilde{G}(s)\right]-s-\overline{p}}\frac{1-\Lambda\left[\tilde{G}(s)\right]}{\lambda\left(1-\tilde{G}(s)\right)}$$

$$W_d(s)=\frac{p\left\{1-\frac{1}{1-v(\overline{p})}U(s)[V(s)-v(\overline{p})]\right\}}{\beta(1-s)}$$

Proof We can have the stochastic decomposition by the equation（8）and mean of the additional waiting time

$$E(W_d)=\frac{p}{1-v(\overline{p})}\frac{(1-v(\overline{p}))E\left[U(U-1)\right]+E\left[V(V-1)\right]+2E[U]E[V]}{2\beta}$$

Comparison theorem 1 with theorem 2, we have

$$E(\Pi_d)=p\lambda E(W_d)$$

which he testes Little formula established.

IV. Analysis of Busy Period

As for the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times, we have known that there are two kinds of busy period caused by a single customer and customers of a batch, respectively. Let θ, Θ stand for the busy period caused by a customer and a batch of custom-mers, and θ_i, $\Theta_i, i=1,2,\cdots$, denotes the busy period caused by the ith customer and the ith batch.

The probability, PGFs and means of the number of the customers arrived, denoted Q_b, in the setup time and vacation time are respectively

$$p(Q_b=j)=\sum_{i=1}^{j}C_i^{(v)}C_{j-i}^{(u)}, j=0,1,2,\cdots$$

$$Q_b(z)=\sum_{j=1}^{\infty}p(Q_b=j)z^j =\frac{1}{1-v(\overline{p})}U(\overline{p}+p\Lambda(z))[V(\overline{p}+p\Lambda(z))-v(\overline{p})]$$

$$E(Q_b)=p\lambda E(U)+\frac{p\lambda}{1-v(\overline{p})}E(V)$$

Q_b causes the busy period $\theta_v=\sum_{j=1}^{Q_b}\theta_i$ and its PGF $\theta_v(z)=Q_b(\theta(z))$

The batch customers cause the busy period $\Theta=\sum_{i=1}^{\Lambda}\theta_i$, the corresponding PGF $\Theta(z)=\Lambda(\theta(z))$

Because

$$\theta=A+\Theta_1+\cdots+\Theta_{N(A)}$$

$$\Theta=U_{\xi}+\theta_1+\cdots+\theta_{N(U_{\xi})}$$

where $U_{\xi}=\sum_{i=1}^{\xi}A_i$, so the PGF of busy period caused by a customer is listed as

$$\begin{aligned}\theta(z)&=E\left(z^{\theta}\right)=E\left(z^{A+\Theta_1+\cdots+\Theta_{N(A)}}\right)\\&=\sum_{n=1}^{\infty}z^nE\left(z^{\Theta_1+\cdots+\Theta_{N(A)}}\right)p(A=n)\\&=\sum_{n=1}^{\infty}z^n\sum_{j=1}^{n}\left(\Theta(z)\right)^jC_n^jp^j\overline{p}^{n-j}p(A=n)\\&=\sum_{n=1}^{\infty}\left[z-zp+zp\Theta(z)\right]^np(A=n)\\&=\tilde{G}\left(z-zp+zp\Theta(z)\right)\\&=\tilde{G}\left(z-zp\left(1-\Lambda\left(\theta(z)\right)\right)\right)\end{aligned}$$

Thus we know

$$\theta_v(z)=Q_b\left(\tilde{G}\left(z-zp\left(1-\Lambda\left(\theta(z)\right)\right)\right)\right),$$

$$E(\theta_v)=E(Q_b)E(\theta)=\frac{g}{\alpha}\frac{E(Q_b)}{1-\rho}.$$

V. Special Models

Case 1 The $Geom^X/G/1$ queue with multiple vacation and set-up times

In the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times, when $\alpha=1$, the model changes into the $Geom^X/G/1$ queue with multiple vacation and set-up times, the probability generating function of the steady-state queue length and the stationary batch waiting times:

$$\Pi(z)=\frac{(1-\rho)\left(1-\Lambda(z)\right)\tilde{G}\left(\bar{p}+p\Lambda(z)\right)}{\tilde{G}\left(\bar{p}+p\Lambda(z)\right)-z}\frac{1-\frac{1}{1-v(\bar{p})}U(\bar{p}+p\Lambda(z))[V(\bar{p}+p\Lambda(z))-v(\bar{p})]}{\beta\left(1-\Lambda(z)\right)}$$

$$W(s)=\frac{(1-\rho)(1-s)}{p\Lambda\left[\tilde{G}(s)\right]-s-\bar{p}}\frac{p\left\{1-\frac{1}{1-v(\bar{p})}U(s)[V(s)-v(\bar{p})]\right\}}{\beta(1-s)}\frac{1-\Lambda\left[\tilde{G}(s)\right]}{\lambda\left(1-\tilde{G}(s)\right)}$$

where

$$\tilde{G}\left(\bar{p}+p\Lambda(z)\right)=G\left(\bar{p}+p\Lambda(z)\right),\ \Lambda\left[\tilde{G}(s)\right]=\Lambda\left[G(s)\right].$$

The results are consistent with paper[6].

Case 2 The Bernoulli feedback on $Geom/G/1$ queue with multiple vacation when $\Lambda(z)=z$ and no setup time, the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times turns into the Bernoulli feedback on $Geom/G/1$ queue with multiple vacation, the probability generating function of the steady-state queue length :

$$\Pi(z)=\frac{(1-\rho)(1-z)\tilde{G}(\bar{p}+pz)}{\tilde{G}(\bar{p}+pz)-z}\frac{1-\frac{1}{1-v(\bar{p})}[V(\bar{p}+pz)-v(\bar{p})]}{\beta(1-z)}$$

The results are consistent with paper[7]. The probability generating function of the stationary waiting times:

$$W(s)=\frac{(1-\rho)(1-s)}{p\tilde{G}(s)-s-\bar{p}}\frac{p\left\{1-\frac{1}{1-v(\bar{p})}[V(s)-v(\bar{p})]\right\}}{\lambda\beta(1-s)}$$

where

$$\tilde{G}(s)=\frac{\alpha G(s)}{1-(1-\alpha)G(s)},\beta=\frac{p}{1-v(\bar{p})}E(V).$$

Case 3 The Bernoulli feedback on $Geom/G/1$ queue with Multiple vacation and set-up times. When $\Lambda(z)=z$, the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times turns into the Bernoulli feedback on $Geom/G/1$ queue with multiple vacation and set-up times. the probability generating function of the steady-state queue length and the stationary waiting times:

$$\Pi(z)=\frac{(1-\rho)\left(1-\Lambda(z)\right)\tilde{G}(\bar{p}+pz)}{\tilde{G}(\bar{p}+pz)-z}\frac{1-\frac{1}{1-v(\bar{p})}U(\bar{p}+pz)[V(\bar{p}+pz)-v(\bar{p})]}{\beta(1-z)}$$

$$W(s)=\frac{(1-\rho)(1-s)}{p\tilde{G}(s)-s-\bar{p}}\frac{p\left\{1-\frac{1}{1-v(\bar{p})}U(s)[V(s)-v(\bar{p})]\right\}}{\lambda\beta(1-s)}$$

where

$$\tilde{G}(\bar{p}+pz)=\frac{\alpha G(\bar{p}+pz)}{1-(1-\alpha)G(\bar{p}+pz)}$$

$$\tilde{G}(s)=\frac{\alpha G(s)}{1-(1-\alpha)G(s)}.$$

VI. Numerical Results

In this section, we present some numerical examples to study the effect of the varying parameters on the main performance characteristics of our system. The values of the parameters are chosen so as to satisfy $\rho=\frac{p\lambda g}{\alpha}<1$. And through the above analysis, we obtain the expected queue length and the expe cted busy period in the system. So, in the follow, we present numerical examples in some situations by the expected queue length and the expected busy period to explain that our model is reasonable. We assume the total service number of a customer ξ obeys the geometric distribution of the parameter α without loss of generality, where α change from 0 to 1.Thus, in this system, according the customers arrive in a batch, the average service time, set-up time, vacation time to analyze each performance indicators of the system. Figs. 1-4 illustrate some trends of the system indices in the Bernoulli feedback on $Geom^X/G/1$ queue with multiple vacation and set-up times.

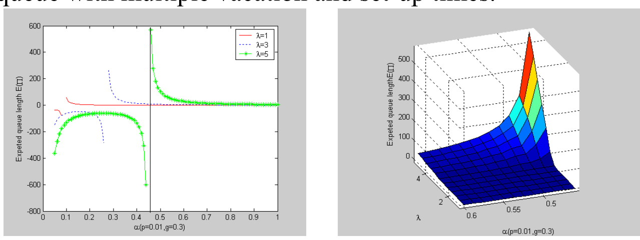

Fig.1 $E(\prod)$ versus α and λ

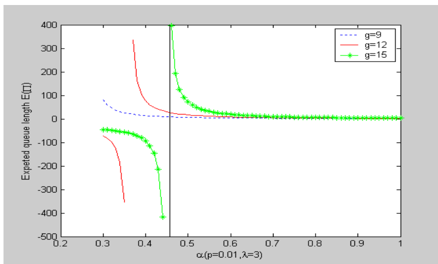

Fig.2 $E(\prod)$ versus α and λ

In Fig. 1, we show the relationship of the parameters α, λ, and $E(\prod)$ through the 2d and 3d images, and pay attention to the curves of the stationary queue length $E(\prod)$ with the change of the probability α of leaving the system forever. Evidently, the queue length decreases with the increase of, and the more customers arrive in batches, the larger the queue length $E(\prod)$ is. Meanwhile, when the para-meter α tends to 1, customers in the queue will have no waiting. Similarly, in Figs. 2 shows the trends observed for the expected queue length $E(\prod)$ under three systems with the average service time $g=9$, $g=12$, $g=5$, respectively. That is, $E(\prod)$ increases as g increases.

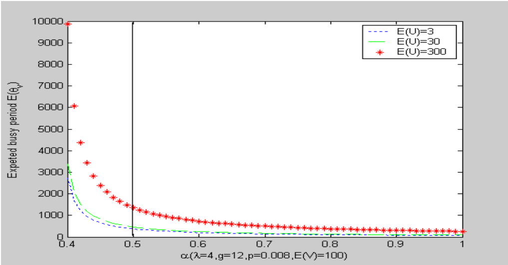

Fig.3 $E\left(\theta_v\right)$ versus α and $E\left(U\right)$

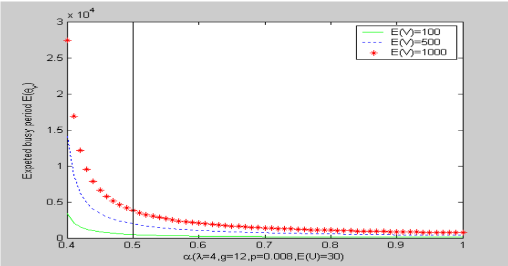

Fig.4 $E\left(\theta_v\right)$ versus α and $E\left(V\right)$

In Fig. 3 and 4, the trend of the expected busy period $E\left(\theta_v\right)$ of the number of the customers in the setup time and vacation time is presented with the changes of two parameters, the average setup time $E\left(U\right)$ and the average vacation time $E\left(V\right)$. Certainly, the expected busy period will increase with the increase of $E\left(U\right)$ or $E\left(V\right)$. But from the figure, we find that, the expected busy period will still decrease with the increase of the parameter α.

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